2. Ordinary Differential Equations of First Order

There are several methods to find the solution of the ordinary differential equation of the first order some of these methods:

a) Separable differential equations
b) Homogenous differential equations
c) Exact differential equations
d) Non-Exact differential equations
e) Linear Equations

2.1 Separable First Order Differential Equation

في حالات كثيرة يمكننا ترتيب المعادلات التفاضلية من المرتبة الأولى (first order) بحيث يصبح المتغيران x, y منفصلين عن بعضهما بحيث ترتبط x ب dx وترتبط دالة المتغير y ب dy وتحل بالتكامل المباشر مع ظهور ثابت واحد، كما في الحالات التالية:

1) $f(x)dx = g(y)dy \quad \rightarrow \quad \int f(x)dx = \int g(y)dy + c$

2) $f(x)G(y)dx = F(x)g(y)dy \quad \rightarrow \quad \frac{f(x)}{F(x)}dx = \frac{g(y)}{G(y)}dy$

$$\int \frac{f(x)}{F(x)}dx = \int \frac{g(y)}{G(y)}dy + c, \qquad F(x), and\ G(y) \neq 0$$

3) $\frac{dy}{dx} = f(x)g(y) \quad \rightarrow \quad \frac{dy}{g(y)} = f(x)dx \quad \rightarrow \quad \int \frac{dy}{g(y)} = \int f(x)dx + c$

$g(y) \neq 0$

Example (1): Solve the following differential equations:

1) $\acute{y} = 5y$

2) $\frac{dy}{dx} = \frac{x+1}{y^4+1}$

Solve:

1) $\acute{y} = 5y$

$$\frac{dy}{dx} = 5y \quad \rightarrow \quad \frac{dy}{y} = 5\,dx$$

$$\int \frac{dy}{y} = \int 5dx + c \qquad \left(\textbf{Note}: \frac{d}{dy}\ln(\text{الدالة}) = \frac{\text{مشتقة الدالة}}{\text{الدالة}}\right)$$

$\ln(y) = 5x + c$ نأخذ e للطرفين

$$y = e^{5x+c} \quad \rightarrow \quad y = e^{5x} \cdot e^{c} \qquad (k = e^{c})$$

$y = k\, e^{5x}$ (general solution)

2) $\dfrac{dy}{dx} = \dfrac{x+1}{y^4+1}$

$$(y^4 + 1)\,dy = (x + 1)\,dx$$

$$\int (y^4 + 1)\,dy = \int (x + 1)\,dx + c$$

$\dfrac{y^5}{5} + y = \dfrac{x^2}{2} + x + c$ (general solution)

Example (2): Find a particular solution for $e^x dx - y\,dy = 0,\ \ if\ \ y(0) = 1$?

Solve:

$$e^x dx = y\,dy$$

$$\int e^x dx = \int y\,dy + c$$

$$e^x = \frac{y^2}{2} + c \quad \rightarrow$$

$y^2 = 2e^x - 2c$ (general solution)

Apply initial condition: $y(0) = 1$

$$1^2 = 2e^0 - 2c \quad \rightarrow \quad c = 1/2$$

$\therefore y^2 = 2e^x - 1$ (particular solution)

Example (3): **Solve the differential equation $dx + xy\,dy = y^2\,dx + y\,dy$ and then find a particular solution when the curve passes through the point $\left(\frac{-7}{5}, \frac{13}{5}\right)$?**

Solve:

$$dx - y^2\,dx = y\,dy - xy\,dy$$

$$(1 - y^2)dx = (1 - x)y\,dy \quad \rightarrow \quad \text{re} - \text{arrangement:}$$

$$\frac{dx}{(1-x)} = \frac{y\,dy}{(1-y^2)}$$

$$\int \frac{dx}{(1-x)} \times \frac{-1}{-1} = \int \frac{y\,dy}{(1-y^2)} \times \frac{-2}{-2} + c$$

$$\left.-\ln(1-x) = \frac{1}{-2}\ln(1-y^2) + c\right\} \times -2$$

$$2\ln(1-x) = \ln(1-y^2) - 2c$$

$$\ln(1-x)^2 - \ln(1-y^2) = -2c$$

$$\ln\frac{(1-x)^2}{(1-y^2)} = -2c \qquad \text{نأخذ } e \text{ للطرفين}$$

$$\frac{(1-x)^2}{(1-y^2)} = e^{-2c} = k$$

$$\because x = \frac{-7}{5}, and\ y = \frac{13}{5} \quad \rightarrow \quad \therefore k = -1$$

$$\therefore \frac{(1-x)^2}{(1-y^2)} = -1$$

Problems:

H.W: Find a general solution for:

1) $\acute{y} = 3x^2(1 + y)$

Answer: $y = k\,e^{x^3} - 1$

2) $(y^2 - 1)\,dx = xy\,dy, \quad if\ x = 2, y = 0$

Answer: $x^2 + 4y^2 = 4$

3)$(y + x^2 \cdot y)dy = (x \cdot y^2 + x)dx$

Answer: $y^2 + 1 = k\,(1 + x^2)$