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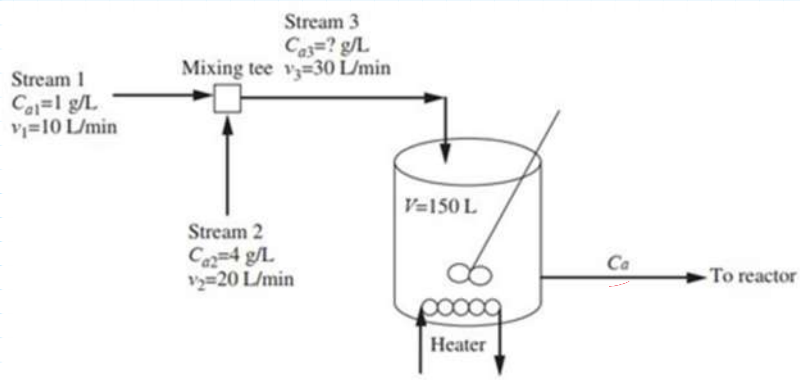
Chapter Two

Modeling Tools for Process Dynamics

Understanding process dynamics (how process variables change with time) will be very important to our studies of process control. In the examples in Chap.1, we saw some of the implications of process dynamics and their relationship to process control. In this chapter we explore process dynamics further and review some mathematical tools for solving the resulting process models.

Process Dynamics: A Chemical Mixing Scenario

Example 1: Consider the following chemical mixing example (Fig. 2–1). Two process streams are mixed to produce one of the feeds for our chemical reactor. After mixing, the blended stream is fed to a heating vessel before being sent to the reactor. The process is running along at a steady state. The concentration of *A* in stream 1 is 1 g/L and in stream 2 is 4 g/L. At 3:00 P.M., the shift changes at the plant. The new operator on our unit misreads the flowmeters for the process and switches the flow rates of the two streams. Stream 1 is switched to 20 L/min, and stream 2 is switched to 10 L/min. At 3:30 P.M., the shift supervisor hurries to the control room to determine the source of the problem now being experiencing with the reactor. Use your knowledge of chemical engineering to determine what has happened to the exit concentration from the heating vessel over the first half-hour of the shift.



Solution:

- Before disturbance on flow (i.e. change of flow rates of stream 1+2)

Mass balance on component A around mixing tee at steady state

Mass In = Mass out

$$v_1 C_{a_1} + v_2 C_{a_2} = v_3 C_{a_3}$$

$$(10)(1) + (20)(4) = v_3 C_{a_3} \quad \text{—— (1)}$$

checking the unit

$$v \quad C_a$$

$$\frac{L}{min} * \frac{g}{L} = \frac{g}{min} \quad \checkmark \text{ ok}$$

since $v_3 = v_1 + v_2 = 10 + 20 = 30 \ \frac{L}{min}$ sub. in to Eq (1)

$$10 + 80 = 30 \ C_{a3}$$

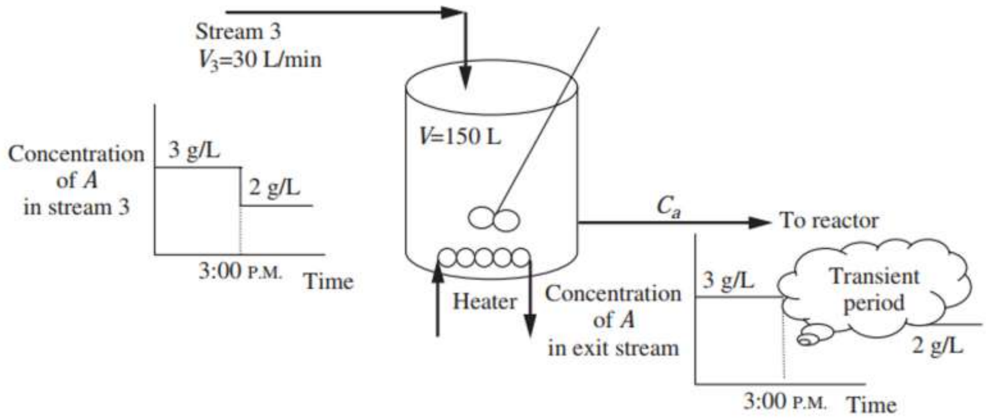
$$90 = 30 \ C_{a3} \Rightarrow \boxed{C_{a3} = 3 \ \frac{g}{L}}$$

- After disturbance on flow (i.e. change of flow rates of stream 1+2)

