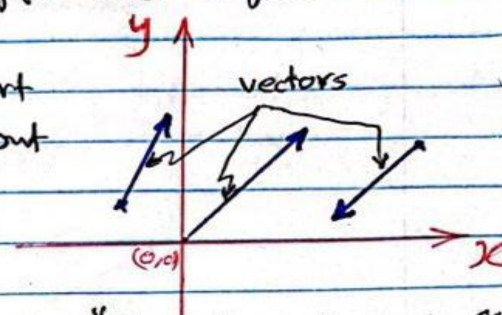
Vectors :- المتجهات

- The vector is only two pieces of information:
1- Direction
2- Length or Magnitude

المتجه عبارة عن مكونين وهما : اتجاه وطول أو كمية .

- We can graph a vector by an arrow that we can visualize on x-y plane and we can capture it by the arrow length and angle

- Vectors on graph could start From not just an origion, but From any where -



"Examples of Vectors"

Examples of Vectors :-

- To answer the question "What is the current temperature?" We use a single number (scalar); likewise the question about a mass ;
- While to answer the question "What is the current velocity of the wind?", we need more than just a single number. We need magnitude (speed) and direction. This where vectors come to handy.

Position, displacement, velocity, acceleration, force, momentum & torque are all physical quantities that can be represented mathmatically by vectors.

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Vector Denoting :-

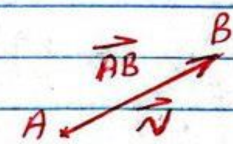
- Vectors are writing with arrow on top on equations.

Ex)

- Velocity vector $\longrightarrow \vec{v}$
- Force vector $\longrightarrow \vec{F}$

Note) Any variable symbol with no arrow on top means scalar.

- A vector can be geometrically represented by a direction line segment with a head & a tail ;



so vector $\vec{AB}$ is a vector from point A to B.

- Also, we can denote vector $\vec{AB}$ by a small case letter $\vec{v}$
- The length of the arrow A B corresponds to the magnitude of the vector.
- The arrow points in the direction of the vector.

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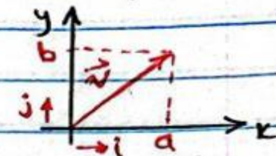
How to represent the vector mathmatically :-

Vector in plane :- كيف تمثل المتجه رياضياً

We can write vectors as Columns. Let us take a very important special vector as example :-

$$i = \begin{pmatrix}1\\0\end{pmatrix} \quad , \quad j = \begin{pmatrix}0\\1\end{pmatrix}$$

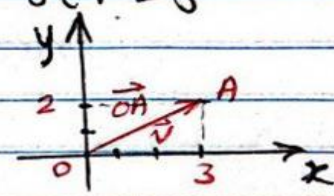


Any vector in the xy-plane can be written in terms of i & j using the triangle law & scalar multiplication.

$$\vec{v} = ai + bj = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$$

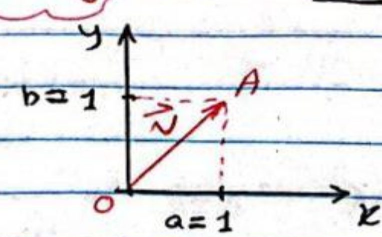
Ex)

$$\begin{pmatrix}3\\2\end{pmatrix} = 3\begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}0\\1\end{pmatrix} = 3i + 2j$$



Notes)

* IF $a = b = 1$, then $\vec{v} = i + j$ is a "unit vector"



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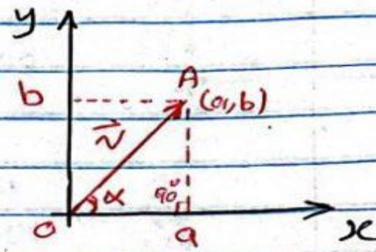
Finding the length/magnitude and the direction of vector :- إيجاد طول (مقدار) و اتجاه المتجه

IF $\vec{V} = a\,i + b\,j$ ——(1), then the length/magnitude of a vector $\vec{V}$ is :-



$$|V| = \sqrt{a^2 + b^2} \quad \text{——(2)}$$

(نظرية فيثاغورس)

