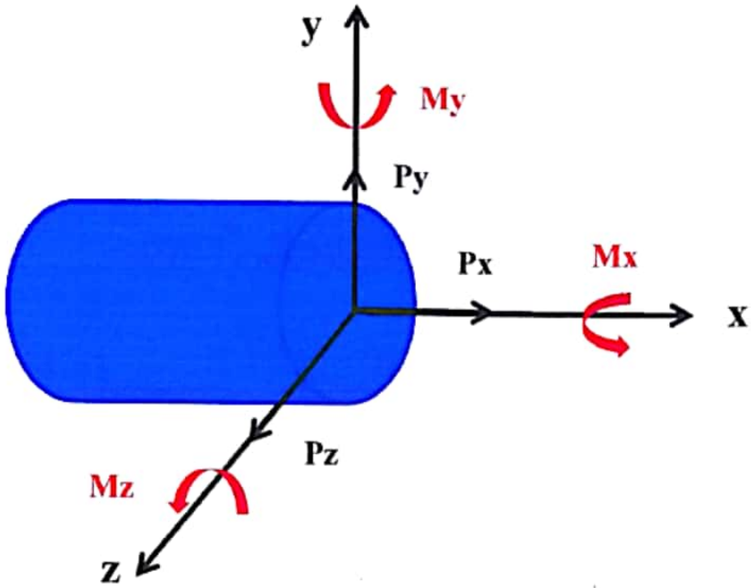
Internal Forces

For any equilibrium body, if we take any section perpendicular to its longitudinal axis, a system of six internal forces will be obtained, as shown in fig. These forces can be classified into four types:



1- Axial force or normal force (Px): represents a tensile or compressive force in (X) direction (acts perpendicular to that cross-section) and usually denoted by (N).

2- Shear force (Py and Pz): acts in plane of that cross-section (i.e. tangential), usually denoted by (V), and tends to slid the two parts of the body.

$$VR = \sqrt{P^2y + P^2z}$$

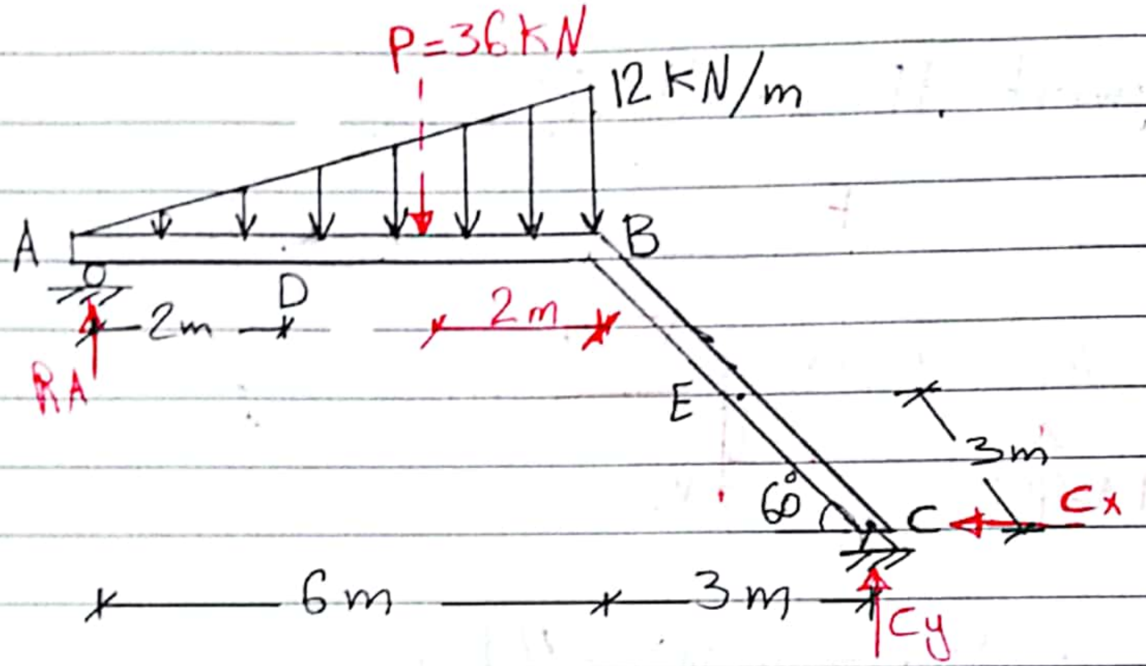
3-Torsional moment (or torque) Mx: tends to twist one segment of the body with respect to other, and usually denoted by (M_t or T).

