جامعة المستقبل
AL MUSTAQBAL UNIVERSITY
كلية العلوم

2^{nd} class

2024- 2025

Numerical Analysis

Lecture 3

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الرياضيات المتقدمة: المرحلة الثانية

مادة التحليل العددي

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Contents

1 Simpsons 3/8 Method

The Simpsons 3/8 method is another numerical integration used to approximate the definite integral of a function. It is similar to Simpsons 1/3 Rule but uses cubic polynomials instead of quadratic polynomials to approximate the function over subintervals.

$$I = \int_a^b f(x)\,dx \approx \frac{3h}{8}\left[f(x_0) + 3\sum_{i\neq 3k} f(x_i) + 2\sum_{i=3k} f(x_i) + f(x_n)\right]$$

such that $a = x_0 < x_1 < x_2 < \ldots < x_n = b$.
$h = \frac{b-a}{n}$, and $x_i = x_{i-1} + h$ are the points at which the function is evaluated.

Examples of Simpsons 3/8 Rule

Example 1.1. Let us approximate the integral $\int_0^2 e^{x^2}\,dx$ using the Simpsons 3/8 Rule with $n = 5$ intervals.

Solution. The interval is $[0, 2]$, and we are dividing it into $n = 5$ subintervals.
Step 1: Calculate h

$$h = \frac{b-a}{n} = \frac{2-0}{5} = 0.4$$

Step 2: Determine the evaluation points x_i, since $n = 5 \Rightarrow x_0, x_1.x_2, x_3, x_4, x_5$.

$$x_0 = a = 0$$

$$x_1 = x_0 + h = 0 + 0.4 = 0.4$$

$$x_2 = x_1 + h = 0.4 + 0.4 = 0.8$$

$$x_3 = x_2 + h = 0.8 + 0.4 = 1.2$$

$$x_4 = x_3 + h = 1.2 + 0.4 = 1.6$$

$$x_5 = x_4 + h = 1.6 + 0.4 = 2 = b$$

Step 3: Evaluate the function at these points

$$f(x_0) = e^{x_0^2} = f(0) = e^{0^2} = 1$$

$$f(x_1) = e^{x_1^2} = f(0.4) = e^{(0.4)^2} = e^{0.16} \approx 1.1735$$

$$f(x_2) = e^{x_2^2} = f(0.8) = e^{(0.8)^2} = e^{0.64} \approx 1.8965$$

$$f(x_3) = e^{x_3^2} = f(1.2) = e^{(1.2)^2} = e^{1.44} \approx 4.2207$$

$$f(x_4) = e^{x_4^2} = f(1.6) = e^{(1.6)^2} = e^{2.56} \approx 12.9358$$

$$f(x_5) = e^{x_5^2} = f(2) = e^{(2)^2} = e^{4} \approx 54.5982$$

Step 4: Apply the Simpsons 3/8 Rule

$$\begin{aligned} I = \int_a^b f(x)\,dx &\approx \frac{3h}{8}\left[f(x_0) + 3\sum_{i\neq 3k} f(x_i) + 2\sum_{i=3k} f(x_i) + f(x_n)\right] \\ &\approx \frac{3h}{8}\left[f_0 + 3(f_1 + f_2 + f_4) + 2(f_3) + f_5\right] \\ &\approx \frac{3\times 0.4}{8}\left[1 + 3(1.1735 + 1.8965 + 12.9358) + 2(4.2207) + 54.5982\right] \\ &\approx \frac{1.2}{8}\left[1 + 48.0174 + 8.4414 + 54.5982\right] = 0.15\,[112.057] \end{aligned}$$