Mass In = Mass out

$$v_1 C_{a_1} + v_2 C_{a_2} = v_3 C_{a_3}$$

$$(20)(1) + (10)(4) = 30 \ C_{a3} \Rightarrow \boxed{C_{a3} = 2 \ \frac{g}{L}}$$



Mass balance on component A around heating vessel at unsteady state

In − out = Acc

$$v_3 C_{a3} - v C_a = \frac{d}{dt} V C_a \quad \text{——} ②$$

checking unit

$$v C_a = \frac{L}{min} \times \frac{g}{L} = \frac{g}{min}$$

$$\frac{d}{dt} V C_a = \frac{1}{min} L \times \frac{g}{L} = \frac{g}{min}$$

Since $v = v_3$ sub. into Eq. ②

$$v_3 C_{a3} - v_3 C_a = V \frac{dC_a}{dt}$$

$$v_3 (C_{a3} - C_a) = V \frac{dC_a}{dt}$$

$$\left[V \frac{dC_a}{dt} = v_3 (C_{a3} - C_a) \right] \div v_3$$

$$\frac{V}{v_3} \frac{dC_a}{dt} = (C_{a3} - C_a) \quad \text{——} ③$$

Since $\frac{V}{v_3} = \tau$ = Residence time of heating vessel $\quad \frac{V}{v_3} = \frac{L}{\frac{L}{min}} = min$

$$\tau \frac{dC_a}{dt} = (C_{a3} - C_a) \quad \text{——} ④$$

Since $\tau = \frac{V}{v_3} = \frac{\cancel{150}^{5}}{\cancel{30}} = 5$ min sub. into Eq. ④

$$5 \frac{dC_a}{dt} = (C_{a3} - C_a)$$, by separation of variable

$$\frac{dC_a}{C_{a3} - C_a} = \frac{1}{5} dt$$

Since $C_{a3} = 2 \frac{g}{L}$

Hence

$$\frac{dC_a}{2 - C_a} = \frac{1}{5} dt$$, by intergaration both sides

$$\int \frac{dC_a}{2-C_a} = \int \frac{1}{5}\, dt \qquad -C_a \Rightarrow -dC_A$$

$$-\ln(2-C_a) = \frac{1}{5}t + C \quad \text{——— } (5)$$

By using initial boundary condition

at $t=0$ $\quad C_a = 3\ \frac{g}{L}$ $\quad$ sub. into Eq. (5)

$$-\ln(2-3) = \frac{1}{5}(0) + C$$

$$-\ln(-1) = C \Rightarrow C = -\ln(-1) \quad \text{sub. into Eq. (5)}$$

$$-\ln(2-C_a) = \frac{1}{5}t - \ln(-1)$$

$$\left(-\ln(2-C_a) + \ln(-1) = \frac{1}{5}t\right) \times -1$$

$$\ln(2-C_a) - \ln(-1) = -\frac{t}{5}$$

since $\ln\frac{a}{b} = \ln a - \ln b$

$$\ln\left(\frac{2-C_a}{-1}\right) = -\frac{t}{5} \quad \text{——— } (6)$$

By taking exponential for both sides (e^x)

