Al-Mustaqbal University

Department of Biomedical Engineering

Third Stage / 1st Course

“Transport Phenomena for BME”

Dr. A.Najah Saud

amir_Najah@uomus.edu.iq

Lecture note 2

Viscosity

Viscosity

- ***Viscosity*** is a measure of a fluid's resistance to flow.
- It describes the internal friction of a moving fluid.
- A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction.
- A fluid with low viscosity flows easily because its molecular makeup results in very little friction when it is in motion.
- Gases also have viscosity, although it is a little harder to notice it in ordinary circumstances.

Viscosity

To understand viscosity, let fluid between two parallel infinite in width and length plates.

See Fig. 1.1. The bottom plate A is fixed and the upper plate B is moveable.

The vertical distance between the two plates is represented by *h*.

A constant force *F* is applied to the moveable plate B causing it to move along at a constant velocity *uB* with respect to the fixed plate.

This behavior is consistent with the definition of a fluid: a material that deforms continuously under the application of a **shearing stress**, regardless of how small the stress is.

After some infinitesimal time *dt*, a line of fluid that was vertical at time *t*= 0 will move to a new position, as shown by the dashed line in Fig. 1.1.

The tan of angle between the line of fluid at *t* = 0 and *t* = *t* + *dt* is defined as the **shearing strain *du/dy***.

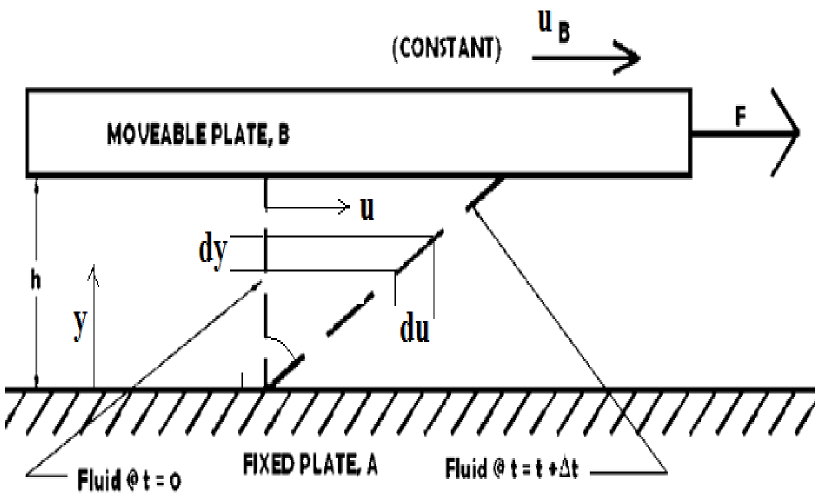

Figure 1.1 Velocity profile in a fluid between two parallel plates.

Viscosity

The fluid that touches plate A has zero velocity $u=0$.

The fluid that touches plate B moves with the same velocity as that of plate B, uB.

That is, the molecules of the fluid adhere to the plate and do not slide along its surface. This is known as the no-slip condition. The **no-slip condition** is important in fluid mechanics.

All fluids, including both gasses and liquids, satisfy this condition.

Let the distance from the fixed plate A to some arbitrary point above the plate be y. The velocity u of the fluid between the plates is a function of the distance above the fixed plate A, or $u=u(y)$. Let us define the velocity gradient as the change in fluid velocity with respect to y.

$$\text{Velocity gradient} \equiv du/dy$$

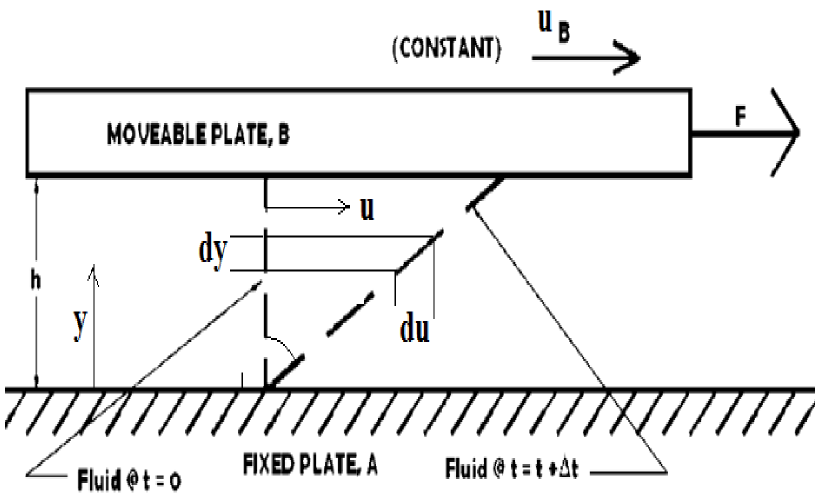

Figure 1.1 Velocity profile in a fluid between two parallel plates.