4- Bending moment (My, Mz): that tends to bend the body about (Y) or (Z) axis.

$$MR = \sqrt{M^2y + M^2z}$$

Ex1: For the structure shown in Fig., determine the internal forces and their types acting on the cross-section of (D) and (E).

Sol



★ find external reactions:

$\sum F_x = 0 \longrightarrow \boxed{CX = 0}$

The whole structure as F.B.D.:

$\circlearrowleft^{+} \sum M_c = 0$

$RA * 9 - P * 5 = 0$

$RA * 9 - (\frac{1}{2} * 6 * 12) * 5 = 0 \longrightarrow \boxed{RA = 20 \text{ KN} \uparrow}$

$+\uparrow \sum F_y = 0$

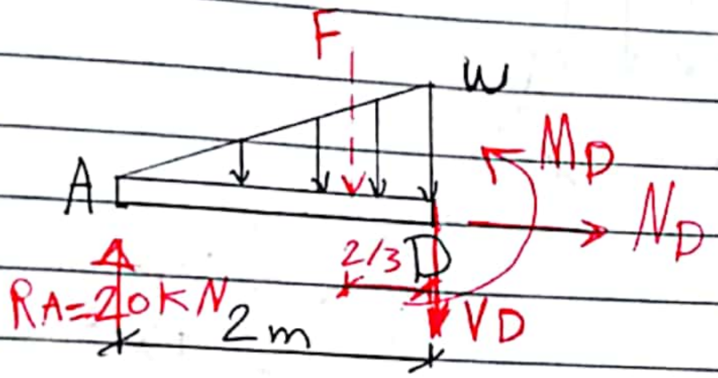
$RA + Cy - P = 0$

$20 + Cy - 36 = 0 \longrightarrow \boxed{Cy = 16 \text{ KN} \uparrow}$

* Take the considered Section and put the internal forces:

1- cross-section of (D):

Segment AD as f.B.D



$$\frac{12}{6} = \frac{w}{2} \Longrightarrow w = 4\,kN$$

$$F = \frac{1}{2} * 2 * 4 = 4\,kN$$

$\Sigma F_x = 0 \Longrightarrow \boxed{N_D = 0}$ (no axial force).

$+\uparrow \Sigma F_y = 0 \Longrightarrow R_A - V_D - F = 0$

$20 - V_D - 4 = 0 \Longrightarrow \boxed{V_D = 16\,kN}$ (shear force)

$\circlearrowleft + \Sigma M_D = 0$

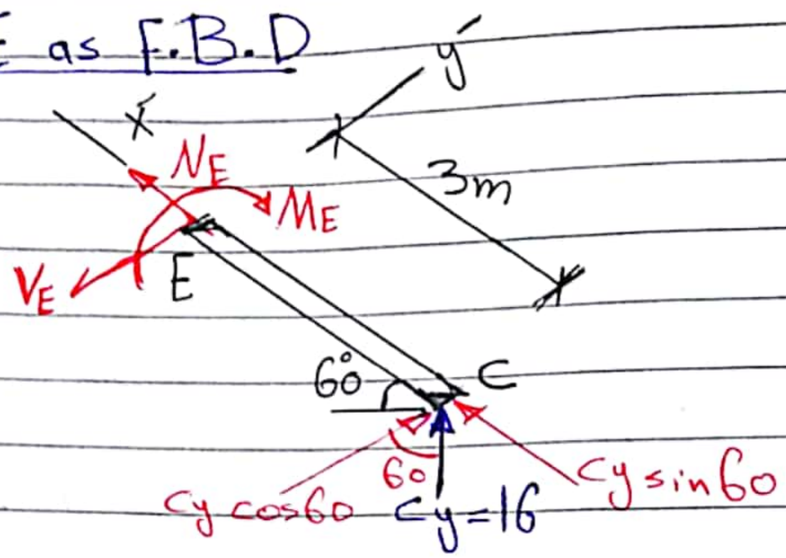
$$M_D - R_A * 2 + F * \frac{2}{3} = 0$$

$$M_D - 20 * 2 + 4 * \frac{2}{3} = 0$$

$\therefore \boxed{M_D = 37.33\,kN.m}$ (bending moment)

2- cross-section of (E):

Segment CE as F.B.D



$+\nwarrow \sum F_{x'} = 0$

$N_E + C_y \cdot \sin 60° = 0$

$\therefore$ $\boxed{N_E = -13.86 \text{ KN}}$ (axial compressive force)