$$e^{\ln\left(\frac{2-C_a}{-1}\right)} = e^{-\frac{t}{5}}$$

since $e^{\ln x} = x$

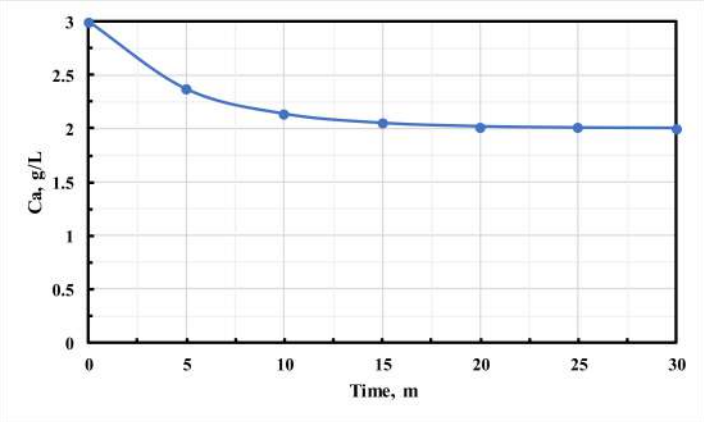
$$\frac{2-C_a}{-1} = e^{-\frac{t}{5}}$$

$$2 - C_a = -e^{-\frac{t}{5}}$$

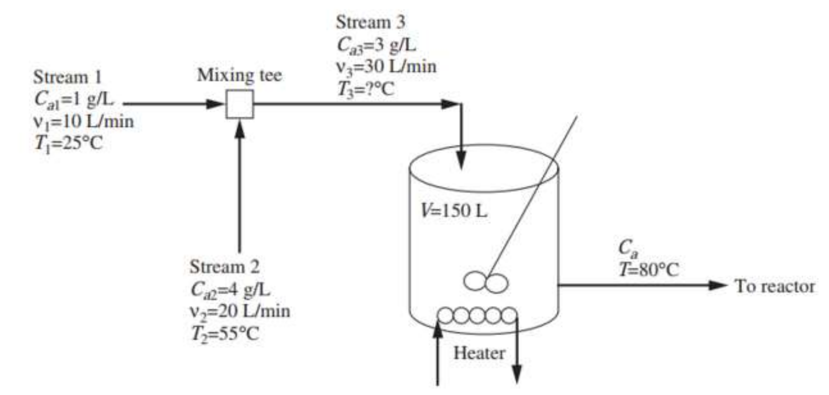
$e^0 = 1$

$$2 + e^{-\frac{t}{5}} = C_a \Rightarrow C_a = 2 + e^{-\frac{t}{5}}$$

t (min)	C_a ($\frac{g}{L}$)
0	3
1	2.8
2	2.67
⋮	
30	2.0



Example 2: Consider the energy balance for the mixing process described above. Prior to 3 P.M. the process conditions are depicted as in Fig. 2–4. Stream 1 (at 25 °C) mixes with stream 2 (at 55 °C), producing stream 3, the feed to the heating vessel. The heater adds energy to the vessel to bring the outlet stream to 80 °C.



$q = mC_p\Delta T$

Solution:-

Energy balance on mixing tee at steady state (before change, before 3:00 P.m)

Energy In = Energy out

$$\rho v_1 C_p (T_1 - T_{ref.}) + \rho v_2 (T_2 - T_{ref}) = \rho v_3 C_p (T_3 - T_{ref}) \quad \text{—— (1)}$$

Since $\left.\begin{array}{l} \rho = 1000 \dfrac{g}{L} \\ C_p = 1 \dfrac{cal}{g \cdot C^o} \end{array}\right\}$ are constant

Hence

$$[\rho v_1 C_p (T_1 - T_{ref.}) + \rho v_2 (T_2 - T_{ref}) = \rho v_3 C_p (T_3 - T_{ref})] \div \rho C_p$$

$$v_1 (T_1 - T_{ref}) + v_2 (T_2 - T_{ref.}) = v_3 (T_3 - T_{ref.})$$

$$v_1 T_1 - v_1 T_{ref.} + v_2 T_2 - v_2 T_{ref.} = v_3 T_3 - v_3 T_{ref.}$$

$$v_1 T_1 + v_2 T_2 - T_{ref.}(v_1 + v_2) = v_3 T_3 - v_3 T_{ref.} \quad \text{—— (2)}$$

Since $v_3 = v_1 + v_2$

Hence, Eq. (2) becomes

$$v_1 T_1 + v_2 T_2 - \cancel{v_3 T_{ref.}} = v_3 T_3 - \cancel{v_3 T_{ref.}}$$

$$v_1 T_1 + v_2 T_2 = v_3 T_3$$

$$(10)(25) + 20(55) = 30(T_3) \Rightarrow \boxed{T_3 = 45\ ^\circ C}$$

Energy balance on mixing tee at steady state (after change in flow rate)

Energy In = Energy out

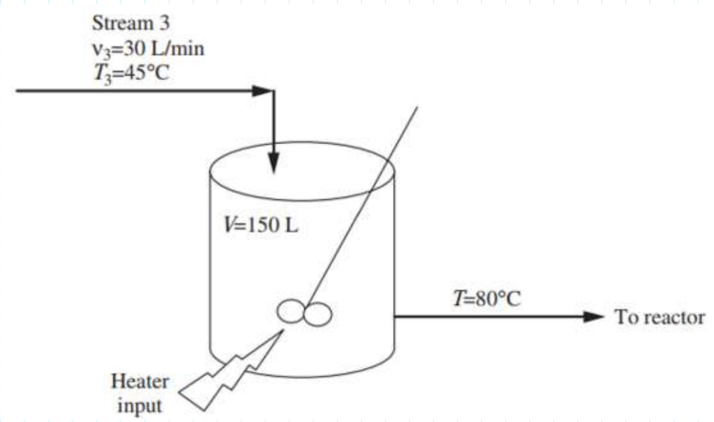
$$\rho v_1 C_p (T_1 - T_{ref.}) + \rho v_2 (T_2 - T_{ref}) = \rho v_3 C_p (T_3 - T_{ref}) \quad \text{—} \; 1$$

we can use directly the following equation

$$v_1 T_1 + v_2 T_2 = v_3 T_3$$

$$(20)(25) + (10)(55) = 30 (T_3) \Rightarrow \boxed{T_3 = 35 \; ^\circ C}$$

Energy balance at steady state on heating vessel



In = out

$$\rho v_3 C_p (T_3 - T_{ref}) + Q = \rho v C_p (T - T_{ref}) \quad \text{—} \; 3$$

since $v_3 = v$

Therefore, the above equation becomes

$$\underline{\rho v_3 C_p} (T_3 - T_{ref}) + Q = \underline{\rho v_3 C_p} (T - T_{ref})$$

