From Fig. 21.5, we see that centrifugal fans with backward curved blades require maximum power. The operating condition which requires the maximum power is close to the combination of volume and static pressure under which the fan operates most efficiently. The fans of this type are said to have non- overloading power characteristic which means that the driving motor cannot be overloaded if the fan and motor are properly selected.

The axial flow fans, as shown in Fig. 21.7, also have non over-loading power characteristics.

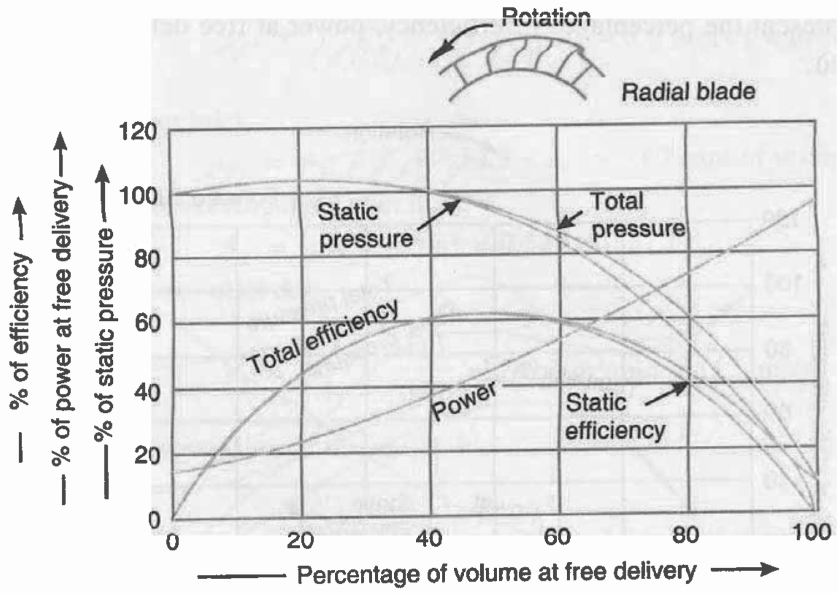


Fig. 21.6. Performance curves for a centrifugal fan with radial blades.

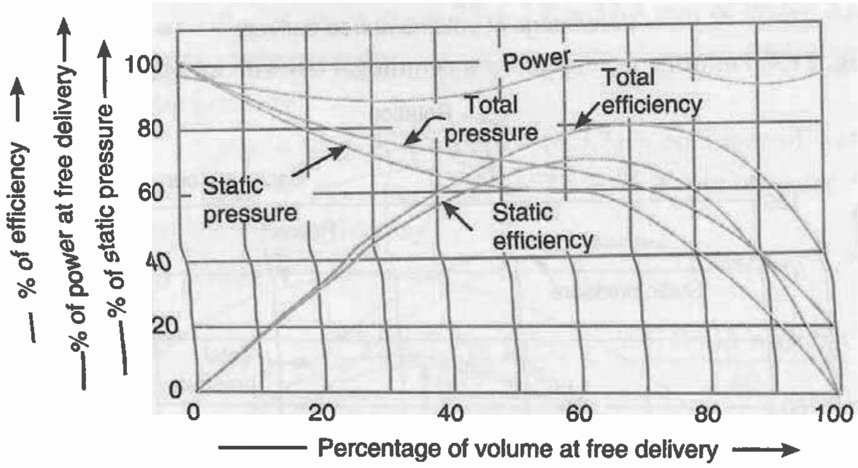


Fig. 21.7. Performance curves for an axial flow air foil-type fan .

21.9 Velocity Triangles for Moving Blades of a Centrifugal Fan

Consider a stream of air entering the backward curved blade at C and leaving it at D, as shown in Fig. 21.8 (*a*). The velocity triangles at the inlet and outlet tips of the blade are shown in Fig. 21.8 (*b*).

Let

V_{b1} = Linear or tangential velocity of the moving blade at inlet (*BA*).

V_1 = Absolute velocity of air entering the blade ($A'C$).