Viscosity

If the velocity of the fluid at any point between the plates varies linearly between $u = 0$ and $u = uB$, the velocity gradient can also be written as

$$Velocity\ gradient = u_B/h$$

The velocity profile is a graphical representation of the velocity gradient.

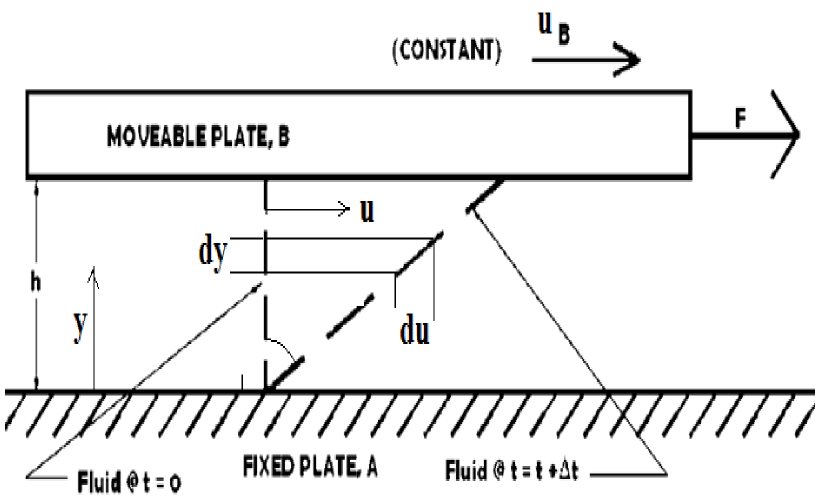

Figure 1.1 Velocity profile in a fluid between two parallel plates.

Viscosity

Figure 1.2 represents the shear stress on an element of the fluid at some arbitrary point between the plates in Figs. 1.1.

The shear stress on the top of the element results in a force that pulls the element “downstream.

The shear stress at the bottom of the element resists that movement.

Since the fluid element shown will be moving at a constant velocity, and will not be rotating, therefore

$$0 \; and \; \tau_A = \tau_B = \tau_{wall}$$

Physically, the shearing stress at the wall may also be represented by

$$\tau_A = \tau_B = {}^{force}/_{plate\ area} = {}^{F}/_{A_{parallel}}$$

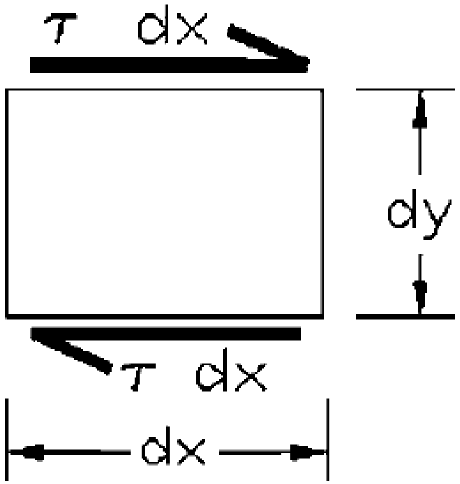

Figure 1.2 Shear stress on an element of the fluid in figure 1.1.

Viscosity

The shear stress on a fluid is related to the rate of shearing strain.

$$\tau \propto du/dy$$

In fact, the relationship between shearing stress and rate of shearing strain is determined by the fluid property known as viscosity sometimes referred to by the name (absolute viscosity or dynamic viscosity) represented by the Greek letter μ (mu).

$$\tau = \mu * du/dy$$

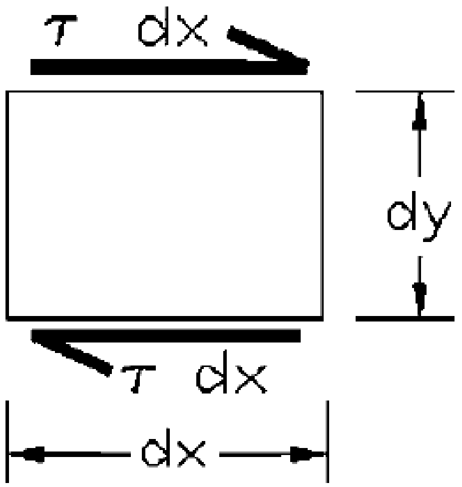

Figure 1.2 Shear stress on an element of the fluid in figure 1.1.

