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المحاضرة الخامسة

Mathematical Representation of the Image

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Mathematical Representation of the Image

Digital images, such as those found on websites or captured with mobile phones, can be represented using matrices. Matrix theory plays a crucial role in various fields of modern engineering, including wireless communications, signal processing, control systems, and biomedical engineering. Mathematically, matrices serve as both a representation of data and a powerful tool for information processing, enabling the efficient and reliable resolution of numerous practical problems. Given their widespread use in engineering and their ability to provide innovative solutions to significant challenges, this lecture will explore key applications of matrix theory in image processing. Image processing involves the mathematical analysis of images, which can be viewed as two-dimensional arrays of data, where each element is called a pixel (picture element). For digital images, we will adopt the following notation:

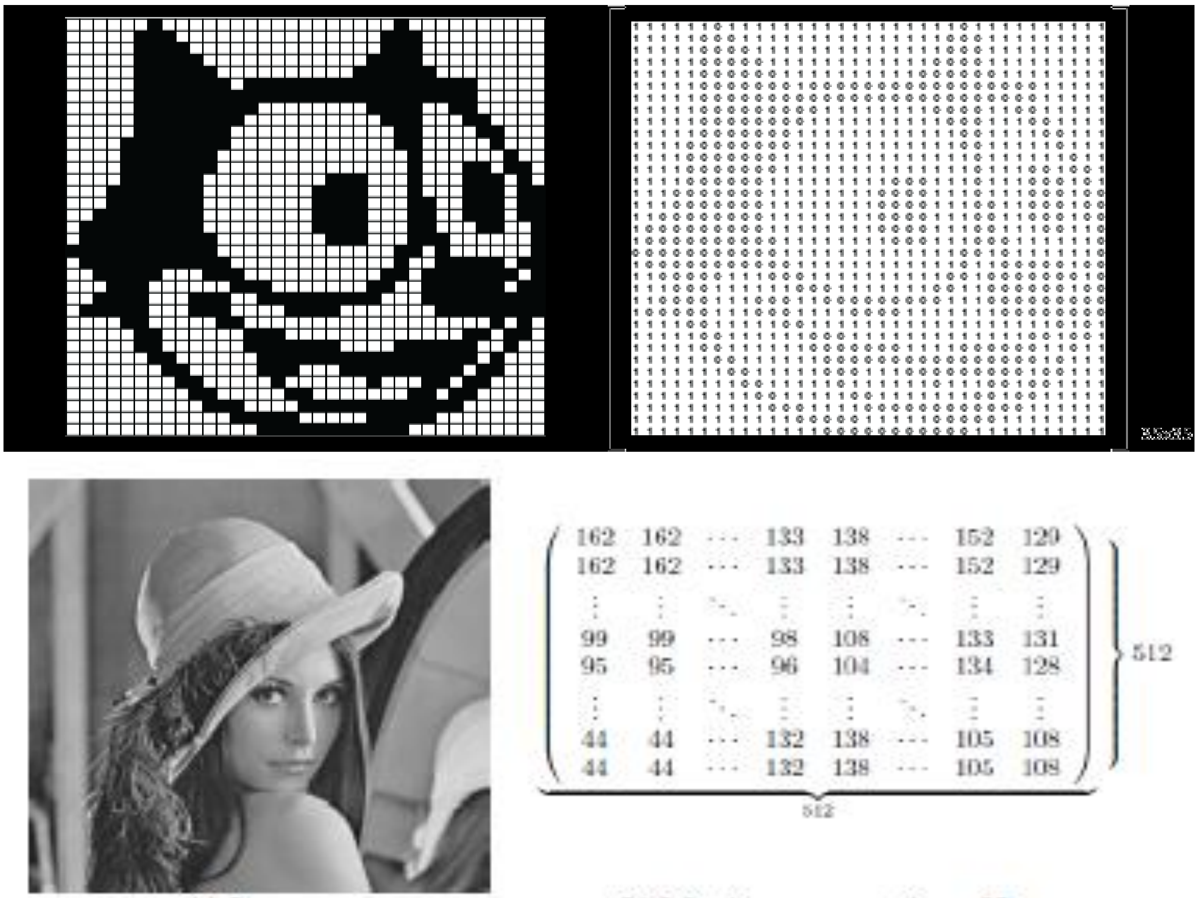
I(r, c) = The brightness of the image at the point (r,c)

Where r= row and c= column.