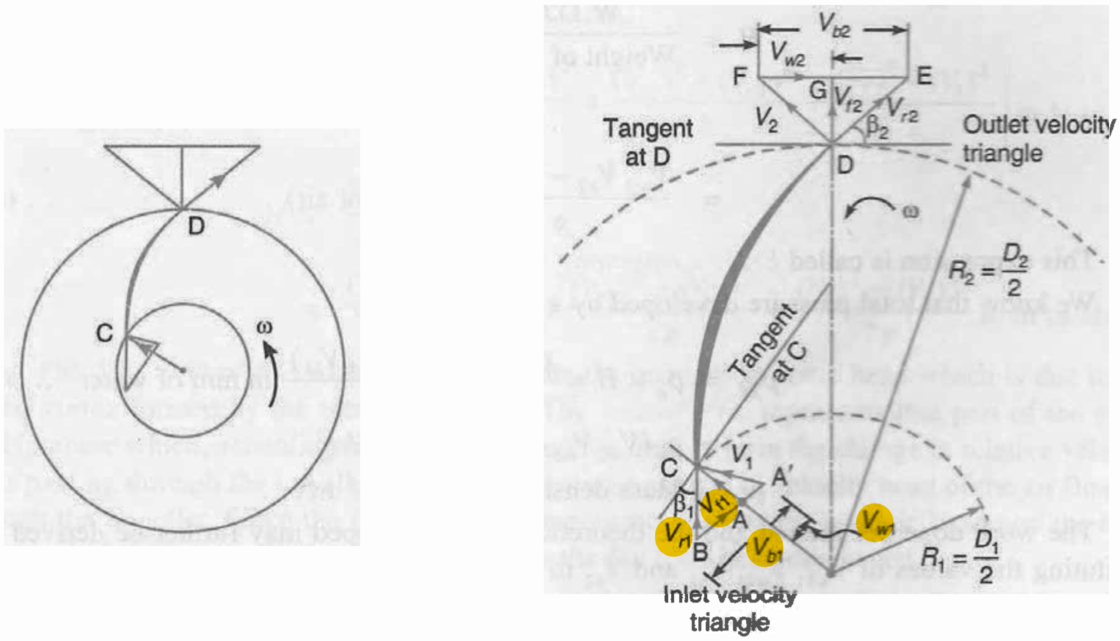
V_{f1} = Velocity of flow at inlet (AC). It is the radial component of V_1.

V_{r1} = Relative velocity of air to the moving blade at inlet (*BC*). It is vectorial difference between V_{b1} and V_1.

V_{w1} = Velocity of whirl at inlet. It is the tangential component of V_1.

β_1 = Blade angle at inlet. It is the angle which the relative velocity (V_{r1}) makes with the tangent at the blade inlet. It is equal to the angle between V_{r1} and V_{b1} (*i.e.* angle *CBA*).

$V_{b2}, V_2, V_{f2}, V_{r2}, V_{w2}, \beta_2$ = Corresponding values at outlet of the blade tip.



(*a*) (*b*)

Fig. 21.8. Velocity triangles for a centrifugal fan.

It may be seen from the above that the suffix 1 stands for the blade inlet and the suffix 2 stands for blade outlet. A little consideration will show that as the air enters and leaves the blades without any shock (or in other words tangentially), therefore the shape of the blades will be such that V_{r1} and V_{r2} are along the tangents to the blades at inlet and outlet respectively.

Let m = Mass of air flowing through the impeller in kg per second,

R_1 = Internal radius of the impeller = $D_1/2$

R_2 = External radius of the impeller = $D_2/2$, and

ω = Angular velocity of the impeller in radians per second

$$= \frac{V_{b1}}{R_1} \text{ or } \frac{V_{b2}}{R_2}$$

We know that the angular momentum entering the impeller per second

= Mass of air flowing per second × Velocity of whirl × Radius of impeller

$$= m\,V_{w1}\,R_1$$

Similarly, angular momentum leaving the impeller per second

$$= m\,V_{w2}\,R_2$$

According to Newton's second law of angular motion, the torque in the direction of motion of blades is equal to the rate of change of angular momentum.

∴ Torque in the direction of motion of blades

$$= mV_{w2}R_2 - mV_{w1}R_1 = m(V_{w2}R_2 - V_{w1}R_1)$$

and work done per second in the direction of motion of blades,

$$\text{W.D./second} = \text{Torque} \times \text{Angular velocity} = m(V_{w2}R_2 - V_{w1}R_1)\,\omega$$

Since $V_{b1} = \omega R_1$ and $V_{b2} = \omega R_2$, therefore

$$\text{W.D./second} = m(V_{w2}V_{b2} - V_{w1}V_{b1}) \quad \text{... (i)}$$

∴ Theoretical total head developed by a centrifugal fan,

$$H = \frac{\text{W.D./second}}{\text{Weight of air / second}} = \frac{m(V_{w2}V_{b2} - V_{w1}V_{b1})}{mg}$$

... (∵ Weight = mg)

$$= \frac{V_{w2}V_{b2} - V_{w1}V_{b1}}{g} \text{ (in m of air)} \quad \text{... (ii)}$$

This expression is called *Euler's equation.*

We know that total pressure developed by a centrifugal fan,

$$p_{TF} = \rho_a \times H = \frac{\rho_a(V_{w2}V_{b2} - V_{w1}V_{b1})}{g} \text{ in mm of water} \quad \text{... (iii)}$$

$$= \rho_a(V_{w2}V_{b2} - V_{w1}V_{b1}) \text{ in N/m}^2$$

where ρ_a = Mass density of air in kg / m³.

