عنوان المحاضرة:

General Review on Derivative and Intergration

مراجعة عامة عن المشتقة والتكامل

"ماتم دراسته في المرحلة السابقة"

Derivative Rules:- قواعد المشتقة

what is a derivative ? ماهي المشتقة ؟

The derivative is Finding a slope at any point.
المشتقة هي ايجاد الميل في أي نقطة.

Before we go over the derivative Rules, lets introduce the definition of the derivative Formula.

Definition of the Derivative Formula :-

By using the limit process as

$$\frac{df}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} \quad \text{---} \circledast$$

Ex① / Find $\frac{df}{dx}$ of the following equation by using the definition of the derivative

$F(x) = 5x - 2$

Sol:-

- $F(x+\Delta x) = 5(x+\Delta x) - 2$
- $F(x) = 5x - 2$

plug the above two eqs into eq $\circledast$

$$\frac{df}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{5(x+\Delta x) - 2 - (5x-2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{5x} + 5\Delta x \cancel{-2} - \cancel{5x} \cancel{+2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5\cancel{\Delta x}}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} 5 = \boxed{5}$$

Ex ② By using the definition of the derivative find $\frac{df}{dx}$ for the following eqs, $F(x) = x^2$

Sol.

$$\frac{df}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x}$$

$$* \; F(x+\Delta x) = (x+\Delta x)^2$$
$$F(x) = x^2$$

$$\therefore \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)(x+\Delta x) - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + \Delta x \cdot x + \Delta x \cdot x + \Delta x^2 - \cancel{x^2}}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} (2x + \Delta x)$$

$$\frac{df}{dx} = 2x + 0 = \boxed{2x}$$

By your own exp. try to find $\frac{df}{dx}$ for the following eqs using the def.

1- $F(x) = \frac{1}{x}$

2- $F(x) = \frac{1}{\sqrt{x}}$

3- $F(x) = \frac{5}{\sqrt{x}}$

4- $F(x) = x^2 - 2x + 4$

the Derivative Rules :- قواعد المشتقة

① Constant derivative مشتقة الثابت

$$F(x) = a \Longrightarrow \frac{dF}{dx} = f'(x) = \text{zero} \quad , \ a = \text{constant}$$

② variable derivative مشتقة المتغير

$$f(x) = x^n \Longrightarrow \frac{df}{dx} = f'(x) = n\,x^{n-1} \quad , \ n = \text{any no.}$$

③ Multi-variable Funs مشتقة الدوال كثيرة الحدود

$$f(x) = h(x) \mp g(x) \Longrightarrow \frac{df}{dx} = f'(x) = h'(x) \mp g'(x)$$

④ Quotient Funs مشتقة قسمة دالتين

$$f(x) = \frac{h(x)}{g(x)} \Longrightarrow \frac{df}{dx} = f'(x) = \frac{g(x)\cdot h'(x) - h(x)\cdot g'(x)}{(g(x))^2}$$

⑤ Product Funs مشتقة ضرب دالتين

$$F(x) = h(x)\cdot g(x) \Longrightarrow \frac{dF}{dx} = f'(x) = h(x)\cdot g'(x) + g(x)\cdot h'(x)$$

⑥ Power raised Fun:- مشتقة دالة مرفوعة لأس

$$f(x) = [h(x)]^n \Longrightarrow \frac{df}{dx} = f'(x) = n\,[h(x)]^{n-1}\cdot h'(x)$$

Examples(3):-

1- $F(x) = 4 \Longrightarrow F'(x) = \text{Zero}$

2- $F(x) = x \Longrightarrow F'(x) = 1$

3- $F(x) = x^4 \Longrightarrow F'(x) = 4x^3$

4- $F(x) = 5x^3 \Longrightarrow F'(x) = 5*3x^2 = 15x^2$

5- $F(x) = x^{-3} \Longrightarrow F'(x) = -3x^{-3-1} = -3x^{-4} = \frac{-3}{x^4}$

6- $F(x) = \sqrt{x} \Longrightarrow F(x) = x^{1/2} \Longrightarrow F'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}}$

7- $F(x) = \sqrt[5]{x^2} \Longrightarrow F(x) = x^{\frac{2}{5}} \Longrightarrow F'(x) = \frac{2}{5}x^{\frac{2}{5}-1} = \frac{2}{5}x^{-\frac{3}{5}} = \frac{2}{5\sqrt[5]{x^3}}$

