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Numerical Analysis

Lecture 7

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الرياضيات المتقدمة: المرحلة الثانية

مادة التحليل العددي

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1 Formation of PDE by Eliminating Arbitrary Constant

A PDE may formed by a eliminating arbitrary constants or arbitrary function from a given relation and other relation obtained by differentiating partially the given relation.

Remark 1.1. Suppose the following relation:

1. $\frac{\partial z}{\partial x} = z_x = p$
2. $\frac{\partial z}{\partial y} = z_y = q$

Example 1.1. Form a Partial Differential Equations from the following equation:

$$z = (x-a)^2 + (y-b)^2 \tag{1}$$

Sol.

$$z_x = 2(x-a) \Rightarrow (x-a) = \frac{z_x}{2} \Rightarrow -a = \frac{z_x}{2} - x \Rightarrow a = x - \frac{z_x}{2}$$
$$z_y = 2(y-b) \Rightarrow (y-b) = \frac{z_y}{2} \Rightarrow -b = \frac{z_y}{2} - y \Rightarrow b = y - \frac{z_y}{2}$$

Eq. (1) become

$$\begin{aligned} z &= (x - (x - \frac{z_x}{2}))^2 + (y - (y - \frac{z_y}{2}))^2 \\ &= (-\frac{z_x}{2})^2 + (-\frac{z_y}{2})^2 \\ &= \frac{z_x^2}{4} + \frac{z_y^2}{4} \Rightarrow 4z = z_x^2 + z_y^2 = p^2 + q^2 \end{aligned}$$

$$\therefore 4z = p^2 + q^2$$

□

Example 1.2. Form a Partial Differential Equations from the following equation:

$$z = f(x^2 + y^2) \tag{1}$$

Sol.

$$z_x = 2x \cdot f'(x^2 + y^2) \Rightarrow f'(x^2 + y^2) = \frac{z_x}{2x} \tag{2}$$

$$z_y = 2y \cdot f'(x^2 + y^2) \Rightarrow f'(x^2 + y^2) = \frac{z_y}{2y} \tag{3}$$

Sub. Eq. (2) in Eq (3)

$$z_y = 2y\frac{z_x}{2x} \Rightarrow \frac{z_y}{z_x} = \frac{y}{x}$$

$$\therefore \frac{q}{p} = \frac{y}{x}$$

□

Example 1.3. Form a Partial Differential Equations from the following equation:

$$z = ax + by + a^2 + b^2 \tag{1}$$

Sol.

$$z_x = a$$

$$z_y = b$$

Eq. (1) become

$$z = z_x x + z_y y + (z_x)^2 + (z_y)^2$$

$$\therefore z = px + qy + p^2 + q^2$$

□

Example 1.4. Form a Partial Differential Equations from the following equation:

$$v = f(x - ct) + g(x + ct) \tag{1}$$

Sol.

$$v_x = f'(x-ct) + g'(x+ct)$$
$$v_t = -cf'(x-ct) + cg'(x+ct)$$
$$v_{xx} = f''(x-ct) + g''(x+ct)$$
$$v_{tt} = c^2 f''(x-ct) + c^2 g''(x+ct)$$
$$v_{tt} = c^2(f''(x-ct) + g''(x+ct))$$
$$v_{tt} = c^2(v_{xx})$$

$$\therefore v_{tt} = c^2 v_{xx}$$

$\square$

Homework of Formation of PDE by Eliminating Arbitrary Constant

Form a Partial Differential Equations from the following:

1. $z = ax + by + a^2 + b^2$
2. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$
3. $z = f(\frac{y}{x})$
4. $f(x-at) + g(x+at)$

2 Method of Separation of Variables

Although there are several methods that can be tried to find particular solutions of a linear PDE, in the method of separation of variables we seek to find a particular solution of the form of a product of a function of x and a function of y,

$$u(x,y) = X(x)Y(y).$$

With this assumption, it is sometimes possible to reduce a linear PDE in two variables to two ODEs. To this end we observe that

$$\frac{\partial u}{\partial x} = X'Y \quad \frac{\partial u}{\partial y} = XY' \quad \frac{\partial^2 u}{\partial x^2} = X''Y \quad \frac{\partial^2 u}{\partial y^2} = XY'',$$

where the primes denote ordinary differentiation.

$$X' = \frac{dX}{dx} \quad Y' = \frac{dY}{dy}$$

Example 2.1. Solve the following Partial Differential Equation with boundary condition

$$\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 0 \tag{1}$$

With boundary condition

$$u(0, y) = 4e^{-2y} - 3e^{-6y} \tag{2}$$

To solve Eq. (1) suppose $u(x, y) = XY$.

Then

$$\frac{\partial u}{\partial x} = X'Y \quad \frac{\partial u}{\partial y} = XY'$$

$$X' = \frac{dX}{dx} \quad Y' = \frac{dY}{dy}$$

Put in Eq. (1)

$$YX' + 3XY' = 0$$

$$\frac{X'}{3X} = -\frac{Y'}{Y}$$

Now let

$$\frac{X'}{3X} = -\frac{Y'}{Y} = c \quad c \text{ constant}$$

$$\frac{X'}{3X} = c, \quad -\frac{Y'}{Y} = c$$

$$\Rightarrow X' = 3cX, \quad Y' = -cY, \Rightarrow X = a_1 e^{3cx}, \quad Y = a_2 e^{-cy},$$

$$\Rightarrow u(x,y) = XY = a_1 e^{3cx} a_2 e^{-cy} = a_1 a_2 e^{3cx-cy} = Be^{c(3x-y)}, \quad B = a_1 a_2$$

Now let

$$u(x,y) = u_1 + u_2 = b_1 e^{c_1(3x-y)} + b_2 e^{c_2(3x-y)}$$

$$\Rightarrow u(0,y) = b_1 e^{c_1(-y)} + b_2 e^{c_2(-y)} = 4e^{-2y} - 3e^{-6y}$$

$$\Rightarrow b_1 = 4, b_2 = -3, c_1 = 2, c_2 = 6$$

$$\therefore u(x,y) = 4e^{2(3x-y)} - 3e^{6(3x-y)}$$

Homework of Method of Separation of Variables

1. Solve the following Partial Differential Equation with boundary condition

$$\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = 0$$

With boundary condition

$$u(0,y) = 4e^{-6y} - 5e^{-y}$$

2. Solve the following Partial Differential Equation with boundary condition

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

With boundary condition

$$u(0,y) = 2e^{-6y} + e^{-2y}$$