The work done per second and the theoretical head developed may further be derived by substituting the values of V_{w1}, V_{w2}, V_{b1} and V_{b2} in equation (*i*) and (*ii*) from the inlet and outlet velocity triangles. From the inlet velocity triangle, we find that

$$V_{b1} = AA' + AB = V_{w1} + V_{r1}\cos\beta_1$$

From the outlet velocity triangle,

$$V_{b2} = FG + GE = V_{w2} + V_{r2}\cos\beta_2$$

or

$$V_{w2} = V_{b2} - V_{r2}\cos\beta_2 \quad \text{... (iv)}$$

We know that

$$(V_2)^2 = (V_{b2})^2 + (V_{r2})^2 - 2(V_{b2})(V_{r2})\cos\beta_2$$

∴

$$\cos\beta_2 = \frac{(V_{b2})^2 + (V_{r2})^2 - (V_2)^2}{2\,V_{b2}\,V_{r2}}$$

Substituting the value of cos β_2 in equation (*iv*), we get

$$V_{w2} = V_{b2} - V_{r2}\left[\frac{(V_{b2})^2 + (V_{r2})^2 - (V_2)^2}{2\,V_{b2}\,V_{r2}}\right]$$

$$= V_{b2} - \left[\frac{(V_{b2})^2 + (V_{r2})^2 - (V_2)^2}{2\,V_{b2}}\right]$$

$$= \frac{2(V_{b2})^2 - (V_{b2})^2 - (V_{r2})^2 + (V_2)^2}{2\,V_{b2}} = \frac{(V_{b2})^2 - (V_{r2})^2 + (V_2)^2}{2\,V_{b2}}$$

Similarly for the inlet velocity triangle,

$$V_{w1} = \frac{(V_{b1})^2 - (V_{r1})^2 + (V_1)^2}{2V_{b1}}$$

Substituting the value of V_{w1} and V_{w2} in equation (*i*), we get

$$\text{W.D./second} = m\left[\frac{(V_{b2})^2 - (V_{r2})^2 + (V_2)^2}{2V_{b2}} \times V_{b2} - \frac{(V_{b1})^2 - (V_{r1})^2 + (V_1)^2}{2V_{b1}} \times V_{b1}\right]$$

$$= m\left[\frac{(V_{b2})^2 - (V_{b1})^2}{2} + \frac{(V_{r1})^2 - (V_{r2})^2}{2} + \frac{(V_2)^2 - (V_1)^2}{2}\right] \text{ in N-m or J}$$

and

$$H = \frac{\text{W.D / second}}{mg}$$

$$= \frac{(V_{b2})^2 - (V_{b1})^2}{2g} + \frac{(V_{r1})^2 - (V_{r2})^2}{2g} + \frac{(V_2)^2 - (V_1)^2}{2g} \text{ in m of air}$$

The *first term* of this expression represents the increase in static head which is due to the forced vortex formed by the rotating impeller. The *second term* represents that part of the static head increase which, according to Bernoulli's equation, results from the change in relative velocity of air passing through the impeller. The *third term* is the change in velocity head of the air flowing through the impeller. Since the first two terms represents the change in static head and the third term represents the velocity head, therefore H is the fan total head developed.

Note : If the speed of impeller is N r.p.m., then the blade velocity at inlet or outlet (V_{b1} or V_{b2}) may be obtained by the relations :

$$V_{b1} = \frac{\pi D_1 N}{60}, \quad \text{and} \quad V_{b2} = \frac{\pi D_2 N}{60}$$

where D_1 and D_2 are the internal and external diameters of the impeller respectively.

21.10 Work Done and Theoretical Total Head Developed by a Centrifugal Fan for Radial Entry of Air

We have seen in the previous article that the work done per second by a centrifugal fan,

$$\text{W.D./second} = m\,(V_{w2} \times V_{b2} - V_{w1} \times V_{b1}) \text{ in N-m or J}$$

and theoretical total head developed by a centrifugal fan,

$$H = \frac{1}{g}(V_{w2} \times V_{b2} - V_{w1} \times V_{b1}) \text{ in m of air}$$

When the air enters the blades at right angles (*i.e.* radially) to the direction of motion of the blade, then

$$V_1 = V_{f1} \quad \text{and} \quad V_{w1} = 0$$

∴ For radial entry of air, workdone per second,

$$\text{W.D./second} = m\,(V_{w2} \times V_{b2}) \text{ in N-m or J} \quad \text{... (i)}$$

and theoretical total head developed,

$$H = \frac{1}{g}(V_{w2} \times V_{b2}) \text{ in m of air} \quad \text{... (ii)}$$

Let Q = Quantity of air flowing through the fan in m³/s,

D_1 = Internal diameter of impeller in metres,
D_2 = External diameter of impeller in metres,
b_1 = Width of impeller at inlet in metres, and
b_2 = Width of impeller at outlet in metres.