8- $F(x) = 3x^5 + 7x \Longrightarrow F'(x) = 3*5x^{5-1} + 7 = 15x^4 + 7$

9- $F(x) = (x^4 - x^2 + 1)(5x^6 - 3x) \Longrightarrow F'(x) = (x^4 - x^2 + 1)(30x^5 - 3) + (5x^6 - 3x)(4x^3 - 2x)$

10- $F(x) = \frac{x^3+1}{x^4+1} \Longrightarrow F'(x) = \frac{(x^4+1)(3x^2) - (x^3+1)(4x^3)}{(x^4+1)^2}$

11- $F(x) = (x^3 + x^2 + x + 1)^5 \Longrightarrow F'(x) = 5(x^3 + x^2 + x + 1)^4 * (3x^2 + 2x + 1)$

12- $F(x) = \sqrt{x^2 - 2x + 1} \Longrightarrow F'(x) = \frac{2x - 2}{2\sqrt{x^2 - 2x + 1}}$

Ex(4)/ Find the derivative of the quotient Fn at $x = 1$, $F(x) = \frac{x^3+1}{x^4+1}$

Sol/

From Ex3 # 10 $\Longrightarrow F'(x)\Big|_{x=1} = \frac{(1^4+1)(3*1^2) - (1^3+1)(4*1^3)}{(1^4+1)^2}$

$= \frac{2*3 - 2*4}{2^2} = \frac{6-8}{4}$

$= \boxed{\frac{-1}{2}}$

Trigonometric Derivatives مشتقات الدوال الدائرية

1- $F(x) = \sin x \Rightarrow F'(x) = \cos x$

2- $F(x) = \cos x \Rightarrow F'(x) = -\sin x$

3- $F(x) = \tan x \Rightarrow F'(x) = \sec^2 x$

4- $F(x) = \cot x \Rightarrow F'(x) = -\csc^2 x$

5- $F(x) = \sec x \Rightarrow F'(x) = \sec x \tan x$

6- $F(x) = \csc x \Rightarrow F'(x) = -\csc x \cot x$

Ex(5) Find the derivative of the eqs
$F(x) = 5\sin x - 4\tan x$

Sol.

$F'(x) = \frac{dF}{dx} = 5\cos x - 4\sec^2 x$

Ex(6) Find $\frac{d}{dx}[8\sec x - 5\cos x]$

Sol.

$F'(x) = 8\sec x \tan x - 5(-\sin x)$
$F'(x) = 8\sec x \tan x + 5\sin x$

Ex(7) Find $\frac{d}{dx}[2\cot x - 7\csc x]$

Sol.

$F'(x) = 2(-\csc^2 x) - 7(-\csc x \cot x)$
$F'(x) = -2\csc^2 x + 7\csc x \cot x$

Chain Rule قاعدة السلسلة

- If $z = f(x)$ & $x = g(u)$, then

$$\frac{dz}{du} = \frac{dz}{dx} * \frac{dx}{du}$$

- And If $z = f(x)$ & $x = g(u)$ & $u = h(w)$ then,

$$\frac{dz}{dw} = \frac{dz}{dx} * \frac{dx}{du} * \frac{du}{dw}$$

So, the chain rule is using whenever we have a nested (متداخلة) Functions, i.e one function inside of another function

تستخدم قاعدة السلسلة عندما يكون لدينا دوال متداخلة، يعني يكون لدينا دالة داخل دالة اخرى.

Ex(8) $y = f(g(x)) \Longrightarrow y'(x) = f'(g(x)) * g'(x)$

Ex(9) $y = f(g(h(x))) \Longrightarrow y'(x) = f'(g(h(x))) * g'(h(x)) * h'(x)$

And so forth ...

Ex(10) Derive $y = (\sin(x^3+6))^2$

Sol.)

$$y'(x) = 2(\sin(x^3+6)) * \cos(x^3+6) * 3x^2$$

هنا تم اشتقاق القوس ومن ثم ما داخل القوس وهو دالة الجيب وبعدها الزاوية تحت دالة الجيب وبهذا تكون [illegible] دوال متداخلة باستخدام قاعدة السلسلة.

