Duct resistance

A single duct-line having lengths of different cross-sections, bends, expanders, reducers, dampers and registers, constitutes a system of resistances in series, in which all the pressure losses occurring at a given volumetric flow rate are added together to give the system resistance or total pressure loss at the given volumetric flow rate.

We know that the pressure loss (frictional or dynamic) is directly proportional to the velocity pressure, therefore total pressure loss in a duct system,

$$p_L = C_1 p_{v1} + C_2 p_{v2} + \dots \qquad \dots (i)$$

We also know that $p_{v1} \alpha (V_1)^2$, $p_{v2} \alpha (V_2)^2$, and so on. Thus, the equation (i) may be written as:

$$p_L = C_1 (V_1)^2 + C_2 (V_2)^2 + \dots \qquad \dots (ii)$$

But for a system in series, the volumetric flow rate (Q) is same. Therefore from the continuity equation,

$$Q = A_1 V_1 = A_2 V_2 = \dots$$

i.e. $V_1 \alpha Q, V_2 \alpha Q$ and so on. Equation (ii) may be written as

$$p_L = K_1 Q^2 + K_2 Q^2 + \dots$$
$$= (K_1 + K_2 + \dots) Q^2 = KQ^2$$

This expression represents the relationship between the pressure loss (or resistance) in the system and the volume flow rate. Thus, system resistance,

$$\boxed{R = KQ^2}$$

Where K is a constant of the system

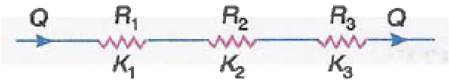
System in series

If a number of systems R_1 R_2, R_3 …etc. having constants K_1, K_2, K_3 ….etc. are connected in series, as shown in Fig. 1(a) , it can be reduced to a single equivalent system as shown in Fig. 1(b). The resistance of a single equivalent system or the overall system resistance for the given flow rate is obtained by adding the individual system resistances, i.e.

$$R_e = R_1 + R_2 + R_3$$

or $$K\,Q^2 = K_1 Q^2 + K_2 Q^2 + K_3 Q^2$$

and $$\boxed{K = K_1 + K_2 + K_3}$$

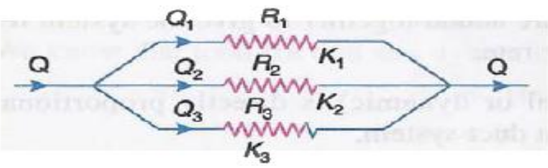


(*a*) Systems in series.

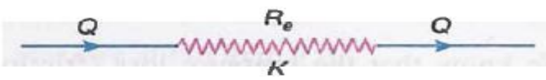
(*b*) Single equivalent system.

System in parallel

If a number of systems R_1, R_2, R, etc. having constants K_1, K_2, K_3…etc. are connected in parallel, as shown in fig.(a), it can be reduced to a single equivalent system as shown in fig.(b). The resistance or a single equivalent system or the overall system resistance for the given now rate may be obtained by calculating the constant K for the equivalent system from the constants K_1, K_2, K_3 etc. of the parallel system.



(*a*) Systems in parallel.



(*b*) Single equivalent system.

Fig. 20.26

We know that $$Q = Q_1 + Q_2 + Q_3$$

$$\sqrt{\frac{R_e}{K}} = \sqrt{\frac{R_1}{K_1}} + \sqrt{\frac{R_2}{K_2}} + \sqrt{\frac{R_3}{K_3}}$$

For the systems in parallel, the pressure loss is same, *i.e.*

$$R_e = R_1 = R_2 = R_3$$

$$\therefore \quad \boxed{\sqrt{\frac{1}{K}} = \sqrt{\frac{1}{K_1}} + \sqrt{\frac{1}{K_2}} + \sqrt{\frac{1}{K_3}}}$$

Example 1: *A duct system is represented diagramatically as shown in Fig. 20.27,*

R_1 = 60 mm of water at 180 m³/min,

R_2 = 18 mm of water at 60 m^3/min, and

R_3 = 30 mm of water at 75 m^3/min.