Since the quantity of air flowing through the impeller is constant, therefore

$$Q = \pi D_1 b_1 V_{f1} = \pi D_2 b_2 V_{f2}$$

or
$$V_{f1} = \frac{Q}{\pi D_1 b_1}; \text{ and } V_{f2} = \frac{Q}{\pi D_2 b_2}$$

Now from the outlet velocity triangle as shown in Fig. 21.8,

$$\cot \beta_2 = \frac{GE}{GD} = \frac{V_{b2} - V_{w2}}{V_{f2}} \quad \text{... (iii)}$$

$$\therefore \quad V_{b2} - V_{w2} = V_{f2} \cot \beta_2 = \frac{Q \cot \beta_2}{\pi D_2 b_2}$$

and
$$V_{w2} = V_{b2} - \frac{Q \cot \beta_2}{\pi D_2 b_2}$$

Substituting the value of V_{w2} in equations (*i*) and (*ii*), we get

$$\text{W.D./second} = m\left[V_{b2} - \frac{Q \cot \beta_2}{\pi D_2 b_2}\right] V_{b2}$$

$$= m\left[(V_{b2})^2 - \frac{Q \cot \beta_2 \times V_{b2}}{\pi D_2 b_2}\right] \text{ in N-m or J} \quad \text{... (iv)}$$

and theoretical total head developed,

$$H = \frac{1}{g}\left[(V_{b2})^2 - \frac{Q \cot \beta_2 \times V_{b2}}{\pi D_2 b_2}\right] \text{ in m of air} \quad \text{... (v)}$$

Notes : 1. We know that total pressure developed by a centrifugal fan,

$$p_{TF} = \rho_a \times H = \frac{\rho_a \times V_{w2} \times V_{b2}}{g} \text{ in mm of water}$$

... [From equation (*ii*)]

$$= \rho_a \times V_{w2} \times V_{b2} \text{ in N/m}^2$$

$$= \frac{\rho_a}{g}\left[(V_{b2})^2 - \frac{Q \cot \beta_2 \times V_{b2}}{\pi D_2 b_2}\right] \text{ in mm of water}$$

... [From equation (*v*)]

$$= \rho_a\left[(V_{b2})^2 - \frac{Q \cot \beta_2 \times V_{b2}}{\pi D_2 b_2}\right] \text{ in N/m}^2$$

where ρ_a = Mass density of air in kg / m³.

2. From equation (*iii*),

$$\cot \beta_2 = \frac{V_{b2} - V_{w2}}{V_{f2}} \quad \text{or} \quad V_{w2} = V_{b2} - V_{f2} \cot \beta_2$$

It may be noted that

(*a*) For radial blades, $\beta_2 = 90°$. Thus $V_{w2} = V_{b2}$.

(*b*) For forward curved blades, β_2 is greater than 90°. Thus V_{w2} is greater than V_{b2}.

(*c*) For backward curved blades, β_2 is less than 90°. Thus V_{w2} is less than V_{b2}.

We know that the total pressure developed by a centrifugal fan is

$$p_{TF} = \rho_a \times V_{w2} \times V_{b2} \text{ in N/m}^2$$

Thus, for a given blade velocity (V_{b2}), the total pressure developed by a centrifugal fan having forward curved blades is greatest.

Example 21.2. *A centrifugal fan delivers 120 m³/min when running at 960 r.p.m. The impeller diameter is 0.7 m and the diameter at the blade inlet is 0.48 m. The air enters the impeller with a small whirl component in the direction of impeller rotation, but the relative velocity meets the blade tangentially. The impeller width at inlet is 160 mm and at outlet is 110 mm. The blades are backward curved making angles of 22.5° and 50° with the tangents at inlet and outlet respectively. Draw the inlet and outlet velocity triangles and determine the theoretical total head developed by the impeller.*

Assuming that the losses at inlet, in the impeller and in the casing amount to 70 per cent of the velocity head at impeller outlet and the velocity head at the fan discharge is 10 per cent of the velocity head at impeller outlet, calculate the fan static pressure in mm of water. Take the mass density of air to be 1.2 kg / m³ and neglect the effect of blade thickness and interblade circulation.

Solution. Given : $Q = 120 \text{ m}^3/\text{min} = 2 \text{ m}^3/\text{s}$; $N = 960$ r.p.m. ; $D_2 = 0.7$ m ; $D_1 = 0.48$ m ; $b_1 = 160 \text{ mm} = 0.16$ m ; $b_2 = 110 \text{ mm} = 0.11$ m ; $\beta_1 = 22.5°$; $\beta_2 = 50°$; $\rho_a = 1.2 \text{ kg / m}^3$

Theoretical total head developed by the impeller

Let H = Theoretical total head developed by the impeller.