$+\swarrow \sum F_{y'} = 0$

$V_E - C_y \cdot \cos 60° = 0$

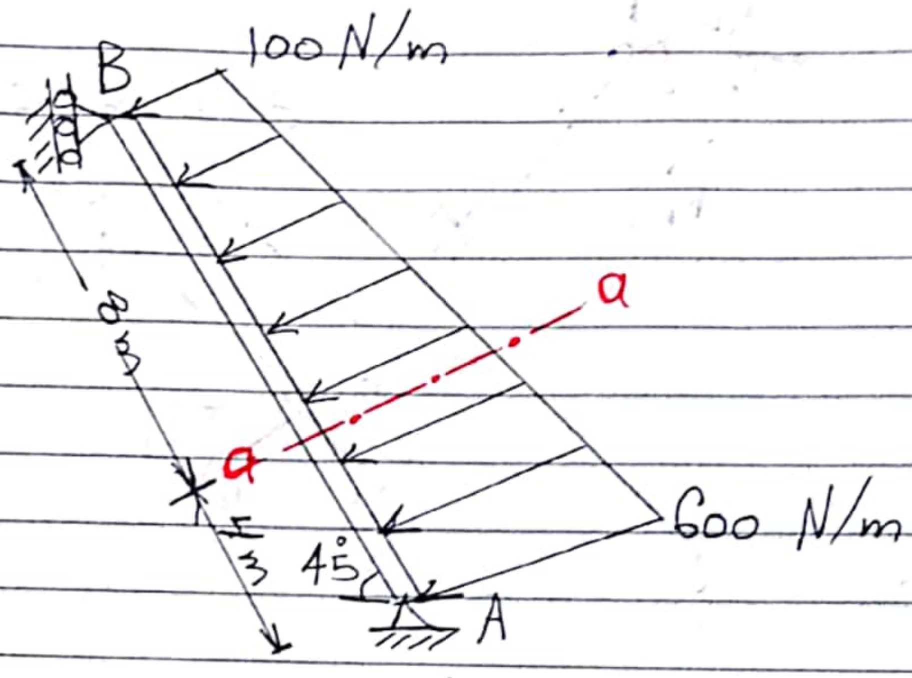
$\therefore$ $\boxed{V_E = 8 \text{ KN}}$ (shear force)

$+\circlearrowright \sum M_E = 0$

$M_E - (C_y \cos 60) \times 3 = 0$

$\therefore$ $\boxed{M_E = 24 \text{ KN.m}}$ (Bending mement).

Ex 2 : Determine the internal forces acting on sec (a-a) of point (C), for the beam shown below.



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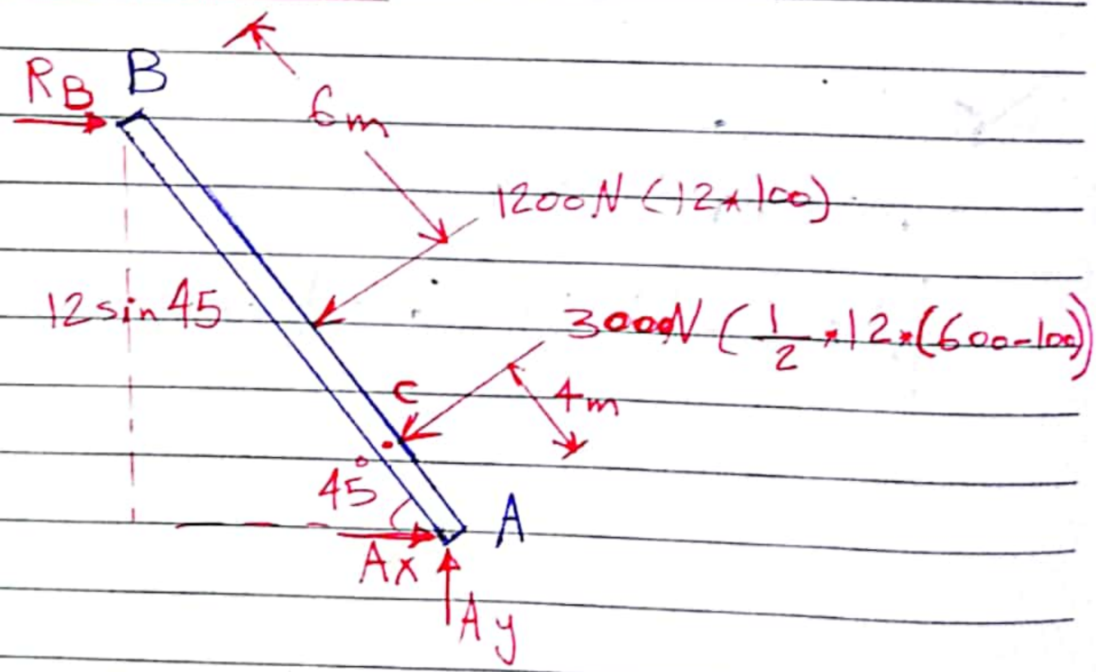
Beam AB as F.B.D