Derivative Applications تطبيقات المشتقات

IF the time is denoted by t, and $s(t)$ is a location or a displacement function (دالة الازاحة او موقع)
Then,

- Velocity $= V(t) = s'(t)$ (السرعة)
- Acceleration $= a(t) = V'(t)$ (التعجيل)

Velocity is gonna have a sign associated with it. either positive or nagative, i-e either moving to the right or to the left or it maybe moving away or back in
السرعة يكون لها اشارة مرافقة. حتى اما موجبة أو سالبة، بمعنى موجبة اذا كان الجسم المتحرك يسير الى جهة اليمين وسالبة اذا عكس ذلك أو موجبة اذا كان الجسم يتحرك مبتعداً وسالبة اذا عكس ذلك.

We have anothe term, named by speed, which is always positive, so we need to take the absolute value velocity to get the speed

speed $= |V(t)|$

المصطلح الثاني الذي نود ذكره هو speed، والذي يكون موجباً دائماً ويكون مساوياً الى مطلق ال Velocity.

Ex) The Following equation of motion descripes the displacement (in meter) of a particle moving in a straight line

$$S = 5t^3 + 3t + 8$$

wher t is measured in seconds.

a- find the velocity after $t=2$ seconds?

b- Find the acceleration after $t=2$ seconds?

[Solution]

a- $V(t) = S'(t) = 5(3t^2) + 3$

$= 15t^2 + 3$

after 2 seconds $\Rightarrow t=2$

$V(2) = 15(2)^2 + 3 = 15 * 4 + 3$

$V(2) = \boxed{63 \ m/sec}$ Ans (a)

b- $a(t) = V'(t) = S''(t) = 15(2t)$

$= 30t$

$\therefore a(2) = 30(2) = \boxed{60 \ m/sec^2}$ Ans (b)

The Derivative of Hyperbolic Functions
مشتقة الدوال الزائدية

① $\frac{d}{dx} \sinh u = \cosh u * u'$

② $\frac{d}{dx} \cosh u = \sinh u * u'$

③ $\frac{d}{dx} \tanh u = \operatorname{sech}^2 u * u'$

④ $\frac{d}{dx} \operatorname{sech} u = -\operatorname{sech} u \tanh u * u'$

⑤ $\frac{d}{dx} \operatorname{cosech} u = -\operatorname{cosech} u \coth u * u'$

⑥ $\frac{d}{dx} \coth u = -\operatorname{cosech}^2 u * u'$

Examples

① Find the derivative of $y = 4\sinh 2x - \frac{3}{7}\cosh 3x$

Sol/

$$\frac{dy}{dx} = 4(\cosh 2x * 2) - \frac{3}{7}(\sinh 3x * 3)$$
$$= \boxed{8\cosh 2x - \frac{9}{7}\sinh 3x}$$

② Derive $y = 5\tanh\frac{x}{2} - 2\coth 4x$

Sol/

$$\frac{dy}{dx} = 5\left(\operatorname{sech}^2\frac{x}{2} * \frac{1}{2}\right) - 2\left(-\operatorname{cosech}^2 4x * 4\right)$$
$$= \boxed{\frac{5}{2}\operatorname{sech}^2\frac{x}{2} + 8\operatorname{cosech}^2 4x}$$

The Derivative of Inverse Hyperbolic Functions :-

مشتقة معكوس الدوال الزائدية

① $\frac{d}{dx}\sinh^{-1}u = \frac{u'}{\sqrt{u^2+1}}$

② $\frac{d}{dx}\cosh^{-1}u = \frac{u'}{\sqrt{u^2-1}}$

③ $\frac{d}{dx}\tanh^{-1}u = \frac{u'}{1-u^2}$

④ $\frac{d}{dx}\operatorname{cosech}^{-1}u = \frac{-u'}{|u|\sqrt{1+u^2}}$

⑤ $\frac{d}{dx}\operatorname{sech}^{-1}u = \frac{-u'}{u\sqrt{1-u^2}}$

⑥ $\frac{d}{dx}\coth^{-1}u = \frac{u'}{1-u^2}$

Examples :-

① Find the derivative of $y = \sinh^{-1}(4x)$

Sol.

$$\frac{dy}{dx} = \frac{4}{\sqrt{(4x)^2+1}} = \boxed{\frac{4}{\sqrt{16x^2+1}}}$$

② Derive $y = \cosh^{-1}(x^3)$

Sol.

$$\frac{dy}{dx} = \frac{3x^2}{\sqrt{(x^3)^2-1}} = \boxed{\frac{3x^2}{\sqrt{x^6-1}}}$$

③ Derive $y = \tanh^{-1}(\sqrt{x})$

Sol.

$$\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{x}}}{1-(\sqrt{x})^2} = \frac{\frac{1}{2\sqrt{x}}}{1+x} = \boxed{\frac{1}{2\sqrt{x}(1+x)}}$$

Integration :- التكامل

The process of integration reverses the process of differentiation.