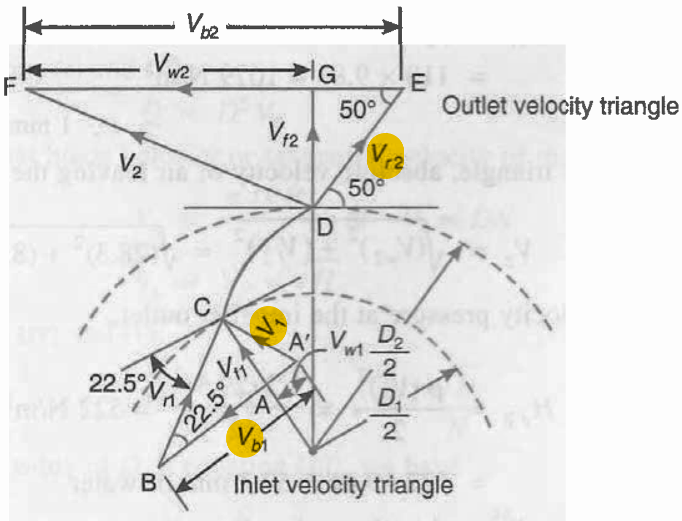


Fig. 21.9

The inlet and outlet velocity triangles are shown in Fig. 21.9. We know that blade velocity at inlet,

$$V_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.48 \times 960}{60} = 24 \text{ m / s}$$

Blade velocity at outlet,

$$V_{b2} = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.7 \times 960}{60} = 35.2 \text{ m / s}$$

Quantity of air delivered

$$Q = \pi D_1 b_1 V_{f1} = \pi D_2 b_2 V_{f2}$$

∴ Velocity of flow at inlet,

$$V_{f1} = \frac{Q}{\pi D_1 b_1} = \frac{2}{\pi \times 0.48 \times 0.16} = 8.3 \text{ m/s}$$

and velocity of flow at outlet,

$$V_{f2} = \frac{Q}{\pi D_2 b_2} = \frac{2}{\pi \times 0.7 \times 0.11} = 8.27 \text{ m/s}$$

From the inlet velocity triangle,

$$AB = AC \cot 22.5° \quad \text{or} \quad V_{b1} - V_{w1} = V_{f1} \cot 22.5°$$

$$\therefore \quad 24 - V_{w1} = 8.3 \times 2.4142 = 20 \quad \text{or} \quad V_{w1} = 24 - 20 = 4 \text{ m/s}$$

From the outlet velocity triangle,

$$GE = GD \cot 50° \quad \text{or} \quad V_{b2} - V_{w2} = V_{f2} \cot 50°$$

$$\therefore \quad 35.2 - V_{w2} = 8.27 \times 0.8391 = 6.9 \quad \text{or} \quad V_{w2} = 35.2 - 6.9 = 28.3 \text{ m/s}$$

We know that the theoretical total head developed by the impeller,

$$H = \frac{1}{g}(V_{w2} \times V_{b2} - V_{w1} \times V_{b1})$$

$$= \frac{1}{9.81}(28.3 \times 35.2 - 4 \times 24) = 91.76 \text{ m of air}$$ **Ans.**

Fan static pressure

We know that the total pressure developed by the impeller or fan,

$$p_{TF} = \rho_a H = 1.2 \times 91.76 = 110 \text{ mm of water}$$

$$= 110 \times 9.81 = 1079 \text{ N/m}^2$$

... ($\because$ 1 mm of water = 9.81 N/m^2)

From the outlet velocity triangle, absolute velocity of air leaving the blade,

$$V_2 = \sqrt{(V_{w2})^2 + (V_{f2})^2} = \sqrt{(28.3)^2 + (8.27)^2} = 29.5 \text{ m/s}$$

∴ *Velocity head or velocity pressure at the impeller outlet,

$$H_{v2} = \frac{\rho (V_2)^2}{2} = \frac{1.2 (29.5)^2}{2} = 522 \text{ N/m}^2$$

$$= 522 / 9.81 = 53.2 \text{ mm of water}$$

Losses at inlet, in the impeller and in the casing

$$= 70\% \text{ of velocity head at outlet}$$ ... (Given)

$$= 0.7 H_{v2} = 0.7 \times 53.2 = 37.24 \text{ mm of water}$$

---

* The velocity head (in mm of water) may be calculated by using the relation as discussed in Chapter 20 on 'Ducts'. We know that the velocity head or velocity pressure,

$$H_{v2} = \left(\frac{V_2}{4.04}\right)^2 = \left(\frac{29.5}{4.04}\right)^2 = 53.2 \text{ mm of water}$$

and velocity head or velocity pressure at the fan discharge

$$= 10\% \text{ of velocity head at outlet}$$

$$= 0.1\, H_{v2} = 0.1 \times 53.2 = 5.32 \text{ mm of water}$$

Now applying Bernoulli's equation to fan inlet and outlet, we have

Total pressure at fan inlet + Total pressure developed by impeller

= Losses + Total pressure at fan outlet

Since the total pressure at the fan inlet is zero and the total pressure at the fan outlet (p_{TF}) is the sum of static pressure at fan outlet or fan static pressure (p_{S2}) and the velocity pressure at fan outlet, therefore

$$0 + 110 = 37.24 + p_{S2} + 5.32$$

$$\therefore \quad p_{S2} = 110 - 37.24 - 5.32 = 67.44 \text{ mm of water} \quad \textbf{Ans.}$$

21.11 Specific Speed of a Centrifugal Fan

The specific speed of a centrifugal fan is defined as the speed of a geometrically similar fan which would deliver 1 m^3 of air per second against a head of 1 m of air. It is usually denoted by N_S.