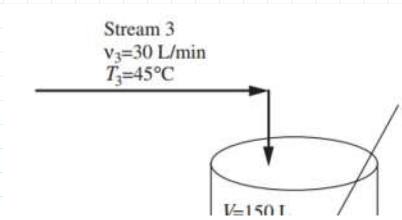
$$\rho v_3 C_p T_3 - \cancel{\rho v_3 C_p T_{ref.}} + Q = \rho v_3 C_p T - \cancel{\rho v_3 C_p T_{ref.}}$$

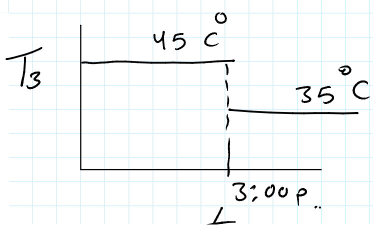
$\therefore$

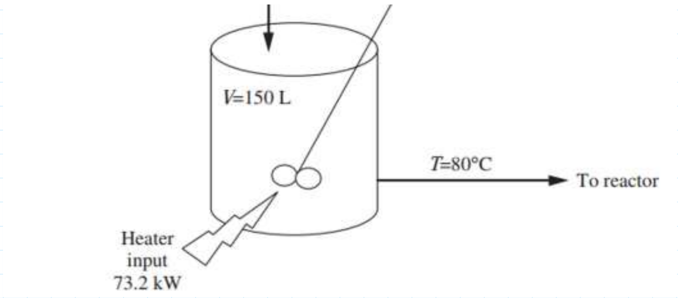
$$Q = \rho v_3 C_p T - \rho v_3 C_p T_3$$

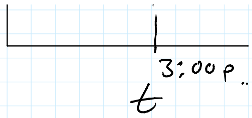
$$Q = \rho v_3 C_p (T - T_3)$$

$$Q = (1000)(30)(1)(80 - 45) \Rightarrow Q = 1.05 \times 10^6 \; \frac{cal}{min} = 73.2 \; kW$$









Energy balance at unsteady state around heating vessel

$$In - out = Acc$$

$$\rho V_3 C_p (T_3 - T_{ref.}) + Q - \rho V_3 C_p (T - T_{ref.}) = \frac{d}{dt} \rho V C_p (T - T_{ref})$$

$$\left[\rho V_3 C_p (T_3 - T_{ref.}) + Q - \rho V_3 C_p (T - T_{ref}) = \rho V C_p \frac{dT}{dt}\right] \div \rho C_p$$

$$\left[V_3 (T_3 - T_{ref}) + \frac{Q}{\rho C_p} - V_3 (T - T_{ref}) = V \frac{dT}{dt}\right] \div V_3$$

$$T_3 - \cancel{T_{ref.}} + \frac{Q}{\rho V_3 C_p} - T + \cancel{T_{ref.}} = \frac{V}{V_3} \frac{dT}{dt}$$

since $\frac{V}{V_3} = \tau = \frac{150}{30} = 5$ min

$$T_3 - T + \frac{Q}{\rho V_3 C_p} = \tau \frac{dT}{dt}$$

$$\tau \frac{dT}{dt} = T_3 - T + \frac{Q}{\rho V_3 C_p}$$

$$5 \frac{dT}{dt} = 35 - T + \frac{1.05 \times 10^6}{(1000)(30)(1)}$$

$$5 \frac{dT}{dt} = 35 - T + 35$$

$5 \frac{dT}{dt} = 70 - T$, by separation of variables

$\frac{dT}{70 - T} = \frac{1}{5} dt$, by integratio

$$-\ln(70 - T) = \frac{1}{5} t + C \qquad \text{—— (4)}$$

By using initial boundary conditi.

at $t = 0$ $T = 80$ °C sub. into Eq. (4)

$$-\ln(70-80)=\frac{1}{5}(0)+c \Rightarrow c=-\ln(-10) \text{ sub. into Eq. } (4)$$

$$-\ln(70-T)=\frac{1}{5}t-\ln(-10)$$

$$-\ln(70-T)+\ln(-10)=\frac{1}{5}t$$

$$\left(-[\ln(70-T)-\ln(-10)]=\frac{1}{5}t\right)\times -1$$

$$\ln(70-T)-\ln(-10)=-\frac{t}{5}$$

$$\ln\left(\frac{70-T}{-10}\right)=-\frac{t}{5}$$

By taking e^x for both sides

$$\frac{70-T}{-10}=e^{-\frac{t}{5}}$$

$$70-T=-10\,e^{-\frac{t}{5}}$$

$$T=70+10\,e^{-\frac{t}{5}} \Rightarrow T=T(t)$$

t (min)	T, °C
0	80
1	78
2	76.7
30	80
40	80
⋮	✓