If $f(x) = 2x^2 \longrightarrow f'(x) = 4x$.
The integration of $4x$ is $2x^2$.

Integration is a process of summation or adding parts together & an elongated S, shown as $\int$, is used to replace the words "the integral of"

Types of Integration :- انواع التكامل

① Indefinite Integrals تكامل غير محدد
Integrals containing an arbitrary constant "C" in their results. This constant needs further infos to be found/calculated.

② Definite Integrals تكامل محدد
Integration limits are applied (يتم تطبيق حدود التكامل)

If an expression is written as $[x]_a^b$, b is called the upper limit and a is the lower limit, where,

$$[x]_a^b = b - a$$

The process of Integration :- عملية التكامل

In integration, the variable of integration is shown by adding d (the derivative) after the function to be integrated.

Thus, $\int 4x\,dx$ means "the integral of $4x$ with respect to x".

and $\int 2t\,dt$ means "the integral of $2t$ with respect to t"

So,

$$\int dx = x + C$$
$$\int dy = y + C$$
$$\int dt = t + C$$

Standard Integrals :- تكاملات قياسية

① Integral of constant $\Longrightarrow \int a\,dx = ax + C$, a-const.

② Power raised variable $\Longrightarrow \int a x^n\,dx = \dfrac{a x^{n+1}}{n+1} + C$

Examples

① $\int 3x^2\,dx \Longrightarrow \int 3x^2\,dx = \dfrac{3x^{2+1}}{2+1} + C = \dfrac{\cancel{3}}{\cancel{3}}x^3 + C = \boxed{x^3 + C}$

② $\int 3x^4\,dx = \dfrac{3x^{4+1}}{4+1} + C = \boxed{\dfrac{3}{5}x^5 + C}$

③ $\int \dfrac{2}{x^2}\,dx = \int 2x^{-2}\,dx = \dfrac{2x^{-2+1}}{-2+1} + C = \dfrac{2x^{-1}}{-1} + C = \boxed{\dfrac{-2}{x} + C}$

④ $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \dfrac{x^{3/2}}{3/2} + C = \boxed{\dfrac{2}{3}\sqrt{x^3} + C}$