Kinematic viscosity

- **Kinematic viscosity** is another fluid property that has been used to characterize flow.
- It is the ratio of absolute viscosity to fluid density and is represented by the Greek character (nu).
- Kinematic viscosity can be defined by the equation:

$$\nu = \mu/\rho$$

- SI unit for absolute viscosity is N.s/m^2 or Pa·s or poise = g/s·cm , and for kinematic viscosity is m^2/s or Stoke = cm2/s.

Newtonian and non-Newtonian Fluids

- Newtonian fluids are the fluids of constant viscosity. For common fluids like oil, water, and air, and are linearly related. (see Fig. 1.5a). And the slope of the stress–shearing rate curve is constant represents the viscosity.
- For non-Newtonian fluids and are not linearly related. For those fluids, viscosity can change as a function of the shear rate (rate of shearing strain). Blood is an important example of a non-Newtonian fluid.
- But we will investigate the condition under which blood behaves as, and may be considered, a Newtonian fluid.
- Shear stress and shear rate are not linearly related for non-Newtonian fluids. Therefore, the slope of the shear stress/shear rate curve is not constant.
- However, we can still talk about viscosity if we define the apparent viscosity as the instantaneous slope of the shear stress/shear rate curve. See Fig. 1.5b.
- Shear thinning fluids are non-Newtonian fluids whose apparent viscosity decreases as shear rate increases. Latex paint is a good example of a shear thinning fluid.
- It is a positive characteristic of the paint that the viscosity is low when one is painting, but that the viscosity becomes higher and the paint sticks to the surface better when no shearing force is present.
- At low shear rates, blood is also a shear thinning fluid. However, when the shear rate increases above 100 s–1, blood behaves as a Newtonian fluid.

Newtonian and non-Newtonian Fluids

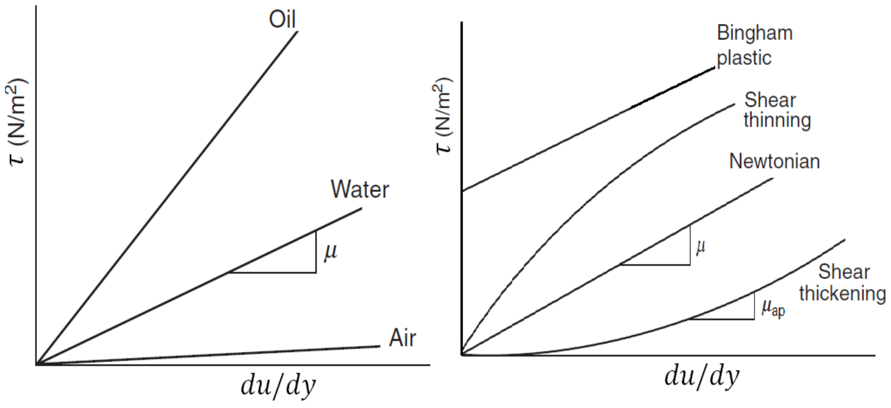

Figure 1.5 relationship between τ and du/dy (a) Newtonian fluid examples, (b) non-Newtonian fluid compared with Newtonian fluid

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- **For non-Newtonian fluids** and are not linearly related. For those fluids, viscosity can change as a function of the shear rate (rate of shearing strain).
- **Blood** is an important example of a non-Newtonian fluid.
- Shear stress and shear rate are not linearly related for non-Newtonian fluids. Therefore, the slope of the shear stress/shear rate curve is not constant.
- **Shear thinning fluids** are non-Newtonian fluids whose apparent viscosity decreases as shear rate increases.
- At **low shear rates**, blood is also a shear thinning fluid. However, when the shear rate increases above 100 s^{-1}, blood behaves as a Newtonian fluid.