- It's a Pythagorean theorem

$$\left.\begin{aligned} a &= |V| \cos\alpha \\ b &= |V| \sin\alpha \end{aligned}\right\} \text{——(3)}$$

$$\tan\alpha = \frac{b}{a}$$

Substitute eq. (3) in (1) yields ;

$$\vec{V} = |V| (\cos\alpha\, i + \sin\alpha\, j)$$

$\vec{V}$ — vector symbol
$|V|$ — vector length
i, j — unit vector components (basis/Fundamental vector components)
α — vector angle with x-axis

Ex/ Find a vector in plane of length (7 units) & makes angle (35°) with x-axis ?

Solution

since $|\vec{V}| = 7$ & $\alpha = 35^\circ$

$\therefore \vec{V} = 7 \times (\cos 35\, i + \sin 35\, j)$

$\therefore \boxed{\vec{V} = 5.7\, i + 4\, j}$ Ans

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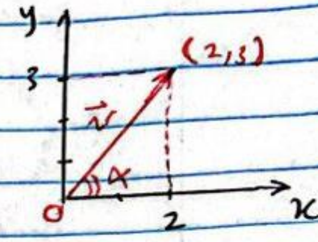
Ex) Find the angle between the vector $\vec{v} = 2i + 3j$ and the x-axis?

Solution



$|\vec{v}| = \sqrt{a^2+b^2}$; $a = 2$, $b = 3$

$|\vec{v}| = \sqrt{2^2+3^2} = \sqrt{13}$

$a = |v| \cos\alpha \Longrightarrow \cos\alpha = \dfrac{a}{|v|} = \dfrac{2}{\sqrt{13}}$

$\therefore \alpha = \cos^{-1} \dfrac{2}{\sqrt{13}} = \boxed{56.3^\circ}$ Ans

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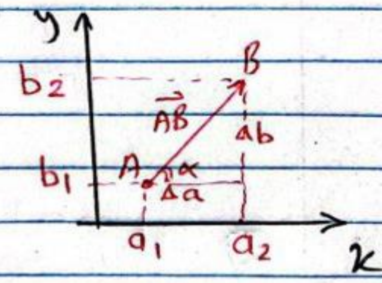
Vectors with tail not in origin :-

المتجهات بنقطة ابتداء ليس ع (0,0)

Vectors can be start not from the origin, but from any where, like A to B



$\therefore \vec{AB} = \Delta a\, i + \Delta b\, j$

$\vec{AB} = (a_2 - a_1)\, i + (b_2 - b_1)\, j$

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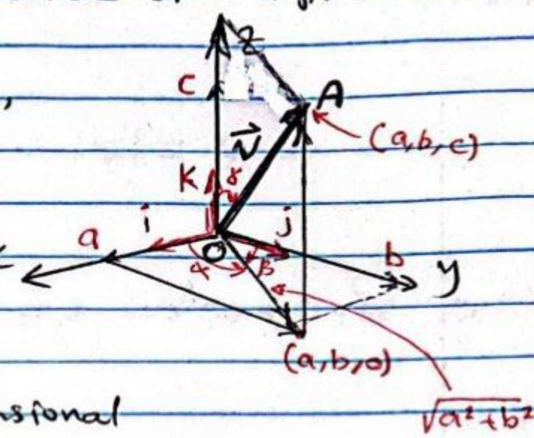
Vectors in a space :-

- This could be in three or higher dimensions



- Similar to the 2-D case, but we now have three basis vectors i, j & k.
- From these three components unit vectors we can describe any vector in three-dimensional space.

$$\vec{V} = \vec{OA} = a\,i + b\,j + c\,K,$$

where :-

i, j, K = Basis or Fundamental unit vector.
a, b, c = Directional numbers (scalars).
α, β, γ = Directional angles.

$$|V| = |OA| = \sqrt{a^2+b^2+c^2},$$

$$a = |V| * \cos\alpha$$
$$b = |V| * \cos\beta$$
$$c = |V| * \cos\gamma$$

$$\frac{\vec{V}}{|V|} = \cos\alpha\, i + \cos\beta\, j + \cos\gamma\, K$$ = unit vector with direction of $\vec{V}$

And,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Ex) Find a vector in space of length (5 units) that makes angles (70°) with x-axis, (85°) with y-axis?