⑤ $\int (3x + 2x^2 - 5)\,dx = \int 3x\,dx + \int 2x^2\,dx - \int 5\,dx$

$= 3\dfrac{x^{1+1}}{1+1} + 2\dfrac{x^{2+1}}{2+1} - 5x + C$

$= \boxed{\dfrac{3x^2}{2} + \dfrac{2x^3}{3} - 5x + C}$

Integrals of the Trigonometric functions :-

تكاملات الدوال المثلثية

① $\int \sin ax\,dx = -\dfrac{1}{a}\cos ax + C$

② $\int \cos ax\,dx = \dfrac{1}{a}\sin ax + C$

③ $\int \sec^2 ax\,dx = \dfrac{1}{a}\tan ax + C$

④ $\int \csc^2 ax\,dx = -\dfrac{1}{a}\cot ax + C$

⑤ $\int \csc ax \cot ax\,dx = -\dfrac{1}{a}\csc ax + C$

⑥ $\int \sec ax \tan ax\,dx = \dfrac{1}{a}\sec ax + C$

Examples

① $\int [8\cos x + 3\sin x]\,dx$

Sol

$= 8\sin x + 3(-\cos x) + C$

$= \boxed{8\sin x - 3\cos x + C}$

② $\int [4\sec^2 x - \sec x \tan x]\,dx$

Sol

$= \boxed{4\tan x - \sec x + C}$

③ $\int \csc x(\cot x - \csc x)\,dx = \int (\csc x \cot x - \csc^2 x)\,dx$

$= -\csc x - (-\cot x) + C = \boxed{\cot x - \csc x + C}$

④ $\int \cos^3 x\, dx = \int \cos^2 x \cos x\, dx$

$= \int (1-\sin^2 x) \cos x\, dx$

$= \int \cos x\, dx - \int \sin^2 x \cos x\, dx$

$= \boxed{\sin x - \frac{1}{3}\sin^3 x + C}$

⑤ $\int \cos^5 x\, dx = \int \cos^4 x \cos x\, dx$

$= \int (\cos^2 x)^2 \cos x\, dx$

$= \int (1-\sin^2 x)^2 \cos x\, dx$

let $u = \sin x$

$du = \cos x\, dx$

$\therefore = \int (1-u^2)^2\, du = \int (1-u^2)(1-u^2)\, du$

$= \int (1 - 2u^2 + u^4)\, du$

$= u - \frac{2}{3}u^3 + \frac{u^5}{5} + C$

replace u with sin x & du with cos x dx

$= \boxed{\sin x - \frac{2}{3}\sin^3 x + \frac{\sin^5 x}{5} + C}$

Integration of Inverse Trigonometric Functions

تكامل عكس الدوال المثلثية

① $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$

② $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

③ $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C$

④ $\int \frac{-du}{\sqrt{a^2-u^2}} = \cos^{-1}\left(\frac{u}{a}\right) + C$

⑤ $\int \frac{-du}{a^2+u^2} = \frac{1}{a}\cot^{-1}\left(\frac{u}{a}\right) + C$

⑥ $\int \frac{-du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\csc^{-1}\left(\frac{u}{a}\right) + C$

Examples

① $\int \frac{dx}{\sqrt{16-x^2}} \xrightarrow{\text{sol.}} a^2=16 \rightarrow a=4$

$u=x$

$\therefore \int \frac{dx}{\sqrt{16-x^2}} = \boxed{\sin^{-1}\frac{x}{4}+c}$

② $\int \frac{3}{25+x^2}dx \xrightarrow{\text{sol.}} a^2=25 \rightarrow a=5$

$u=x$

$\therefore \int \frac{3\,dx}{25+x^2} = 3\int\frac{dx}{25+x^2} = 3\times\frac{1}{5}\tan^{-1}\frac{x}{5}+c$

$= \boxed{\frac{3}{5}\tan^{-1}\frac{x}{5}+c}$

③ $\int \frac{8}{x\sqrt{4x^2-1}}dx \xrightarrow{\text{sol}} a^2=1 \rightarrow \boxed{a=1}$

$u^2=4x^2 \rightarrow \boxed{u=2x}$

$\therefore \int \frac{8}{x\sqrt{4x^2-1}}dx = 8\int\frac{\cancel{2}dx}{\cancel{2}x\sqrt{4x^2-1}} = \boxed{8\sec^{-1}(2x)+c}$

Another method $\quad du=2dx \rightarrow dx=\frac{du}{2}, \boxed{x=\frac{u}{2}}$

$8\int\frac{dx}{x\sqrt{4x^2-1}} = 8\int\frac{du/\cancel{2}}{\frac{u}{\cancel{2}}\sqrt{u^2-1}} = 8\int\frac{du}{u\sqrt{u^2-1}}$

$= 8\sec^{-1}u+c$

$= \boxed{8\sec^{-1}(2x)+c}$

Standard integration of inverse Tri Fun's so

① $\int \sin^{-1}x\,dx = x\sin^{-1}x+\sqrt{1-x^2}+c$

② $\int \cos^{-1}x\,dx = x\cos^{-1}x-\sqrt{1-x^2}+c$

③ $\int \tan^{-1}x\,dx = x\tan^{-1}x-\frac{1}{2}\ln|1+x^2|+c$

④ $\int \sec^{-1}x\,dx = x\sec^{-1}x-\ln|x+\sqrt{x^2-1}|+c$

⑤ $\int \csc^{-1}x\,dx = x\csc^{-1}x+\ln|x+\sqrt{x^2-1}|+c$

⑥ $\int \cot^{-1}x\,dx = x\cot^{-1}x+\frac{1}{2}\ln|1+x^2|+c$

Integration of Logarithm & Exponential Fns

تكامل الدوال اللوغارتمية والاسية

① $$\int \log_a u \, du = \frac{u \cdot \log_a\left(\frac{u}{e}\right)}{u'} + c$$ (u = linear Function)

Ex ① $\int \log_4 x \, dx$

Sol.)