When we have the data in digital form, we can use the software to process the data. The digital image is 2D- array as:

$$\begin{pmatrix} I(0,0) & I(0,1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots I(0,N-1) \\ I(1,0) & I(1,1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots I(1,N-1) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ I(N-1,0) & I(N-1,1) \ldots\ldots\ldots\ldots\ldots\ldots I(N-1,N-1) \end{pmatrix}$$

For instance, the small image of Felix the Cat (on the left) can be represented by a 35 x 35 matrix where the elements are either 0 or 1. These values define the color of each pixel—the smallest graphical element in a matrix image, which can display only one color at a time. In this case, 0 represents black, while 1 represents white. Digital images that utilize only these two colors are referred to as binary or Boolean images.



(a) Lena (b) Matrix representation of Lena

Fig 5.1 Matrix Representation

In any image matrix, the image size is (N×N) [matrix dimension] then:

$N_g = 2^m$ ……….. (1)

Where N_g denotes the number of gray levels m is the no. of bits contains in the digital image matrix.

Example: If we have (6 bit) in a 128 × 128 image. Find the no. of gray levels to represent it, and then find the no. of bit in this image?

Solution:

N_{gray}= 2^6= 64 Gray Level
N_{bit}= = 128 * 128 * 6= 98304 bit =12KB.

5.1 Image Types

Digital images can broadly be classified under several categories:

1. Binary Images
2. Grayscale Images
3. Color Images.
4. Multispectral Images

1. Binary Image

Binary images are the most basic type of images, consisting of two values—typically black and white, or '0' and '1'. They are known as 1 bit/pixel images because each pixel is represented by a single binary digit. These images are commonly used in computer vision applications where the primary information needed is general shapes or outlines.

Examples:

1. To position a robotic gripper to grasp an object
2. To check a manufactured object for deformations
3. FAX
4. OCR (Optical character Recognition)

Binary images are often created from gray-scale images via a threshold value (it turned to white (1) if larger than threshold and black (0) if less).

Fig 5.2. Binary Image

2. Grayscale Images

Grayscale images, also known as monochrome or single-color images, contain only brightness information without any color data. A typical grayscale image has 8 bits per pixel, allowing for 256 different brightness levels (ranging from 0 to 255). This 8-bit representation is standard because a byte, which consists of 8 bits, is the fundamental unit of data in digital computing.

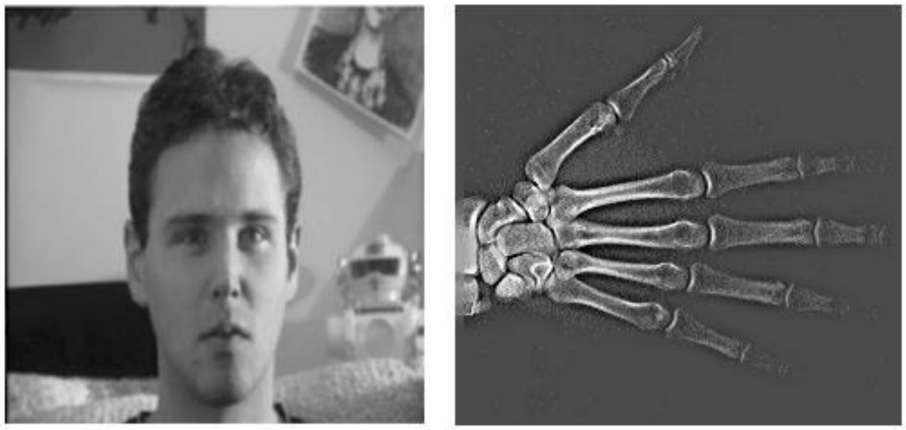


Fig 5.3 Gray Scales Image

3. Colour Images.

Colour image can be modelled as three band monochrome image data, where each band of the data corresponds to a different colour.

Fig 5.4 Colour Image

The actual information stored in the digital image data is brightness information in each spectral band. When the image is displayed, the corresponding brightness information is displayed on the screen by picture elements that emit light energy corresponding to that particular colour.

RGB Typical colour images are represented as Red, Green, and Blue images. Using the 8-bit monochrome standard as a model, the corresponding colour image would have 24 bit/pixel – 8 bit for each colour bands (red, green and blue and about 16 million colours). The following figure denotes the representation of a typical RGB colour image.

4. Multispectral Images

Multispectral images typically contain information outside the normal human perceptual range. This may include infrared, ultraviolet, X-ray, acoustic or radar data. Source of these types of image include satellite systems underwater sonar systems and medical diagnostics imaging systems.

