Mass Transfer

Fall 2024

Lecture #3

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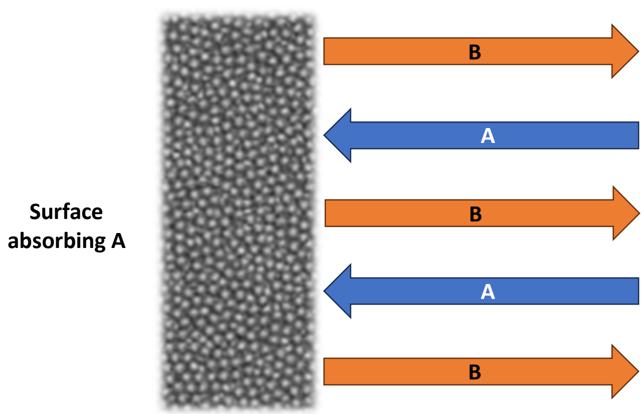
Molecular Diffusion

a) ii- Non-Equimolar- counter diffusion (NEMD)

Mass transfer rates of the two components are not equal but in opposite directions.



حالات

- تفاعلات كيميائية
- تُذكر في السؤال

Molecular Diffusion

a) ii- Non-Equimolar- counter diffusion (NEMD)

Example 5: Drive the rate of diffusion of reaction of producing carbon dioxide from oxygen and charcoal particle.

Solution $O_2 + 2Co \rightarrow 2Co_2$

(A) (B)

$2N_A = -N_B$ sub in (3)

$$N_A = \frac{P_A}{P_t}(N_A + N_B) - \frac{D_{AB}}{R.T} \cdot \frac{dP_A}{dZ} \quad \text{----(3)}$$

$$N_A = \frac{P_A}{P_t}(N_A + (-2N_A)) - \frac{D_{AB}}{R.T} \cdot \frac{dP_A}{dZ}$$

$$N_A = \frac{P_A}{P_t}(-N_A) - \frac{D_{AB}}{R.T} \cdot \frac{dP_A}{dZ}$$

$$N_A - \frac{P_A}{P_t}(-N_A) = -\frac{D_{AB}}{R.T} \cdot \frac{dP_A}{dZ}$$

$$N_A\left(1 + \frac{P_A}{P_t}\right) = -\frac{D_{AB}}{R.T} \cdot \frac{dP_A}{dZ}$$

$$N_A . dZ = -\frac{D_{AB}}{R.T} \cdot \frac{dP_A}{\left(1+\frac{P_A}{P_t}\right)}$$

$$N_A . dZ = -\frac{D_{AB} . P_t}{R.T} \cdot \frac{dP_A}{(P_t+P_A)} \qquad \text{By integration}$$

$$N_A . \int_{Z_1}^{Z_2} dZ = -\frac{D_{AB} . P_t}{R.T} . \int_{P_{A1}}^{P_{A2}} \frac{dP_A}{(P_t+P_A)}$$

$$N_A . (Z_2 - Z_1) = -\frac{D_{AB} . P_t}{R.T} . \ln\frac{(P_t+P_{A2})}{(P_t+P_{A1})}$$

$$N_A = -\frac{D_{AB} . P_t}{R.T} . \frac{\ln\frac{(P_t+P_{A2})}{(P_t+P_{A1})}}{Z_2 - Z_1}$$

لكل حالة يتم ايجاد معادلة من خلال التكامل

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Molecular Diffusion

b) Diffusion of A in stagnant B, or unimolecular diffusion ($N_B = 0$)

Species A diffuses in stagnant non-diffusing B.

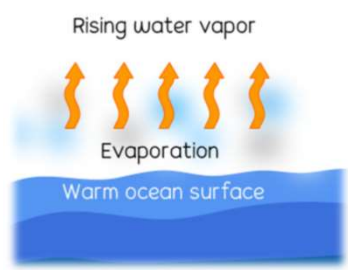
Examples of Counter Diffusion:

- Liquid evaporated to air
- Gas in air being absorbed into water (absorption column).