Calculate the constant of an equivalent resistance of the above system and hence the pressure loss if the volume flow rate through R_1 is 120 m^3/min.

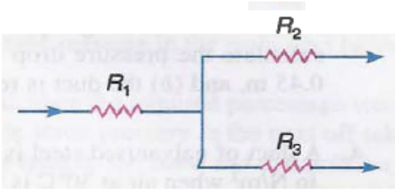


Fig. 20.27

Solution. Given : R_1 = 60 mm of water ; Q_1 = 180 m^3/min; R_2 = 18 mm of water ; Q_2 = 60 m^3/min ; R_3 = 30 mm of water ; Q_3 = 75 m^3/min ; Q_1' = 120 m^3/min

We know that the constant for the resistance R_1,

$$K_1 = \frac{R_1}{(Q_1)^2} = \frac{60}{(180)^2} = 1.85 \times 10^{-3}$$

Now, let us find the constant for the single equivalent resistance (R_e) for the parallel resistances R_2 and R_3. We know that the constant for the resistance R_2,

$$K_2 = \frac{R_2}{(Q_2)^2} = \frac{18}{(60)^2} = 5 \times 10^{-3}$$

Similarly, constant for the resistance R_3,

$$K_3 = \frac{R_3}{(Q_3)^2} = \frac{30}{(75)^2} = 5.3 \times 10^{-3}$$

Let K = Constant for the single equivalent resistance R_e.

$$\therefore \quad \frac{1}{\sqrt{K}} = \frac{1}{\sqrt{K_2}} + \frac{1}{\sqrt{K_3}} = \frac{1}{\sqrt{5\times10^{-3}}} + \frac{1}{\sqrt{5.3\times10^{-3}}}$$

$$= 14.14 + 13.7 = 27.84$$

or

$$K = \frac{1}{(27.84)^2} = 1.3 \times 10^{-3}$$

Since the resistance R_1 of constant K_1 is connected with single equivalent resistance R_e of constant K in series, therefore constant for the whole system,

$$K' = K_1 + K$$

$$= 1.85 \times 10^{-3} + 1.3 \times 10^{-3} = 3.15 \times 10^{-3}$$

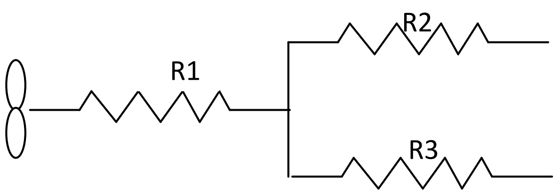
and the equivalent system resistance,

$$R_{e'} = K' (Q_{1'})^2 = 3.15 \times 10^{-3} (Q_{1'})^2 \text{ Ans.}$$

Since the system resistance means the pressure loss, therefore pressure loss for a volume flow rate of Q_1' = 120 m^3/min

$$= 3.15 \times 10^{-3} (120)^2 = 45.36 \text{ mm of water Ans.}$$

Example2: A fan delivers air to a system as shown in fig.. where R_1= 50mm of water for a volume flow of 180 m^3/min and R_2=R_3= 17.5mm of water for a volume flow of 60 m^3/min. The fan performance is as follows:



Volume flow rate (m^3/min)	120	150	180	210
Fan total pressure (mm of water)	70	55	40	18

Find the volume flow rate by the fan. By what percentage would it increase if a third branch of resistance R4=R2 is connected in parallel with the existing two branches.