Let Q = Total quantity of air flowing through the fan,

D = Diameter of impeller,

b = Width of impeller,

V_f = Velocity of flow,

N = Speed of impeller, and

H = Head developed by the fan.

We know that $$Q = \pi D b V_f \quad \text{or} \quad Q \propto D b V_f \quad \ldots (i)$$

Also $$D \propto b \quad \ldots (ii)$$

$\therefore$ From equations (*i*) and (*ii*),

$$Q \propto D^2 V_f \quad \ldots (iii)$$

We also know that blade velocity or tangential velocity of the impeller,

$$V_b = \frac{\pi D N}{60} \quad \text{or} \quad V_b \propto DN \quad \ldots (iv)$$

Also $$V_b \propto V_f \propto \sqrt{H} \quad \ldots (v)$$

From equations (*iv*) and (*v*),

$$DN \propto \sqrt{H} \quad \text{or} \quad D \propto \frac{\sqrt{H}}{N} \quad \ldots (vi)$$

Substituting the value of D in equation (*iii*), we have

$$Q \propto \frac{H}{N^2} \times V_f \quad \text{or} \quad Q \propto \frac{H}{N^2} \times \sqrt{H} \propto \frac{(H)^{3/2}}{N^2}$$

$$\ldots (\because V_f \propto \sqrt{H})$$

$$\therefore \quad Q = \frac{K(H)^{3/2}}{N^2} \quad \ldots (vii)$$

where K is constant of proportionality. According to the definition, if $Q = 1$ m^3/s and $H = 1$ m, then $N = N_S$. Substituting these values in equation (*vii*), we have

$$1 = \frac{K \times 1^{3/2}}{(N_S)^2} \quad \text{or} \quad K = (N_S)^2$$

Now from equation (*vii*), Q/Show that the spesific speed of centrifugal fan is

$$Q = \frac{(N_S)^2 (H)^{3/2}}{N^2} \quad \text{or} \quad N_S = \frac{N\sqrt{Q}}{H^{3/4}}$$

21.12 Fan Similarity Laws

The two fans are said to be geometrically similar when all of their wheel dimensions have the same proportionate ratios.

For any series of geometrically similar fans and for any point on their characteristic curves, the following fan laws hold :

1. The volume flow rate or capacity (Q) of a fan is directly proportional to the fan speed (N) and cube of the impeller diameter (D). In other words

$$Q \propto N \propto D^3$$

or

$$\frac{Q_1}{N_1} = \frac{Q_2}{N_2} \quad \text{and} \quad \frac{Q_1}{(D_1)^3} = \frac{Q_2}{(D_2)^3}$$

It may also be written as

$$\frac{Q_1}{N_1 (D_1)^3} = \frac{Q_2}{N_2 (D_2)^3} = \text{Constant}$$

where suffix 1 represents the actual fan and suffix 2 represents the geometrically similar fan.

2. The total pressure developed by a fan (p_{TF}) is directly proportional to the square of the fan speed (N^2), square of the impeller diameter (D^2), and density of the air (ρ_a). In other words,

$$p_{TF} \propto N^2 \propto D^2 \propto \rho_a$$

or

$$\frac{p_{TF1}}{(N_1)^2} = \frac{p_{TF2}}{(N_2)^2} \quad \text{and} \quad \frac{p_{TF1}}{(D_1)^2} = \frac{p_{TF2}}{(D_2)^2}$$

Also

$$\frac{p_{TF1}}{\rho_{a1}} = \frac{p_{TF2}}{\rho_{a2}}$$

It may also be written as

$$\frac{p_{TF1}}{(N_1)^2 (D_1)^2 \rho_{a1}} = \frac{p_{TF2}}{(N_2)^2 (D_2)^2 \rho_{a2}} = \text{Constant}$$

This is also applicable to fan static pressure (p_{SF}) and fan velocity pressure (p_{VF}).

We know that the total pressure developed by the fan,

$$p_{TF} = \rho_a H$$

where H is the total head developed by the fan,

$$\therefore \quad \frac{\rho_{a1} H_1}{(N_1)^2 (D_1)^2 \rho_{a1}} = \frac{\rho_{a2} H_2}{(N_2)^2 (D_2)^2 \rho_{a2}} \quad \text{or} \quad \frac{H_1}{(N_1)^2 (D_1)^2} = \frac{H_2}{(N_2)^2 (D_2)^2} = \text{Constant}$$

3. The power (P) of a fan is directly proportional to the cube of the fan speed (N^3), fifth power of the impeller diameter (D^5) and density of the air (ρ_a). In other words,

$$P \propto N^3 \propto D^5 \propto \rho_a$$

or

$$\frac{P_1}{(N_1)^3} = \frac{P_2}{(N_2)^3} \quad \text{and} \quad \frac{P_1}{(D_1)^5} = \frac{P_2}{(D_2)^5}$$