Solution

$\alpha = 70^\circ$, $\beta = 85^\circ$, $|v| = 5$

$\gamma = ?$, $\vec{v} = ?$

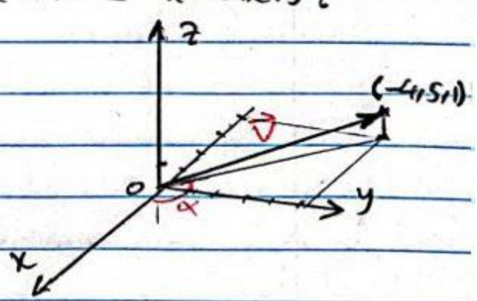
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \cos^2 70 + \cos^2 85 + \cos^2\gamma = 1$$

$$\therefore \boxed{\cos\gamma = 0.935}$$

$$\therefore \vec{v} = |v|(\cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k)$$
$$= 5(\cos 70\, i + \cos 85\, j + 0.935\, k)$$
$$\boxed{\vec{v} = 1.7\, i + 0.436\, j + 4.675\, k}$$ Ans

Ex) Find the angle between the vector $\vec{v} = -4i + 5j + k$ and the x-axis?

Solution



$a = -4$, $b = 5$, $c = 1$

$$|v| = \sqrt{a^2 + b^2 + c^2}$$

$$|v| = \sqrt{(-4)^2 + (5)^2 + (1)^2} = \sqrt{42}$$

$$\cos\alpha = \frac{a}{|v|} \Rightarrow \alpha = \cos^{-1}\frac{a}{|v|} = \cos^{-1}\frac{-4}{\sqrt{42}}$$

$$\boxed{\alpha = 128^\circ}$$

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Scalar product :- (Dot product)
ضرب القيم العددية

Let $\vec{A} = a_1 i + a_2 j + a_3 k$

And $\vec{B} = b_1 i + b_2 j + b_3 k$

Then, $\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta$

where θ — is the angle between $\vec{A}$ & $\vec{B}$

Properties :-

1- $\vec{A} \cdot \vec{A} = |A|^2$

2- $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = a_1 b_1 + a_2 b_2 + a_3 b_3$

3- $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|A|\,|B|} = \dfrac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} + \sqrt{b_1^2 + b_2^2 + b_3^2}}$

4- $\vec{A} \perp \vec{B} \Longrightarrow \vec{A} \cdot \vec{B} = 0$ [Orthogonal Vectors]
متجهات متعامدة

5- $ai + bj \perp bi - aj$

EX) Find the angle "θ" between $\vec{A} = i - 2j - 2k$ & $\vec{B} = 6i + 3j + 2k$?

Solution

$\vec{A} \cdot \vec{B} = (1 * 6) + (-2 * 3) + (-2 * 2) = \boxed{-4}$

$|A| = \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{9} = \boxed{3}$

$|B| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = \boxed{7}$

$\Longrightarrow |A|\,|B| = \boxed{21}$

$\therefore \cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|A|\,|B|} \Longrightarrow \theta = \cos^{-1} \dfrac{-4}{21} \approx \boxed{101^\circ}$ Ans

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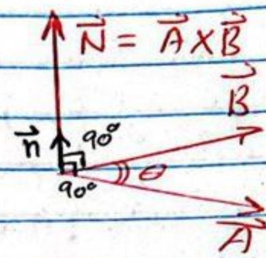
Vector Product :- (Cross product)

ضرب المتجهات

Normal vector is what yields from vector product or cross product.



$$\boxed{\vec{N} = \vec{A} \times \vec{B} = \vec{n}\,|A|\,|B| \sin\theta}$$

where $\vec{n}$ -- is a normal unit vector

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$; where, $\vec{A} = a_1 i + a_2 j + a_3 k$, $\vec{B} = b_1 i + b_2 j + b_3 k$

Properties :-

1- $\vec{A} \times \vec{A} = 0 \longrightarrow$ "$\sin 0 = 0$"

2- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

3- $\vec{A} \parallel \vec{B} \Longrightarrow \vec{A} \times \vec{B} = 0 \longrightarrow \sin 0 = 0$

4- Area of $\Delta ABC = \frac{1}{2} |\vec{A} \times \vec{B}|$

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Ex) Find $\vec{A} \times \vec{B}$ & $\vec{B} \times \vec{A}$ if

$\vec{A} = 2i + j + k$

$\vec{B} = -4i + 3j + k$

Solution)

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 1 \\ -4 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ -4 & 3 \end{vmatrix} k$$

$$= (1\times1 - (3\times1))\, i - (2\times1 - (-4\times1))\, j + (2\times3 - (-4\times1))\, k$$

$$\boxed{\vec{A} \times \vec{B} = -2i - 6j + 10k}$$

but,

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = \boxed{2i + 6j - 10k}$$

Triple Product :- الضرب الثلاثي المكرر

A- Scalar triple product :-

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C}$$

Note)

1- Box Volume is $\Longrightarrow$ $V_{box} = |\vec{A} \cdot \vec{B} \times \vec{C}|$

2- Pyramid Volume is $\longrightarrow$ $V_{Py} = \frac{1}{6} |\vec{A} \cdot \vec{B} \times \vec{C}|$

B- Vector triple Product :-

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \times B - (\vec{A} \cdot \vec{B})\, \vec{C}$$

Note)

$i \cdot i = j \cdot j = k \cdot k = 1$; $i \times i = j \times j = k \times k = 0$

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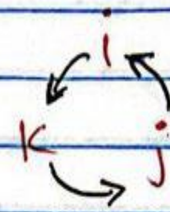
$i \cdot j = j \cdot k = k \cdot i = 0$



$i \times j = k$

$j \times k = i$

$k \times i = j$

HW#2

1- Find the Length & direction of these vectors & the angles make with the x-axis?

a- $5i + 12j$

b- $\sqrt{3}i + j$

2- Find a vector 6 units long in the direction of $\vec{A} = 2i + 2j - k$

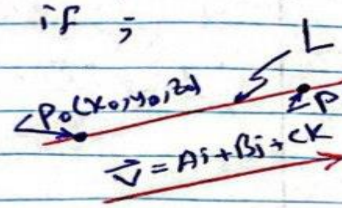
3- Find the area of the triangle whose vertices are $A(1,-1,0)$, $B(2,1,-1)$, & $C(-1,1,2)$?

4- If $\vec{A} = 2i - j$ & $B = i + 3j - 2k$, find $A \times B$, then calculate $(A \times B) \cdot A$?

Equation of line in a space معادلة الخط المستقيم في الفضاء

If L is a line in a space that passes through a point $P_0(x_0, y_0, z_0)$ & it parallel to a vector $\vec{V} = Ai + Bj + CK$, Then $P(x, y, z)$ is any point lies on L only if ;

$$\overrightarrow{P_0P} = t\vec{V} \quad \text{———} \quad (1)$$



Where ;
t — time parameter

So eq. (1) can be written as ;

$$(x - x_0)i + (y - y_0)j + (z - z_0)K = t(Ai + Bj + CK)$$

$$x - x_0 = At \longrightarrow x = At + x_0$$
$$y - y_0 = Bt \longrightarrow y = Bt + y_0$$
$$z - z_0 = Ct \longrightarrow z = Ct + z_0$$

Eq. of line in a space

Ex1

Find parametric equations for the line through the point $(-2, 0, 4)$ parallel to the vector $\vec{V} = 2i + 4j - 2K$

Solution

$$P_0(x_0, y_0, z_0) = (-2, 0, 4)$$

$$Ai + Bj + CK = 2i + 4j - 2K$$

$\therefore$ $\boxed{x = 2t - 2}$

$\boxed{y = 4t}$, $\boxed{z = -2t + 4}$

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Ex)

Find parametric equations for line through the points $P(-3,2,-3)$ & $Q(1,-1,4)$?