حالات

انتشار مادة في وسط لاينتشر

- برج امتصاص
- تبخيرسائل
- تُذكر في السؤال



Deriving the Case of A diffuses in stagnant non-diffusing B ($N_B = 0$).

$$N_A = \frac{P_A}{P_t}(N_A + N_B) - \frac{D_{AB}}{R.T} \cdot \frac{dP_A}{dZ} \quad ; \quad N_B = 0$$

$$N_A = \frac{P_A}{P_t}(N_A + 0) - \frac{D_{AB}}{R.T} \cdot \frac{dP_A}{dZ} \quad \Rightarrow \quad N_A\left(1 - \frac{P_A}{P_t}\right) = -\frac{D_{AB}}{R.T} \cdot \frac{dP_A}{dZ}$$

$$N_A\left(\frac{P_t - P_A}{P_t}\right) = -\frac{D_{AB}}{R.T} \cdot \frac{dP_A}{dZ} \quad \Rightarrow \quad N_A \int_{Z_1}^{Z_2} dZ = -\frac{D_{AB} . P_t}{R.T} . \int_{P_{A1}}^{P_{A2}} \frac{dP_A}{(P_t - P_A)}$$

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Molecular Diffusion

b) Diffusion of A in stagnant B. $(N_B = 0)$

$$N_A \int_{Z_1}^{Z_2} dZ = -\frac{D_{AB}\,.\,P_t}{R\,.T}\,.\int_{P_{A1}}^{P_{A2}} \frac{dP_A}{(P_t - P_A)} \quad \Rightarrow \quad N_A(Z_2 - Z_1) = -\frac{D_{AB}\,.\,P_t}{R\,.T}\,.\left(-\ln\frac{(P_t - P_{A2})}{(P_t - P_{A1})}\right)$$

$$N_A = \frac{D_{AB}\,.\,P_t}{R\,.T}\,.\frac{\ln\frac{(P_t - P_{A2})}{(P_t - P_{A1})}}{(Z_2 - Z_1)} \quad ; \quad P_t - P_{A1} = P_{B1} \quad ; \quad P_t - P_{A2} = P_{B2}$$

$$N_A = \frac{D_{AB}\,.\,P_t}{R\,.T}\,.\frac{\ln\frac{P_{B2}}{P_{B1}}}{(Z_2 - Z_1)} \quad ; Log\ mean\ partial\ pressure\ P_{BM} = \frac{P_{B2} - P_{B1}}{\ln\frac{P_{B2}}{P_{B1}}}$$

$$N_A = \frac{D_{AB}}{R\,.T}\left(\frac{P_t}{P_{BM}}\right).\frac{P_{B2} - P_{B1}}{(Z_2 - Z_1)} \quad \Rightarrow \quad N_A = -\frac{D_{AB}}{R\,.T}\left(\frac{P_t}{P_{BM}}\right).\frac{P_{A2} - P_{A1}}{(Z_2 - Z_1)}$$

Example 6 : Water in the bottom of a narrow metal tube is held at a constant temperature (293 K). The total pressure of dry air is 1.0132 * 10^5 Pa at T= 293 K. Water evaporates and diffuses through the air, the diffuses path is(0.15 m) long. **Calculate** the rate of evaporation at a steady state in $Kmol/\,m^2.sec$ Given that D_{AB} = $0.25 * 10^{-4}\ m^2/sec$ and water vapor pressure at (20 ℃) = 17.54mm Hg.

Solution

The water is diffused into the air but the air is not diffusing in the water, so $(N_B = 0)$

$$N_A = -\frac{D_{AB}}{R\,.T}\left(\frac{P_t}{P_{BM}}\right).\frac{P_{A2} - P_{A1}}{Z_2 - Z_1}$$

$P_{A1} = 17.54\ mm\ Hg$ = $17.54\ mm\ Hg * \frac{1\ atm}{760\ mm\ Hg}$ = 0.023 atm

$P_{A2} = 0\ atm$ (***Pure air and larg bulk volume***)

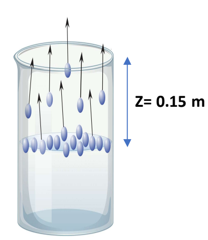
$P_{B1} = P_t - P_{A1} = 1 - 0.023 = 0.977$ atm