Newtonian and non-Newtonian Fluids

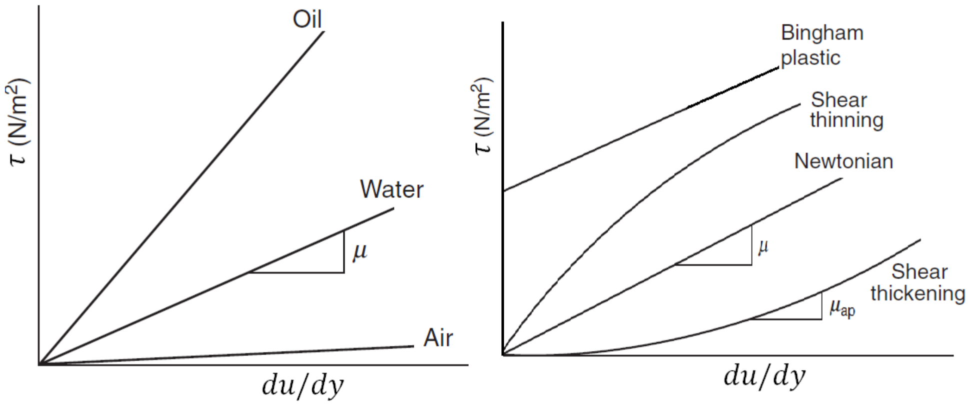
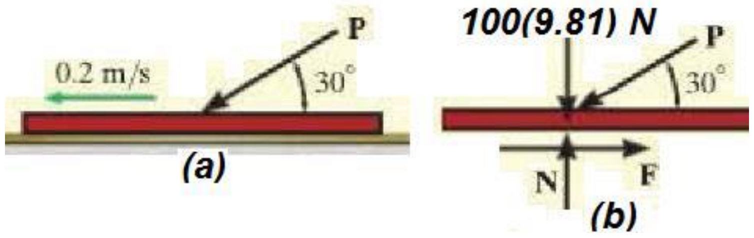


Figure 1.5 relationship between τ and du/dy (a) Newtonian fluid examples, (b) non-Newtonian fluid compared with Newtonian fluid

- A Bingham plastic is neither a fluid nor a solid.
- A Bingham plastic can withstand a finite shear load and flow like a fluid when that shear stress is exceeded Toothpaste and mayonnaise are examples of Bingham plastics.
- Blood is also a Bingham plastic and behaves as a solid at shear rates very close to zero.
- The yield stress for blood is very small, approximately in the range from 0.005 to 0.01 N/m^2.

Examples

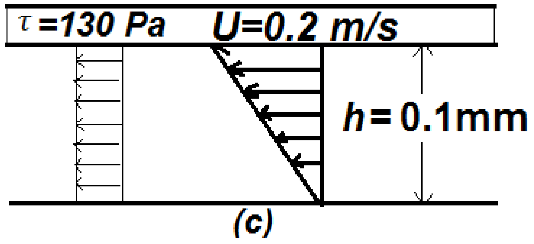
- The 100-kg plate in Fig. l-3a is resting on a very thin film of oil, which has a viscosity of μ _= 0.0652 N s/m2. Determine the force P that must be applied to the center of the plate to slide it over the oil with a constant velocity of 0.2 m/s. Assume the oil thickness is 0.1 mm, and the velocity profile across this thickness is linear. The bottom of the plate has a contact area of 0.75 m2 with the oil.



SOLUTION

Assumption: The oil is a Newtonian fluid, and so Newtonian's law of viscosity can be applied

Analysis: Because the plate moves with constant velocity, the force equation of equilibrium in the horizontal direction applies.



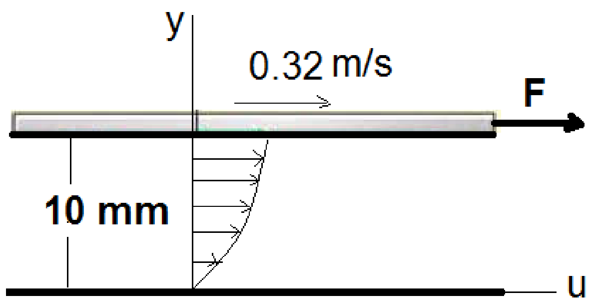
$$\sum F_x = 0 \qquad F - P\cos 30° = 0 \rightarrow F = 0.8660P$$

$$\tau = \frac{F}{A} = \frac{0.8660P}{0.75} = 1.155P$$

$$\tau = \mu\frac{du}{dy} = \mu\frac{U}{h} \Rightarrow 1.155P = 0.0652 * \left(\frac{0.2}{0.0001}\right) \Rightarrow P = 113N$$