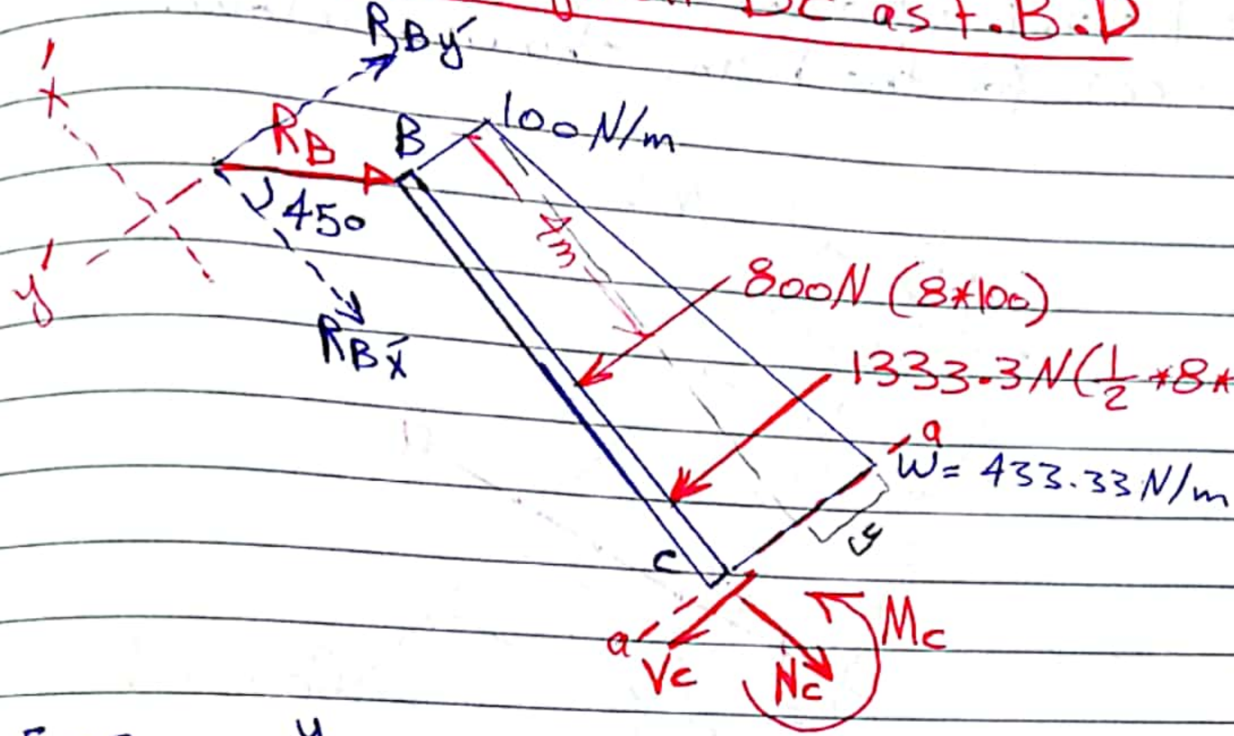


$\circlearrowleft^{+} \sum M_A = 0$

$R_B (12 * \sin 45°) - 1200 * 6 - 3000 * 4 = 0$

$\therefore \boxed{R_B = 2263.10 \text{ N}}$

For Sec (a-a) Segment BC as F.B.D



$$\frac{500}{12} = \frac{y}{8} \Rightarrow y = 333.33 \ N/m$$

$$w = y + 100 \Rightarrow \boxed{w = 433.33 \ N/m}$$

$+\searrow \sum F_{x'} = 0$

$$N_c + R_B * \cos 45° = 0$$

$\boxed{N_c = -1600 \ N}$ (axial compressive force)

$\nearrow^{+} \sum F_{y'} = 0$

$R_B \cdot \sin 45° - V_C - 800 - 1333.3 = 0$

$\boxed{V_C = -533.30\,N}$

$\circlearrowleft^{+} \sum M_C = 0$

$M_C + 1333.3\left(\overbrace{8 - \left(\frac{2}{3} * 8\right)}^{\frac{1}{3} * 8}\right) + 800 * 4 - \left(R_B * 8 \cdot \sin 45\right) = 0$

$M_C + 1333.3(8 - 5.333) + 800 * 4 - (2263.1 * 8 * \sin 45) = 0$

$\boxed{M_C = 6044.20\,N \cdot m}$