Work # 1. Sampling Theorem and Aliasing

Theory basics

The idea of the sampling theorem is the following: the continuous real signal with the spectrum limited by the frequency range $0 < f < f_m$ can be completely reconstructed from its discreet samples if the sampling frequency $fs > 2f_m$.

The aliasing effect means that in case of sampling theorem violation (i.e. in case of too low sampling frequency), some frequency components of the signal cannot be differentiated from each other.

Main tasks of the work

- To acquire elementary skills of work with MATLAB package in command and program modes (with the use of m-files). To learn how to calculate simple functions and represent them in graphical form.
- To examine the aliasing effect using artificially generated test signals and real electrocardiogram (ECG) signal.

Supplement #1: Table W.1

Variant	*F*, Hz	*A*	*tmax*, s	*Fs*1, Hz	*Fs*2, Hz	File	*C1*	*C2*
1	13	2	0.5	200	45	W1_01.txt	3	10
2	15	5	0.3	300	50	W1_02.txt	2	15
3	24	20	0.2	350	80	W1_03.txt	4	25
4	26	25	0.2	400	90	W1_04.txt	5	16
5	28	3	0.15	450	95	W1_05.txt	6	20
6	30	10	0.1	500	100	W1_06.txt	2	25
7	35	50	0.1	600	110	W1_07.txt	3	16
8	35	6	0.1	800	130	W1_08.txt	5	20
9	40	10	0.1	900	150	W1_09.txt	4	15
10	45	7	0.1	1000	160	W1_10.txt	6	16

We will use **Variant Nº 2**

Part 1: Calculation and graphical representation of functions

Code

```
x=0:0.02:2*pi;
y1=sin(3*x);
figure(1);
plot(x,y1);
title ('signal');
hold on
y2=2*cos(5*x);
plot(x,y2);
hold off
```

Obtained graph

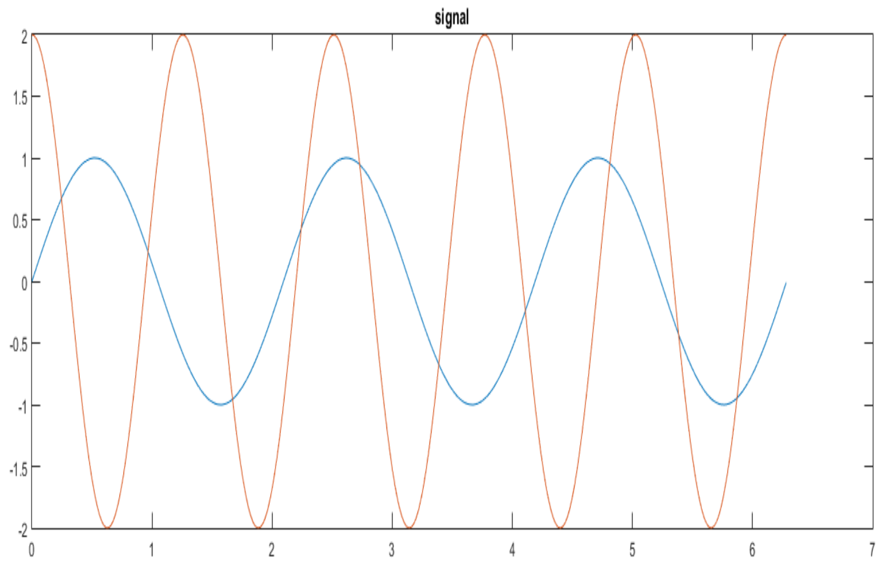

Figure 1: Calculation and graphical representation of functions

<u>Code</u>

```
subplot(1,2,1);
x=0:0.02:2*pi;
y1=sin(3*x);
figure(1);
plot(x,y1);
title ('signal 1');
subplot(1,2,2);
y2=2*cos(5*x);
plot(x,y2);
title ('signal 2');
```