Examples

The plate in Fig.1.4 rests on top of the thin film of water; the viscosity of water at a temperature of 25oC is μ = 0.897(10- 3) N · s/m2. When a small force F is applied to the plate, the velocity profile across the thickness of the fluid can be described as *u* = (40y - 800y2) m/ s. where *y* is in meters. Determine the shear stress acting on the fixed surface and on the bottom of the plate.



Solution:

Assumption: water is a Newtonian fluid, and so Newton's law of viscosity applies.

Analysis: Before applying Newton's law of viscosity, we must first obtain the velocity gradient

$$\frac{du}{dy} = \frac{d}{dy}(40y - 800y^2) = 40 - 1600y$$

Therefore, at the fixed surface, y = 0,

$$\tau = \mu \frac{du}{dy}\Big|_{y\,=\,0} = 0.897 * 10^{-3} * (40 - 0) = 35.88 * 10^{-3}\,\frac{N}{m^2} = 35.88\,mPa$$

And, at the bottom of the moving plate, y = 0.01 m,

$$\tau = \mu \frac{du}{dy}\Big|_{y\,=\,0.01} = 0.897 * 10^{-3} * (40 - 1600(0.01)) = 21.5\,mPa$$

Examples

An experimental test using human blood at T= 30°C indicates that it exerts a shear stress of = 0. 15 N/ m' on a surface A, where the measured velocity gradient at the surface is 16.85 . Since blood is a non-Newtonian fluid. Determine its apparent viscosity at the surface.

Solution: $\mu_{apparent} = {}^{\tau}/_{(du/dy)} = 0.15/16.8 = 8.9 * 10^{-3}\ Pa.s$

Examples

- The force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A / 2$, where V is the wind speed, the density of the air, A the cross-sectional area of the building, and CD is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

$$F = \frac{C_D \rho V^2 A}{2}$$

Where

$$F = mg = \text{kg} \times \frac{m}{s^2} = MLT^{-2}$$

$$\rho = \frac{mass}{volume} = \frac{kg}{m^3} = ML^{-3}$$

$$V = \frac{m}{s} = LT^{-1}$$

$$A = m^2 = L^2$$

$$C_D = \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})(L^2)} = M^0 L^0 T^0$$

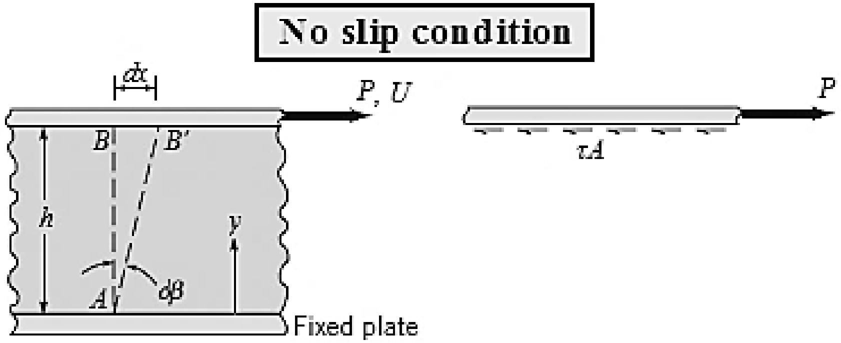
Hence, C_D is dimensionless.

H.W

1- Determine the mass of air in a 2 m3 tank if the air is at room temperature, 20 °C, and the absolute pressure within the tank is 200 kPa.

2- For flowing water, what is the magnitude of the velocity gradient needed to produce a shear stress of 1.0 N/m2?

3-For a parallel plate arrangement of the type shown in Figure it is found that when the distance between plates is 2 mm, a shearing stress of 150 Pa develops at the upper plate when it is pulled at a velocity of 1 m/s. Determine the viscosity of the fluid between the plates. Express your answer in SI units.



4- A layer of water flows down an inclined fixed surface with the velocity profile shown in Figure. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for *U* = 2 m/s and *h* = 0.1 m.