Fig 5.5 Multispectral images

5.2 Thresholding

In many vision applications, it is useful to be able to separate out the regions of the image corresponding to objects in which we are interested, from the regions of the image that correspond to the background. Thresholding often provides an easy and convenient way to perform this segmentation on the basis of the different intensities or colors in the foreground and background regions of an image.

In addition, it is often useful to be able to see what areas of an image consist of pixels whose values lie within a specified range, or *band* of intensities (or colors). Thresholding can be used for this as well.

How It Works

Thresholding is the simplest method of image segmentation and the most common way to convert a grayscale image into a binary image. In thresholding, we select a threshold value and then all the gray level value which is below the selected threshold value is classified as 0 (black i.e background) and all the gray level which is equal to or greater than the threshold value are classified as 1 (white i.e foreground or the object).

The input to a thresholding operation is typically a grayscale or color image. In the simplest implementation, the output is a binary image representing the segmentation. Black pixels correspond to the background and white pixels correspond to the foreground (or vice versa). In simple implementations, the segmentation is determined by a single parameter known as the intensity threshold. In a single pass, each pixel in the image is compared with this threshold. If the pixel's intensity is higher than the threshold, the pixel is set to, say, white in the output. If it is less than the threshold, it is set to black.

Single value thresholding can be given as follow:

$$g(x,y)=\begin{cases}1 \ if \ \ f(x,y)>T \\ 0 \ if \ \ f(x,y)\le T\end{cases}$$

5.3 Digital Image File Format

Why do we need so many different types of image file format?

1. There are many different types of images and application with varying requirements.

2. The market share proprietary information, and a lack of coordination within the imaging industry.

Many image types can be converted to one of other type by easily available image conversion software.

Typically, images have two parts, one called header and the other called data. The header information contain:

1. The number of rows (height)

2. The number of columns (Width)

3. The number of bands.

4. The number of bit per pixel.

5. The file type

6. Additionally, with some of the more complex file formats, the header may contain information about the type of compression used and other necessary parameters to create the image, I(r,c).

Following is an abbreviation list of few image file formats:

Format	Extension
Microsoft Windows DIB	.bmp .dib .rle
Microsoft Palette	.pal
Autocad format 2D	.dxf
JPEG	.jpg
Windows Meta file	.wmf
Portable network graphic	.png
Compuserve gif	.gif
Apple Macintosh	.pict .pic .pct

5.4 Spatial Domain and Frequency Domain

Imagine a vector in two-dimensional space $(x,)$, having defined in standard basis (1,0) and (0,1) also generally known as X and Y axis. The same vector can also be described in other (orthogonal/orthonormal) basis like $(-1,0)$ and $(0,-1)$ so that the component values becomes $(-x,-y)$.

It is like the same object viewed from a different point of view, change of coordinate modifies the apparent values of the object's components but there's no information loss (it can be transformed back to original view). In digital Image processing, each image is either a 2D-matrix (as in case of gray-scale images) or a 3D vector of 2D matrices (as in case of RGB color images). These matrices are a measurement of intensity of gray-scale / redcomponent / green-component / blue-component etc.

1. Spatial Domain

A digital image is a grid of pixels. A pixel is the smallest element in an image. Each pixel corresponds to any one value called pixel intensity. Now the intensity of an image varies with the location of a pixel. Let I be an image and $(x,)$ is the location (or coordinate) of any pixel then the image is represented as a function of location: $(x,)$, where x and y are integers. Thus an image $(x,)$ is a matrix of pixels.

The term spatial refers to space. In an image, this space is a $2D$ plane ($x\,y$ - plane). So, the spatial domain refers to the image plane itself and methods in the spatial domain are based on directly modifying the value of the pixels.

Spatial domain processes are represented as $I\,1(x\,,y\,)=T\,[I\,(x\,,y\,)]$

Where $I\,1$ is the modified image and the value of a pixel with coordinates $(x\,,y\,)$ in $I\,1$ is the result of performing some operation T on the pixels in the neighborhood of $(x\,,y\,)$ in the original image I .

2. **Frequency Domain**

The frequency domain is in contrast with the spatial domain. Basically, the frequency domain represents the rate of change of spatial pixels and hence gives an advantage when the problem you are dealing with relates to the rate of change of pixels which is very important in image processing.

Simplified, it is to study the change in pixel values in the image. This change in frequency is a characteristic of change in geometry of the image (spatial distribution). For example, high frequency in the frequency domain.