$$K_1 = \frac{R_1}{(Q_1)^2} = \frac{50}{(180)^2} = 1.54 \times 10^{-3}$$

Similarly, constant for R_2,

$$K_2 = \frac{R_2}{(Q_2)^2} = \frac{17.5}{(60)^2} = 4.86 \times 10^{-3}$$

and constant for R_3,

$$K_3 = \frac{R_3}{(Q_3)^2} = \frac{17.5}{(60)^2} = 4.86 \times 10^{-3}$$

We know that

$$\sqrt{\frac{1}{K_e}} = \sqrt{\frac{1}{K_2}} + \sqrt{\frac{1}{K_3}}$$

or

$$\sqrt{\frac{1}{K_e}} = \sqrt{\frac{1}{4.86 \times 10^{-3}}} + \sqrt{\frac{1}{4.86 \times 10^{-3}}} = 14.34 + 14.34 = 28.68$$

Squaring both sides, we have

$$\frac{1}{K_e} = (28.68)^2 = 822.5$$

or

$$K_e = \frac{1}{822.5} = 1.2 \times 10^{-3}$$

The whole system now reduces to a system in series as shown in Fig. 21.14. If Q is the volume flow rate of air in m^3/min, then the system resistance,

$$R' = R_1 + R_e = K_1 Q^2 + K_e Q^2$$
$$= 1.54 \times 10^{-3} Q^2 + 1.2 \times 10^{-2} Q^2 = 2.74 \times 10^{-3} Q^2$$

Fig. 21.14

In order to draw the system characteristic curve, tabulate the values of R' for different values of Q as given below :

Q (m^3/min)	120	130	140	150
R' (mm of water)	39.45	46.3	53.7	61.65

From the above values of Q and R', draw the system characteristic curve as shown in Fig. 21.15. This curve intersects the fan performance curve plotted for the given values, at point P as shown in Fig. 21.15. The point P is the operating point of the fan.

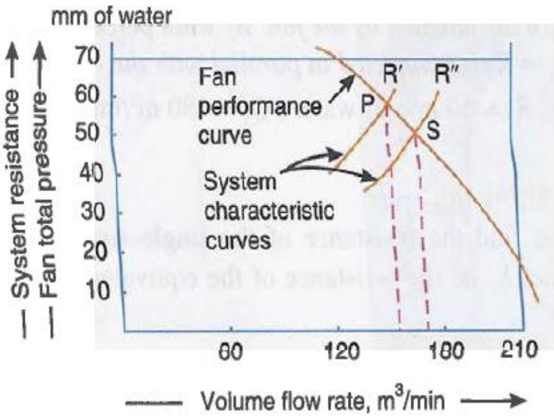


Fig. 21.15

∴ Volume of air handled by the fan

$= $ Volume of air at $P = 145$ m^3/min **Ans.**

Percentage increase in volume

When a third branch of resistance $R_4 = R_2$ is connected in parallel, with the existing two branches as shown in Fig. 21.16, then the constant for the equivalent resistance R'_e is given by

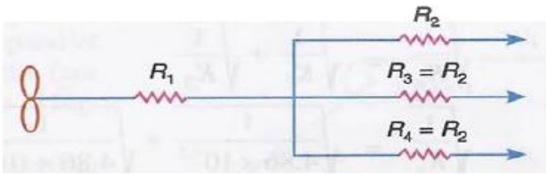


Fig. 21.16

$$\sqrt{\frac{1}{K'_e}} = \sqrt{\frac{1}{K_2}} + \sqrt{\frac{1}{K_3}} + \sqrt{\frac{1}{K_4}}$$

Since $R_2 = R_3 = R_4$, therefore $K_2 = K_3 = K_4 = 4.86 \times 10^{-3}$

$$\therefore \quad \sqrt{\frac{1}{K'_e}} = 3 \times \sqrt{\frac{1}{K_2}} = 3 \times \sqrt{\frac{1}{4.86 \times 10^3}} = 43.02$$