(Solution)

$$\overrightarrow{PQ} = (1-(-3))\,i + (-1-2)\,j + (4-(-3))\,k$$

$$\boxed{\overrightarrow{PQ} = 4i - 3j + 7k} = \boxed{Ai + Bj + Ck}$$

* let $(x_0, y_0, z_0) = (-3, 2, -3)$

$$\therefore \boxed{x = 4t - 3} \; ; \; \boxed{y = -3t + 2} \; ; \; \boxed{z = 7t - 3}$$

or;

* let $(x_0, y_0, z_0) = (1, -1, 4)$

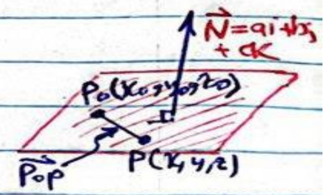
$$\therefore \boxed{x = 4t + 1} \; ; \; \boxed{y = -3t + 1} \; ; \; \boxed{z = 7t + 4}$$

Equation of the plane : معادلة المستوي

To find the eq^n of the plane that passes through the point $P_0(x_0, y_0, z_0)$ & it's normal vector is $\vec{N} = ai + bj + ck$

Let $p(x,y,z)$ be any point in the plane

$$\overrightarrow{P_0P} = (x-x_0)\,i + (y-y_0)\,j + (z-z_0)\,k$$



$$\overrightarrow{P_0P} \perp N \Longrightarrow \overrightarrow{P_0P} \cdot \vec{N} = 0$$

$$\Longrightarrow a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$\Longrightarrow ax + by + cz = ax_0 + by_0 + cz_0$$

or

$$ax + by + cz = d \longleftarrow \text{Equation of the plane}$$

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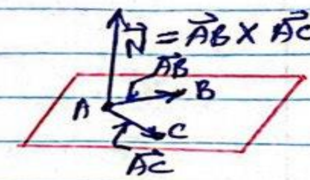
Ex)

Find the equation of the plane having the points $A(2,3,5)$, $B(7,2,1)$ & $C(1,1,1)$

Solution



في هذا المثال توجد نقاط ثلاث ،
من هذه النقاط الثلاث يمكننا ايجاد
متجهين اثنين ، وليكونا $\vec{AC}$ & $\vec{AB}$

ولكننا نحتاج $\vec{N}$ فهنا نستطيع ايجاد
$\vec{N}$ من خلال عمل Cross prod. أو vector prod.
بين $\vec{AB}$ و $\vec{AC}$ وكما في التالي:

$$\vec{AB} = (7-2)i + (2-3)j + (1-5)k$$

$$\boxed{\vec{AB} = 5i - j - 4k}$$

$$\vec{AC} = (1-2)i + (1-3)j + (1-5)k$$

$$\boxed{\vec{AC} = -i - 2j - 4k}$$

$$\therefore \vec{N} = \vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 5 & -1 & -4 \\ -1 & -2 & -4 \end{vmatrix} = i\begin{vmatrix} -1 & -4 \\ -2 & -4 \end{vmatrix} - j\begin{vmatrix} 5 & -4 \\ -1 & -4 \end{vmatrix} + k\begin{vmatrix} 5 & -1 \\ -1 & -2 \end{vmatrix}$$

$$\vec{N} = \boxed{-4i + 24j - 11k}$$

$\therefore$ To find the value of "d", we do a substitution of any point A, B, or C as follows;

$ax + by + cz = d$; $d = ax_0 + by_0 + cz_0$

let $P_0 \longrightarrow C(1,1,1) \Longrightarrow d = (-4 * 1) + (24 * 1) + (-11 * 1)$

$$\boxed{d = -9}$$

$\therefore -4x + 24y - 11z = -9$ ⟵ Eqs of the plane

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Questions for discussions :-

(1) Find the parametric eqns for the lines for

a/ The line through the point $P(3,-4,-1)$ parallel to the vector $\vec{V} = i + j + k$

b/ The line through $P(-2,0,3)$ & $Q(3,5,-2)$

c/ The line through the origin parallel to vector $\vec{V} = 2j + k$

d/ The line through the point $(3,-2,1)$ parallel to the line $x = 1+2t$, $y = 2-t$ & $z = 3t$