$P_{B2} = 1\ atm$

$$P_{BM} = \frac{P_{B2} - P_{B1}}{\ln\frac{P_{B2}}{P_{B1}}} \Rightarrow P_{BM} = \frac{1 - 0.977}{\ln\frac{1}{0.977}} = 0.988 atm$$

$$N_A = -\frac{0.25*10^{-4}}{0.082*293}\left(\frac{1}{0.988}\right).\frac{0 - 0.023}{0.15}$$

$$N_A = 1.64 * 10^{-7}\ Kmol/\,m^2.sec$$



Note: *in case* $(P_{B2} \approx P_{B1})$ *then* P_{BM} *can be equal the average of* P_{B2} & P_{B1}

Example 7: Ammonia (A) diffuses through a stagnant layer of air (B), 1cm thick, at 25 ℃ and 1 atm total pressure. The partial pressures of ammonia on the two sides of the air layer are: P_{A0} =0.9 atm and P_{A1} =0.1 atm respectively. Air is non-diffusing. **Calculate: a)** The molar flux of ammonia in $Kmol/\ m^2.sec$. **b)** Molar and mass average velocity for each species at Z = 0 . c) Plot the partial pressure distribution path.

Given that D_{AB} = $0.214 * 10^{-4}\ cm^2/sec$

Solution

a) For stagnant layer of air (B), $(N_B = 0)$

$$N_A = -\frac{D_{AB}\ .\ P_t}{R\ .T} \cdot \frac{\ln\frac{(P_t - P_{A1})}{(P_t - P_{A0})}}{(Z_1 - Z_0)}$$

$P_{A0} = 0.9\ atm$; $P_{A1} = 0.1\ atm$; $P_t = 1\ atm$

D_{AB} = $0.214 * 10^{-4}\ cm^2/sec$

$$N_A = -\frac{0.214 * 10^{-8}\left(\frac{m^2}{sec}\right).(1atm)}{0.082\ \left(\frac{m^3.atm}{Kmol.K}\right).298\ K} \cdot \frac{\ln\frac{(1-0.1)}{(1-0.9)}}{(0.01\ m - 0)}$$

$N_A = 1.92 * 10^{-8}\ Kmol/\ m^2.sec$

Example 7:

Solution

b) Molar and mass average velocity for each species

- Molar average velocity (U)

$$U = \frac{1}{C_t}\ (C_A\ .u_A + C_B\ .u_B) = \frac{1}{C_t}\ (N_A + N_B) = \frac{1}{C_t}\ (N_t)$$

for $(N_B = 0)$

$$U = \frac{1}{C_t}\ (N_A + 0) = \frac{N_A}{C_t} \quad \text{----------(*)}$$

$$C_t = \frac{P_t}{R.T} = \frac{1\ atm}{0.082\ \left(\frac{m^3.atm}{Kmol.K}\right).298\ K} = 0.041\ Kmol/m^3 \quad \text{sub in (*)}$$

$$U = \frac{1.92 * 10^{-8}\ Kmol/\ m^2.sec}{0.041\ Kmol/m^3} = 4.6 * 10^{-7} m/\ sec$$

C_A varies along diffusion path, so u_A varies as well

at z = 0 ; P_{A0} =0.9 atm $\Rightarrow y_{A0} = \frac{P_{A0}}{P_t} = \frac{0.9}{1} = 0.9$

$$U = \frac{1}{C_t}\ (C_{A0}\ .u_{A0} + C_{B0}\ .u_{B0}) \quad ;\ C_B\ .u_B = N_B = 0$$

$$U = \frac{1}{C_t}\ .(C_{A0}\ .u_{A0}) = \frac{C_{A0}}{C_t}\ .(u_{A0}) = y_{A0}.(u_{A0})$$

$$u_{A0} = \frac{U}{y_{A0}} = \frac{4.6 * 10^{-7} m/\ sec}{0.9} = 5.1 * 10^{-7} m/\ sec$$

$u_B = 0$ (B stationary)

Example 7:

Solution

b) Molar and mass average velocity for each species

- Mass average velocity (u)

$$u = \frac{1}{\rho_t}\left(\rho_A . u_A + \rho_B . u_B\right) \quad ; \quad \rho_B . u_B = 0$$

$$u = \frac{\rho_A . u_A}{\rho_t} \quad ; \quad \rho_i = \frac{P_i . Mwt_i}{R.T}$$

$$u = \frac{\frac{P_A . Mwt_A}{R.T} . u_A}{\frac{P_t . Mwt_t}{R.T}} = \frac{P_A . Mwt_A . u_A}{P_t . Mwt_t} = \frac{y_A . Mwt_A . u_A}{Mwt_t}$$

$$u = \frac{y_{A0} . Mwt_A . u_{A0}}{Mwt_t} \quad \text{------}(**)$$

At Z = 0

$$y_A = 0.9 \quad ; \quad y_B = 1 - y_A = 1 - 0.9 = 0.1$$

$$Mwt_t = y_A . Mwt_A + y_B . Mwt_B$$

$$Mwt_t = 0.9 * 17 + 0.1 * 29 = 18.2 \quad \text{sub in } (**)$$

$$u = \frac{0.9 * 17 * 5.1*10^{-7} m/sec}{18.2} = 4.3 * 10^{-7} m/sec$$

Example 7:

Solution

c) Pressure distribution Plot

$$N_A = 1.92 * 10^{-8}\ Kmol/m^2.sec = -\frac{D_{AB} . P_t}{R.T} . \frac{\ln\frac{(P_t - P_A)}{(P_t - P_{A0})}}{(Z - Z_0)}$$

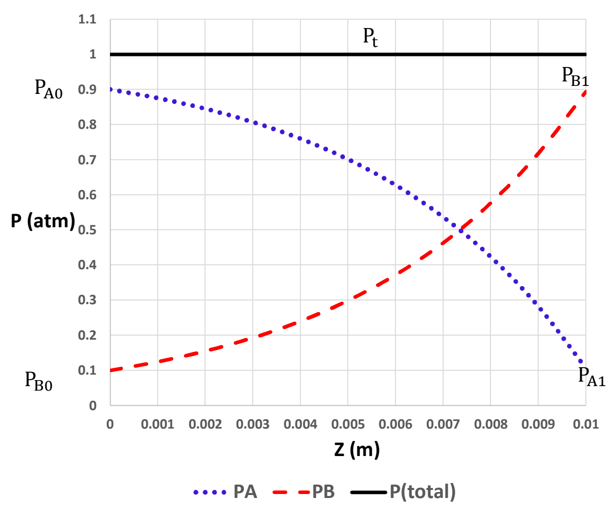
$$1.92 * 10^{-8} = -\frac{0.214*10^{-8}\left(\frac{m^2}{sec}\right).(1atm)}{0.082\left(\frac{m^3.atm}{Kmol.K}\right).298K} . \frac{\ln\frac{(1-P_A)}{(1-0.9)}}{(Z - 0)}$$

$$219\ Z = \ln\frac{(1-P_A)}{(0.1)} \quad \text{by exponential both sides}$$

$$exp^{(219\ Z)} = \frac{(1-P_A)}{(0.1)}$$

$$\boldsymbol{P_A} = 1 - (0.1)\ exp^{(219\ Z)}$$

$$\boldsymbol{P_B} = (P_t - P_A)$$



HW 3: Due date Saturday, October 26th

Q1 Oxygen (A) diffuses through a stagnant layer of nitrogen (B) at 20°C and 1 atm total pressure. The diffusion path is 1 cm thick. The partial pressure of oxygen on one side of the layer is 0.8 atm, and on the other side, it is 0.2 atm. Nitrogen does not diffuse. Given the diffusivity of oxygen in nitrogen is D_{AB} =0.18* $10^{-4}\ m^2/sec$, **calculate**:

1.The molar flux of oxygen.

2.The molar and mass average velocities for each species.

Q2 Ammonia (NH_3) diffuses through a stagnant layer of nitrogen (N_2) at 300 K and 1 atm total pressure. The thickness of the stagnant nitrogen layer is L=0.01 m. The partial pressure of ammonia at the surface (Z_0 = 0) is P_{A0} =0.8 atm, and at the far side (Z=L) it is P_{A1} =0.2 atm. The diffusion coefficient D_{AB} = 0.16* $10^{-4}\ m^2/sec$. **Find** the partial pressure of ammonia at a distance Z=0.005 m from the surface.