Squaring both sides, we have

$$\frac{1}{K'_e} = (43.02)^2 = 1850.7$$

or

$$K'_e = \frac{1}{1850.7} = 0.54 \times 10^{-3}$$

The whole system now reduces to a system in series as shown in Fig. 21.17. If Q is the volume flow rate of air in m^3/min, then the system resistance,

$$R'' = R_1 + R'_e = K_1 Q^2 + K'_e Q^2$$

$$= 1.54 \times 10^{-3} Q^2 + 0.54 \times 10^{-3} Q^2 = 2.04 \times 10^{-3} Q^2$$

Fig. 21.17

Now tabulate the values of R'' for different values of Q as given below :

Q (m³/min)	140	150	160	170	180
R″ (mm of water)	40	45.9	52.2	58.9	66.1

From these values, draw the system characteristic curve as shown in Fig. 21.15. This curve intersects the fan performance curve at point S. At this point, volume of air handled by the fan

$$= 158 \text{ m}^3/\text{min}$$

intersects the fan performance curve at point S. At this point, volume of air handled by the fan

$$= 158 \text{ m}^3/\text{min}$$

$\therefore$ Percentage increase in volume

$$= \frac{158 - 145}{145} \times 100 = 8.96\% \textbf{ Ans.}$$

Example 21.6. *The fans A and B supply equal volumes of air to a system as shown in Fig. 21.18.*

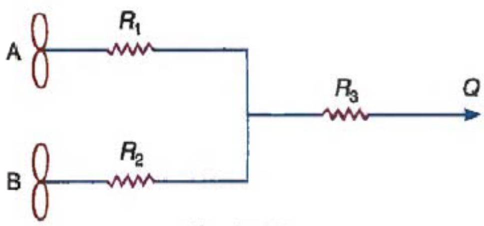


Fig. 21.18

Example 3: *The performance of the two fans is given below :*

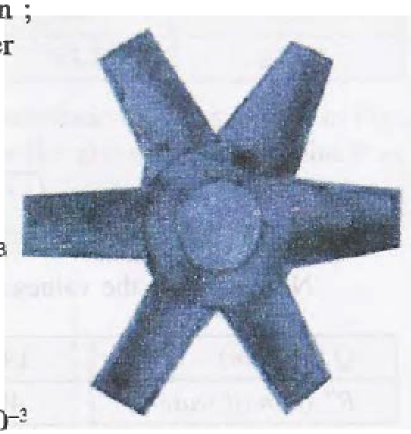
Volume flow (m^3/min)	*Fan total pressure (mm of water)*	
	Fan A	*Fan B*
60	70	76.5
90	69.5	65.5
120	67	48
150	60	26
180	48	—

The resistances of the system are

$$R_1 = 15 \text{ mm of water at } 90 \text{ m}^3/\text{min} ;$$
$$R_2 = 7.5 \text{ mm of water at } 120 \text{ m}^3/\text{min, and}$$
$$R_3 = 2.5 \text{ mm of water at } 60 \text{ m}^3/\text{min.}$$

Determine the operating points of each of the two fans and the volume flowing through the common branch.

Solution. Given : $R_1 = 15$ mm of water ; $Q_1 = 90$ m^3/min ; $R_2 = 7.5$ mm of water ; $Q_2 = 120$ m^3/min ; $R_3 = 2.5$ mm of water $Q_3 = 60$ m^3/min



Tube axial flow fans impeller.

Operating points of each of the two fans

We know that constant for R_1,

$$K_1 = \frac{R_1}{(Q_1)^2} = \frac{15}{(90)^2} = 1.85 \times 10^{-3}$$

Similarly, constant for R_2,

$$K_2 = \frac{R_2}{(Q_2)^2} = \frac{7.5}{(120)^2} = 0.52 \times 10^{-3}$$

and constant for R_3, $$K_3 = \frac{R_3}{(Q_3)^2} = \frac{2.5}{(60)^2} = 0.7 \times 10^{-3}$$