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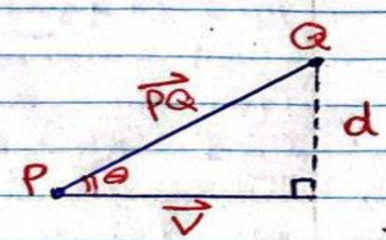
The Distance From a Point to a Line :

ايجاد المسافة من نقطة الى خط مستقيم بواسطة تطبيق المتجهات

To find a distance from a point Q to a line that passes through a point P parallel to a vector $\vec{V}$, we use the following eqs :

$$d = \frac{|\vec{PQ} \times \vec{V}|}{|\vec{V}|}$$



Ex) Find the distance from point $P(1,1,5)$ to the line $x = 1+t$, $y = 3-t$, & $z = 2t$

Solution

First of all, we need to find vector $\vec{V}$ from the eqs of the line

$$\left.\begin{aligned} x &= 1+t \\ y &= 3-t \\ z &= 2t \end{aligned}\right\} \xrightarrow[\text{with}]{\text{Compare}} \left.\begin{aligned} x &= x_0 + At \\ y &= y_0 + Bt \\ z &= z_0 + Ct \end{aligned}\right\} \Longrightarrow \vec{V} = Ai + Bj + Ck$$

$\therefore A = 1$, $B = -1$, $C = 2$.

$\therefore \vec{V} = i - j + 2K$

To find $Q(x_0, y_0, z_0)$, put $t = 0$ $\longrightarrow$
$Q = (1, 3, 0)$

$\therefore \vec{PQ} = (1-1)i + (3-1)j + (0-5)K = 2j - 5K$

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$$\vec{PQ} \times \vec{V} = \begin{vmatrix} i & j & k \\ 0 & 2 & -5 \\ 1 & -1 & 2 \end{vmatrix} = (2\times 2 - (-1\times 5))i - (0\times 2 - (1\times 5))j + (0\times(-1) - 1\times 2)k$$

$$\vec{PQ} \times \vec{V} = -i - 5j - 2k$$

$$|\vec{PQ} \times \vec{V}| = \sqrt{(-1)^2 + (-5)^2 + (-2)^2} = \sqrt{30}$$

$$|\vec{V}| = \sqrt{1^2 + (-1)^2 + (2)^2} = \sqrt{6}$$

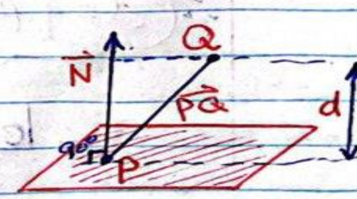
$$\therefore d = \frac{|\vec{PQ} \times \vec{V}|}{|\vec{V}|} = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{\frac{\cancel{30}^{5}}{\cancel{6}}} = \boxed{\sqrt{5}}$$

The Distance from a point to a plane:

ايجاد المسافة من نقطة الى مستوي بواسطة المتجهات

If P is a point with normal $\vec{N}$, then the distance from any point (Q) to the plane is the length of the vector projection of $\vec{PQ}$ onto $\vec{N}$

$$d = |\text{Proj}_{\vec{N}} \vec{PQ}| = \left| \frac{\vec{PQ} \cdot \vec{N}}{|\vec{N}|} \right|$$



Ex//

Find the distance from the point $P(3, 1, 3)$ to the plane whose eq. is $3x - 5y + z = 4$

Solution//

First we need to find a point on a plane, so, put $x = y = 0$ into the given eq. of the plane yields

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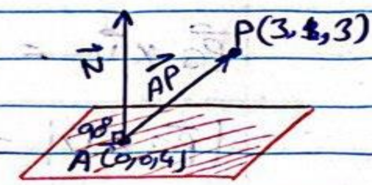
$3(0) - 5(0) + z = 4 \longrightarrow \boxed{z = 4}$

$\therefore$ point $A(0, 0, 4)$ is satisfied the eq. of the plane.