Also $$\frac{P_1}{\rho_{a1}} = \frac{P_2}{\rho_{a2}}$$

It may also be written as

$$\frac{P_1}{(N_1)^3 (D_1)^5 \rho_{a1}} = \frac{P_2}{(N_2)^3 (D_2)^5 \rho_{a2}} = \text{Constant}$$

4. The efficiency (η) is constant. In other words,

$$\eta_1 = \eta_2 = \text{Constant}$$

Example. 21.3. *A fan for the ventilation plant is to be exported to an area where the air density is 0.96 kg / m³ and is scheduled to deliver 6 m³/s against a static pressure of 50 mm of water, with a static efficiency of 65 per cent.*

If it is driven by a constant speed motor, calculate the static pressure and shaft power in the maker's works where the air density is 1.2 kg / m³.

Solution. Given : ρ_1 = 0.96 kg / m³ ; Q_1 = 6 m³/s ; p_{SF1} = 50 mm of water ; η_S = 65% = 0.65 ; ρ_2 = 1.2 kg / m³

Static pressure in the maker's works

case (2)= Ps(static presure) is properational with p(density)

Let p_{SF2} = Static pressure in the maker's works.

We know that $$\frac{p_{SF1}}{\rho_1} = \frac{p_{SF2}}{\rho_2}$$

$$\therefore \quad p_{SF2} = p_{SF1} \times \frac{\rho_2}{\rho_1} = 50 \times \frac{1.2}{0.96} = 62.5 \text{ mm of water}$$

$$= 62.5 \times 9.81 = 613 \text{ N/m}^2 \quad \textbf{Ans.}$$

... ($\because$ 1 mm of water = 9.81 N/m²)

Shaft power in the maker's works

Since the speed (N) and diameter of impeller (D) is constant, therefore from the relation

$$\frac{Q_1}{N_1 (D_1)^3} = \frac{Q_2}{N_2 (D_2)^3}, \text{ we get } Q_1 = Q_2 = 6 \text{ m}^3/\text{s}$$

We know that static fan air power,

$$P_{aS2} = Q \times p_{SF2} = 6 \times 613 = 3678 \text{ N-m / s or W}$$

... ($\because$ 1 N-m/s = 1 W)

$\therefore$ Shaft power in the maker's works,

$$P_{S2} = \frac{P_{aS2}}{\eta_S} = \frac{3678}{0.65} = 5660 \text{ W} = 5.66 \text{ kW} \quad \textbf{Ans.}$$

Important{

Example 21.4. *A fan of diameter 0.7 m running at 1500 r.p.m. delivers 140 m³/min of air at 15°C against 75 mm of water of total pressure when its total efficiency is 86 per cent. Determine the volume of air delivered, total pressure developed and power consumed, if*

(a) the air temperature is 50°C,

(b) the air temperature is 50°C and the fan speed is increased to 1700 r.p.m., and

(c) the conditions are same as in (b) but a 0.6 m diameter, geometrically similar fan is used.

Solution. Given : D_1 = 0.7 m ; N_1 = 1500 r.p.m ; Q_1 = 140 m³/min ; t_1 = 15°C ; p_{TF1} = 75 mm of water ; η_T = 86% = 0.86 ; t_2 = 50°C

First of all, let us find the ratio of the densities at temperatures 15°C and 50°C.

Let ρ_{a1} = Density of air at 15°C, and

ρ_{a2} = Density of air at 50°C.

We know that for the constant barometric pressure, the density of air at 50°C,

$$\rho_{a2} = \rho_{a1}\left(\frac{273+t_1}{273+t_2}\right)$$

$$\therefore \quad \frac{\rho_{a1}}{\rho_{a2}} = \frac{273+t_2}{273+t_1} = \frac{273+50}{273+15} = 1.12$$

(a) Volume of air delivered, total pressure developed and power consumed when the air temperature is 50°C

Let Q_2 = Volume of air delivered,

p_{TF2} = Total pressure developed, and

P_2 = Power consumed.

We know that

$$\frac{Q_1}{N_1(D_1)^3} = \frac{Q_2}{N_2(D_2)^3}$$

Since there is no change in speed or diameter, therefore

$$Q_2 = Q_1 = 140 \text{ m}^3/\text{min} \textbf{ Ans.}$$

Now $$\frac{p_{TF1}}{p_{TF2}} = \frac{\rho_{a1}}{\rho_{a2}}$$

$$\therefore \quad p_{TF2} = p_{TF1} \times \frac{\rho_{a2}}{\rho_{a1}}$$

$$= 75 \times \frac{1}{1.12} = 67 \text{ mm of water}$$

$$= 67 \times 9.81 = 657.3 \text{ N/m}^2 \textbf{ Ans.}$$

Inline centrifugal fans.