$$\therefore I \approx 16.8086$$

$\square$

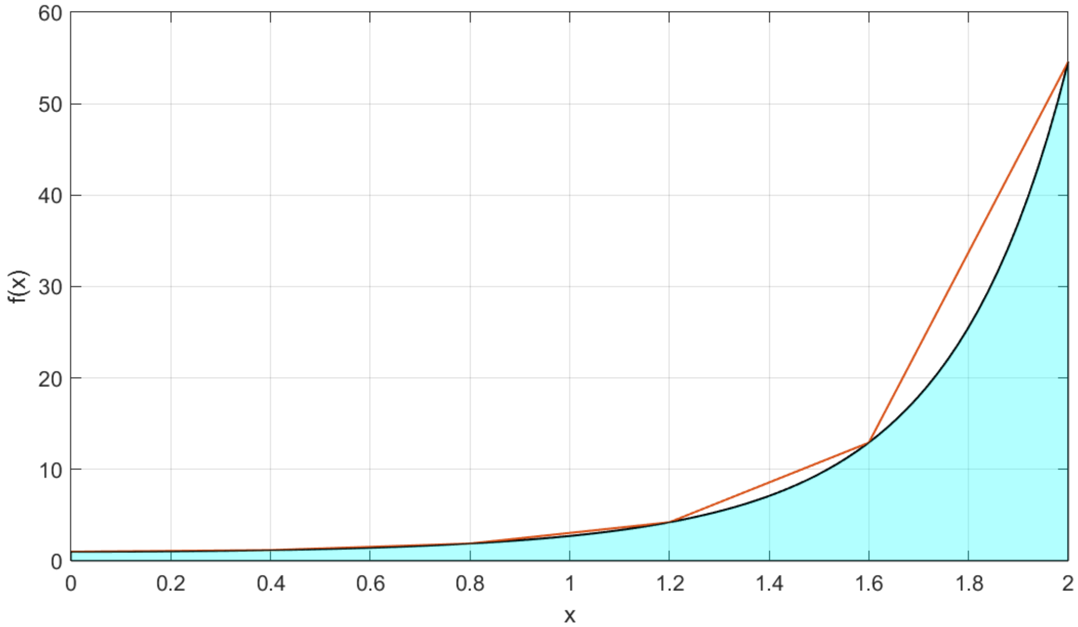

Figure 1: Simpsons 3/8 Method Approximation for $f(x) = e^{x^2}$

Example 1.2. Evaluate the integral $\ln(x)$ by Simpsons 3/8 rule dividing the interval $[1, 2]$ into four equal parts.

Solution. The interval is $[1, 2]$, and we are dividing it into $n = 4$ subintervals.

Step 1: Calculate h

$$h = \frac{b-a}{n} = \frac{2-1}{4} = 0.25$$

Step 2: Determine the evaluation points x_i, since $n = 4 \Rightarrow x_0, x_1.x_2, x_3, x_4$.

$$x_0 = a = 1, \quad x_1 = x_0 + h = 1 + 0.25 = 1.25$$

$$x_2 = x_1 + h = 1.25 + 0.25 = 1.5 \quad x_3 = x_2 + h = 1.5 + 0.25 = 1.75$$

$$x_4 = x_3 + h = 1.75 + 0.25 = 2 = b$$

Step 3: Evaluate the function at these points

$$f_0 = \ln(x_0) = f(1) = \ln(1) = 0, \quad f_1 = \ln(x_1) = f(1.25) = \ln(1.25) \approx 0.2231$$

$$f_2 = \ln(x_2) = f(1.5) = \ln(1.5) \approx 0.4055$$

$$f_3 = \ln(x_3) = f(1.75) = \ln(1.75) \approx 0.5596, f_4 = \ln(x_4) = f(2) = \ln(2) \approx 0.6931$$

Step 4: Apply the Simpsons 3/8 Rule

$$\begin{aligned} I = \int_a^b f(x)\,dx &\approx \frac{3h}{8}\left[f(x_0) + 3\sum_{i\neq 3k} f(x_i) + 2\sum_{i=3k} f(x_i) + f(x_n)\right] \\ &\approx \frac{3h}{8}\,[f_0 + 3(f_1 + f_2) + 2(f_3) + f_4] \\ &\approx \frac{3\times 0.25}{8}\,[0 + 3(0.2231 + 0.4055) + 2(0.5596) + 0.6931] \\ &\approx \frac{0.75}{8}\,[0 + 1.8858 + 1.1192 + 0.6931] = 0.3467 \end{aligned}$$

□

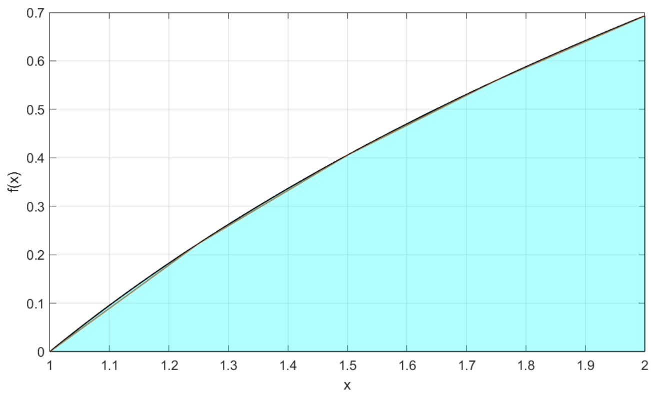

Figure 2: Simpsons 3/8 Method Approximation for $f(x) = \ln(x)$

Homework of Simpsons 3/8 Rule

1. Approximate $\int_0^1 \sin(x^2)dx$, by the Simpsons 3/8 rule, with $n = 6$.
2. Evaluate the integral $\sqrt{x^3+1}$ by Simpsons 3/8 rule dividing the interval $[0, 3]$ into five equal parts.

3. Find $\int_0^1 4x^3 dx$, with $n = 4$ by:

   (a) The exact value

   (b) The Trapezoidal rule

   (c) The Simpsons 1/3 rule

   (d) The Simpsons 3/8 rule

   (e) Compare all solutions

Methods	Solutions
Exact	
Trapezoidal	
Simpsons 1/3	
Simpsons 3/8	