Obtained graph

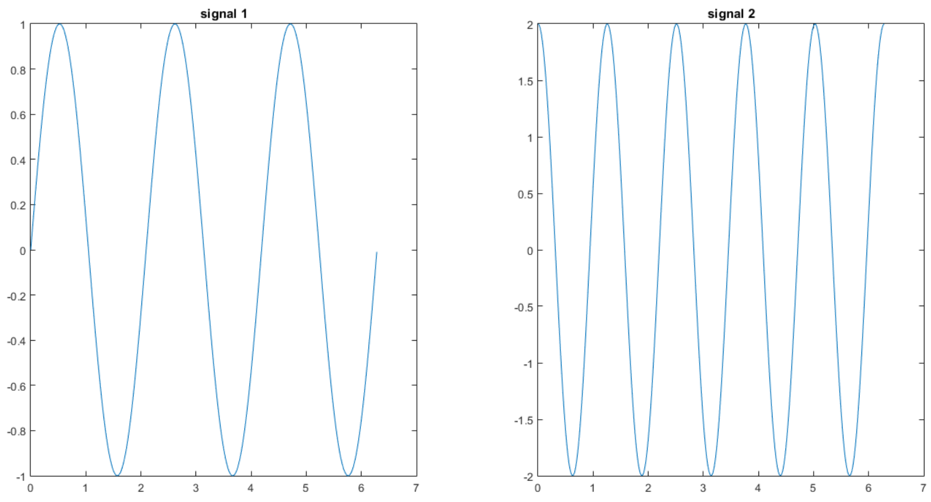

Figure 2: Subplot

Part 2: Examination of the aliasing effect with the use of test signals

Code

```
F=15;
A=5;
tmax=0.3;
Fs1=300;
Fs2=50;
C1=2;
C2=25;
T1=1/Fs1;
t1=(0:T1:tmax);
T2=1/Fs2;
t2=(0:T2:tmax);
subplot(3,1,1);
Y1=A*sin(2*pi*F*t1);
plot(t1,Y1)
hold on
Y2=A*sin(2*pi*F*t2);
plot(t2,Y2,'r')
subplot(3,1,2)
Y3=A*sin(2*pi*(Fs2+F)*t1);
plot(t1,Y3)
hold on
```

```
Y4=A*sin(2*pi*(Fs2+F)*t2);
plot(t2,Y4)
subplot(3,1,3);
Y5=A*sin(2*pi*(Fs2-F)*t1);
plot(t1,Y5)
hold on
Y6=A*sin(2*pi*(Fs2-F)*t2);
plot(t2,Y6,'r')
```

Conclusions

<u>Obtained graph</u>

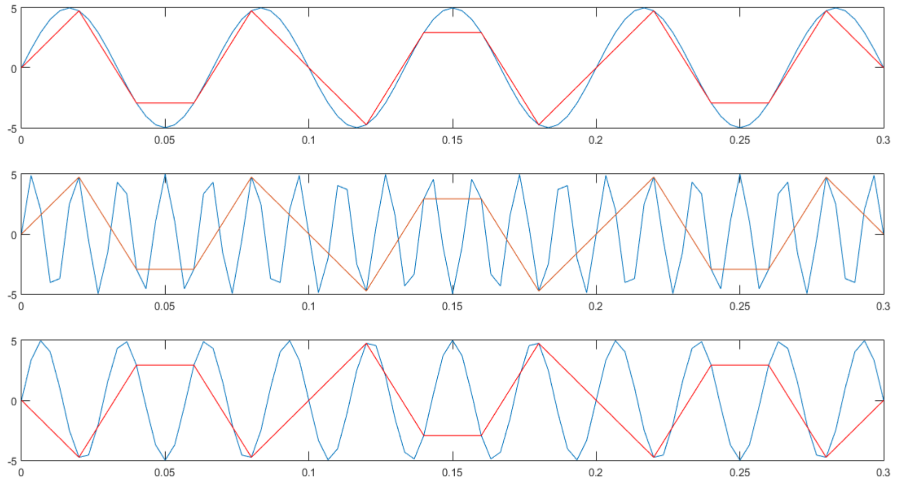

Figure 3: Examination of the aliasing effect (Fs_2-F)