Let Q_A and Q_B be the volume of air supplied by the fans A and B. Since the volume of air supplied by the fans A and B is equal, therefore volume of air flowing through the common duct,

$$Q = 2Q_A = 2Q_B \qquad \dots (i)$$

We know that system resistance with fan A,

$$R_A = R_1 + R_3 = K_1 (Q_A)^2 + K_3 Q^2$$
$$= K_1 (Q_A)^2 + K_3 (2Q_A)^2 = (K_1 + 4K_3)(Q_A)^2 \quad \dots [\because Q = Q_A]$$
$$= (1.85 \times 10^{-3} + 4 \times 0.7 \times 10^{-3})(Q_A)^2 = 4.65 \times 10^{-3} (Q_A)^2$$

Similarly, system resistance with fan B,

$$R_B = R_2 + R_3 = K_2 (Q_B)^2 + K_3 Q^2$$
$$= K_2 (Q_A)^2 + K_3 (2Q_A)^2$$
$$= (K_2 + 4K_3)(Q_A)^2 \qquad \dots [\text{ From equation } (i)]$$
$$= (0.52 \times 10^{-3} + 4 \times 0.7 \times 10^{-3})(Q_A)^2$$
$$= 3.32 \times 10^{-3} (Q_A)^2$$

Now tabulate the values of R_A and R_B for different values of Q_A, as given below :

$Q_A (m^3/min)$	100	110	120	130
R_A *(mm of water)*	46.5	56.25	67	78.6
R_B *(mm of water)*	33.2	40.17	48	56.1

From these values, draw the system characteristic curves as shown in Fig. 21.19. The curve for R_A intersects the fan performance curve for fan *A*, at point *P* and the curve for R_B intersects the fan performance curve for fan *B*, at point *S*. The points *P* and *S* are the operating points of fans *A* and *B* respectively. From Fig. 21.19, we find that the operating point *P* for fan *A* lies at 120 m³/min and 67 mm of water. The operating point *S* for fan *B* lies at 120 m³/min and 48 mm of water. Ans.

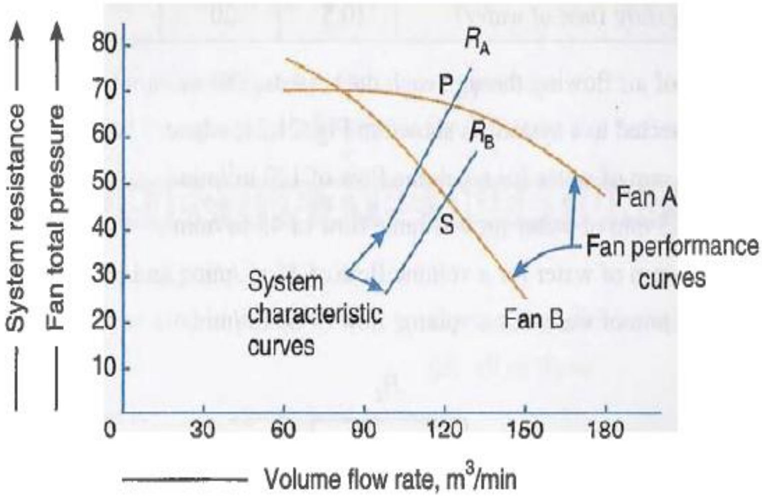


Fig. 21.19

Volume flowing through the common branch

Volume flowing through the common branch,

$$Q = \text{Volume at } P + \text{Volume at } S$$
$$= 120 + 120 = 240 \text{ m}^3/\text{min} \text{ Ans.}$$

Point of Operation

- **Fan and system have variable performance characteristics which can be represented graphically.**
- **"Point of operation" is the single point at the intersection of fan curve and system curve.**

Fan Performance Curves:

- **Curve represents fan performance variables plotted against flow rate.**
- **Curve is specific to a fan of given size operating at a single rotation rate (RPM).**
- **Even with size and rotation rate fixed, power and pressure requirements vary over a range of flow rates.**

System Requirement Curves:

- **Duct system pressure varies with volumetric flow rate.**
- **Curve represents the variation of pressure plotted against volumetric flow rate.**