- lets Find a vector $\overrightarrow{AP}$



$$\overrightarrow{AP} = (3-0)\,i + (1-0)\,j + (3-4)\,k$$

$$\boxed{\overrightarrow{AP} = 3i + j - k}$$

From the eq. of the plane (given), we can Find normal vector $\vec{N}$, as Follows

The typical eq. of the plane is

$$ax + by + cz = d \quad \& \quad \vec{N} = ai + bj + ck$$

$\because$ the eq. of the plane is $\rightarrow 3x - 5y + z = 4$

$$\vec{N} = 3i - 5j + k$$

$$\therefore d = \left|\text{proj}_{\vec{N}} \overrightarrow{AP}\right| = \left|\frac{\overrightarrow{AP} \cdot \vec{N}}{|\vec{N}|}\right|$$

$$= \frac{|(3 \times 3) + (1 \times -5) + (-1 \times 1)|}{\sqrt{(3)^2 + (-5)^2 + (1)^2}} = \boxed{\frac{3}{\sqrt{35}}}$$

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Vector Functions:

If $\vec{F}(x,y,z) = f_1(x,y,z)\,i + f_2(x,y,z)\,j + f_3(x,y,z)\,k$

Then;

$$\vec{R}(t) = x(t)\,i + y(t)\,j + z(t)\,k$$

$\vec{R}(t)$: is the position vector.

$$\vec{V}(t) = \frac{d\vec{R}(t)}{dt} = \frac{dx}{dt}\,i + \frac{dy}{dt}\,j + \frac{dz}{dt}\,k$$

$\vec{v}(t)$: is the velocity vector.

$$\vec{a}(t) = \frac{d^2\vec{R}}{dt^2} = \frac{d^2x}{dt^2}\,i + \frac{d^2y}{dt^2}\,j + \frac{d^2z}{dt^2}\,k$$

$\vec{a}(t)$: is the acceleration vector

$$|\vec{v}| = \sqrt{\left(\frac{dR}{dt}\right)^2} = \text{speed}$$

$$\frac{v}{|v|} = \frac{dR/dt}{\sqrt{(dR/dt)^2}} = \text{Direction}$$

Ex) The vector $\vec{R}(t) = (3\cos t)\,i + (3\sin t)\,j + t^2\,k$ gives the position of a moving body at time. Find the body's speed & direction when $t = 2$.

Solution)

$$\vec{v}(t) = \frac{d\vec{R}(t)}{dt} = -(3\sin t)\,i + (3\cos t)\,j + 2t\,k$$

$$\text{speed} = |\vec{v}(t)| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (2t)^2}$$

$$|\vec{v}(t)|_{t=2} = \sqrt{(-3\sin 2)^2 + (3\cos 2)^2 + (2\times 2)^2} = \boxed{5} \text{ unit of speed}$$

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$$\text{Direction}\Big|_{t=2} = \frac{\vec{v}(2)}{|\vec{v}(2)|} = \frac{(-3\sin 2)i + (3\cos 2)j + (2*2)k}{5}$$

$$\boxed{\text{Direction} = \frac{1}{5}(-0.105i + 3j + 4k)} \quad \underline{\text{Ans}}$$

H.Ws :-

① Find the acute angles between the lines
a- $3x + y = 5$; $2x - y = 4$
b- $12x + 5y = 1$; $2x - 2y = 3$

② a- Find the area of the triangle determined by $P(1,1,1)$, $Q(2,13)$, & $R(3,-1,1)$?
b- Find a unit vector perpendicular to plane PQR ?

③ Let $u = 5i - j + k$, $v = j - 5k$ & $w = -15i + 3j - 3k$
which vectors are (a) perpendicular (b) parallel ?

④ Find the point where the line $x = \frac{8}{3} + 2t$, $y = -2t$ & $z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$?

⑤ Find the distance from $S(1,1,3)$ to the plane $3x + 2y + 6z = 6$

" نهاية محاضرة --Vectors, Vectors in Space, Unit Vector, Scalar Product, Vector Product, line & plane eqs, plane-tangent-perpendicular line-vector function المتجهات، المتجهات فى الفضاء، وحدة العددية، ضرب المتجه، معادلة الخط والمستوى، الخط المماس المتجه، ضرب القيمة والعمودى على المستوى، دالة المتجه"--