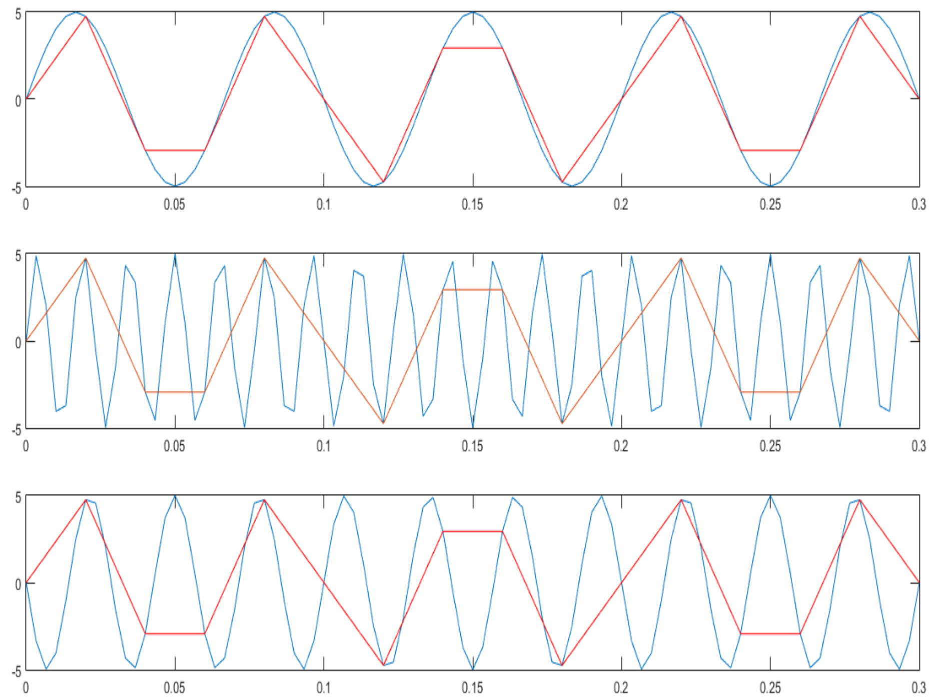

Figure 4: Examination of the aliasing effect (F-Fs_2)

Part 3: Examination of the aliasing effect with the use of the ECG signal

Code

```
subplot(3,1,1);
Y=load('W1_02.txt');
LY=length(Y);
Fs=1200;
T=1/Fs;
tmax=LY*T;
t=0:T:tmax-T;
for i=1:LY;
    Y1(i)=Y((i-1)+1);
end
plot(t,Y1)
hold on
subplot(3,1,2);
C1=2;
Fs1=Fs/C1;
T1=1/Fs1;
t1=0:T1:tmax-T1;
LY1=LY/C1;
for i=1:LY1;
    Y2(i)=Y((i-1)*C1+1);
end
plot(t1,Y2)
hold on
subplot(3,1,3);
C2=25;
Fs2=Fs/C2;
T2=1/Fs2;
t2=0:T2:tmax-T2;
LY2=LY/C2;
for i=1:LY2;
    Y3(i)=Y((i-1)*C2+1);
end
plot(t2,Y3)
```

Obtained graph

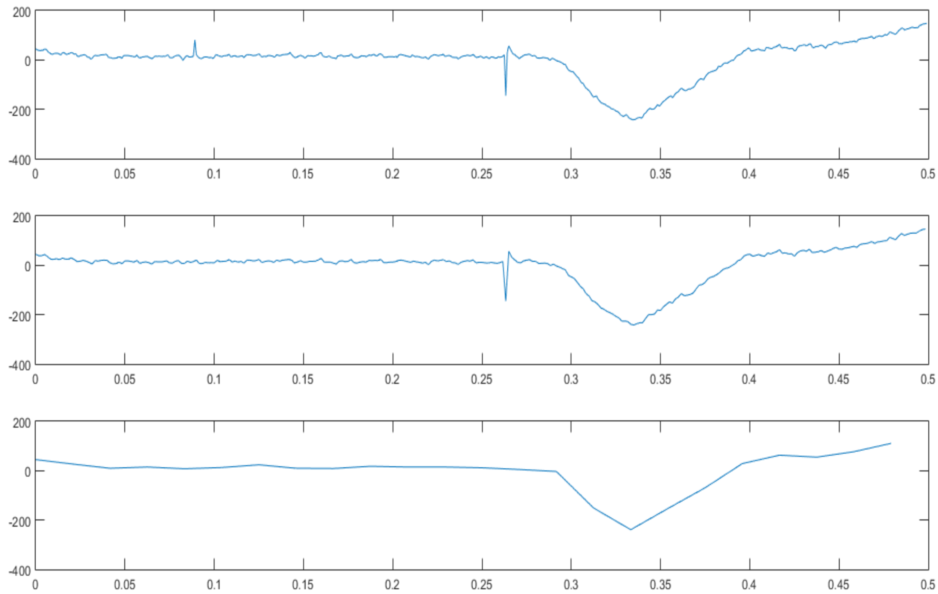

Figure 5: Decimation of ECG signal

Conclusions

- Continues signals, such as Biological Signals have a certain value at every instance of time. The Nyquist sampling theorem underlies all situations where continuous signals are sampled and is especially important where patterns are to be digitized and analyzed by computers.
- Aliasing is an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled. Figure 4 shows three sinewave signals which have different frequencies. The sampling points of the sinewaves in the threes graphs are identical. This effect will make the distinguish between these signals is impossible. The use of (Fs-F) yields a mirror of the required signal as shown in figure 3. This due to the negative sign of the angle but by changing the sequence (F-Fs) the required signal was achieved.

- Decimation is technique used to reduce the down sampling of a signal to reduce the size of the data and the computational time. The first graph in figure 5 shows sampling of an ECG signal. The second and third graph shows the decimated signal by using 2 and 25 respectively. The decimations by 2 removed some features from the original signal (the small peak to the right), while decimation by 25 removed all high frequency amplitudes from the signal. The decimation is working as low pass filter and need to be used carefully because it may remove important events from the digital signal.