We know that power consumed,

$$P_2 = Q_2 \times p_{TF2} \times \frac{1}{\eta} = 140 \times 657.3 \times \frac{1}{0.86}$$

$$= 107\,002 \text{ N-m/min} = 107\,002 / 60 = 1783.4 \text{ W} \textbf{ Ans.}$$

(b) Volume of air delivered, total pressure developed and power consumed when air temperature is 50°C and speed is increased to 1700 r.p.m.

Let Q_3 = Volume of air delivered,

p_{TF3} = Total pressure developed,

P_3 = Power consumed, and

N_3 = Increased speed = 1700 r.p.m. ... (Given)

We know that $$\frac{Q_3}{N_3} = \frac{Q_2}{N_2}$$

$$\therefore \quad Q_3 = Q_2 \times \frac{N_3}{N_2} = 140 \times \frac{1700}{1500} = 158.7 \text{ m}^3/\text{min} \textbf{ Ans.}$$

... (Here $N_2 = N_1$)

Now $$\frac{p_{TF3}}{(N_3)^2} = \frac{p_{TF2}}{(N_2)^2}$$

$$\therefore \quad p_{TF3} = p_{TF2}\left(\frac{N_3}{N_2}\right)^2 = 67\left(\frac{1700}{1500}\right)^2 = 86 \text{ mm of water} \textbf{ Ans.}$$

and $$\frac{P_3}{(N_3)^3} = \frac{P_2}{(N_2)^3}$$

∴ $$P_3 = P_2\left(\frac{N_3}{N_2}\right)^3 = 1783.4\left(\frac{1700}{1500}\right)^3 = 2596 \text{ W } \textbf{Ans.}$$

(c) Volume of air delivered, total pressure developed and power consumed for the conditions as in (b) and when diameter is 0.6 m

Let Q_4 = Volume of air delivered,

p_{TF4} = Total pressure developed,

P_4 = Power consumed, and

D_4 = New impeller diameter = 0.6 m . . . (Given)

We know that $$\frac{Q_4}{(D_4)^3} = \frac{Q_3}{(D_3)^3}$$

∴ $$Q_4 = Q_3\left(\frac{D_4}{D_3}\right)^3 = 158.7\left(\frac{0.6}{0.7}\right)^3 = 99.94 \text{ m}^3/\text{min } \textbf{Ans.}$$

... (Here $D_3 = D_1$)

Now $$\frac{p_{TF4}}{(D_4)^2} = \frac{p_{TF3}}{(D_3)^2}$$

∴ $$p_{TF4} = p_{TF3}\left(\frac{D_4}{D_3}\right)^2 = 86\left(\frac{0.6}{0.7}\right)^2 = 63.2 \text{ mm of water } \textbf{Ans.}$$

and $$\frac{P_4}{(D_4)^5} = \frac{P_3}{(D_3)^5}$$

∴ $$P_4 = P_3\left(\frac{D_4}{D_3}\right)^5 = 2596\left(\frac{0.6}{0.7}\right)^5 = 1201 \text{ W } \textbf{Ans.}$$

21.13 Fan and System Characteristic

We have already discussed that all of the duct work elements such as elbows, tees, registers, dampers etc., offer resistance to the flow of air and cause loss in pressure. The change in pressure loss or resistance with the change in flow rate is called *system characteristic*. Any air-conditioning or ventilating system that has a duct work, heating and cooling coils, dampers, registers etc. has a definite system characteristic. The system characteristic is independent of the fan used in that system.

We have seen in the previous chapter that the system resistance or pressure loss of any fixed system varies as the square of the flow rate, *i.e.*

$$R \text{ or } p_L = KQ^2$$

If the resistance of a system is plotted against the varying amounts of flow rates, a

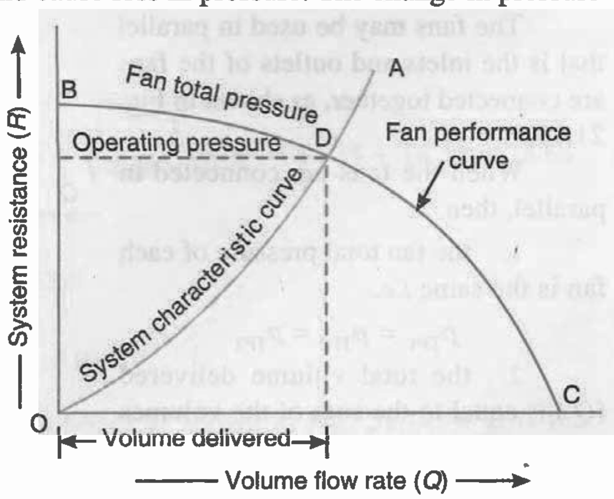


Fig. 21.10. Fan and system characteristic.

System curve (or duct system)represents the variation of pressure plotted against volume flow rate

Fan curve represents fan performance variable plotted against volume flow rate

point of operation is the single point at the intersection of the fan curve and system curve