$u = x \longrightarrow u' = 1$

$a = 4$

$$\therefore \int \log_4 x \, dx = \frac{x \log_4\left(\frac{x}{e}\right)}{1} + c = \boxed{x \log_4\left(\frac{x}{e}\right) + c}$$

Ex ② $\int \log_5 (x+7) \, dx$

Sol.)

$u = x+7 \Longrightarrow u' = 1$

$a = 5$

$$\therefore \int \log_5 (x+7) \, dx = \frac{(x+7) \log_5\left(\frac{x+7}{e}\right)}{1} + c = \boxed{(x+7) \log_5\left(\frac{x+7}{e}\right) + c}$$

Ex ② $\int \log_7 x^4 \, dx$

Sol.)

In such a problem, we need to manipulate the log before going ahead with using the formula

①

$$\int \log_7 x^4 \, dx = 4 \int \log_7 x \, dx \Longrightarrow \left(\begin{array}{l} u = x \rightarrow u' = 1 \\ a = 7 \end{array}\right)$$

$$= \frac{4 \cdot x \log_7\left(\frac{x}{e}\right)}{1} + c$$

$$= \boxed{4x \log_7\left(\frac{x}{e}\right) + c}$$

Ex) $\int \log_2(x^2+8x+16)\,dx$

Sol)

$$\int \log_2(x^2+8x+16)\,dx = \int \log_2 (x+4)^2\,dx$$

$$= 2\int \log_2(x+4)\,dx \qquad , \quad u = x+4 \rightarrow u' = 1, \quad a = 2$$

$$= \boxed{2(x+4)\log_2\left(\frac{x+4}{e}\right) + c}$$

(2) $\int \frac{1}{x}\,dx = \ln|x| + c$

(3) $\int e^{ax}\,dx = \frac{1}{a} e^{ax} + c$

Examples

(1) $\int \frac{7}{x}\,dx = 7\int \frac{1}{x}\,dx = \boxed{7\ln x + c}$

(2) $\int \frac{1}{x+5}\,dx = \boxed{\ln(x+5) + c}$

(3) $\int \frac{5}{6-2x}\,dx = \int \frac{5}{6-2x} * \frac{-2}{-2} = \frac{-1}{2}\int \frac{5(-2)}{6-2x}\,dx$

$$= \boxed{\frac{-5}{2}\ln(6-2x) + c}$$

(4) $\int \frac{x}{x^2-3}\,dx = \int \frac{x\,dx}{x^2-3} * \frac{2}{2} = \frac{1}{2}\int \frac{2x}{x^2-3}\,dx$

$$= \boxed{\frac{1}{2}\ln(x^2-3) + c}$$

(5) $\int e^{2x}\,dx = \int e^{2x}\,dx * \frac{2}{2} = \frac{1}{2}\int e^{2x} * 2\,dx$

$$= \boxed{\frac{1}{2} e^{2x} + c}$$

(6) $\int e^{-5x}\,dx = \boxed{\frac{-1}{5} e^{-5x} + c}$

Integrals of Hyperbolic Function تكامل الدوال الزائدية

تكامل الدوال الزائدية هي تماماً عكس اشتقاق هذه الدوال مع الأخذ بنظر الاعتبار مشتقة القيمة الموجودة تحت الدالة الزائدية.

① $\int \sinh ax \, dx = \frac{1}{a} \cosh ax + C$

② $\int \cosh ax \, dx = \frac{1}{a} \sinh ax + C$

③ $\int \operatorname{sech}^2 ax \, dx = \frac{1}{a} \tanh ax + C$

④ $\int \operatorname{sech} ax \tanh ax \, dx = -\frac{1}{a} \operatorname{sech} ax + C$

⑤ $\int \operatorname{cosech} ax \coth ax = -\frac{1}{a} \operatorname{cosech} ax + C$

⑥ $\int \operatorname{cosech}^2 ax \, dx = -\frac{1}{a} \coth ax + C$

Examples/

① $\int \cosh(2x) \, dx = \frac{1}{2} \sinh(2x) + C$

② $\int \sinh(2x+5) \, dx = \frac{1}{2} \cosh(2x+5) + C$

H.W → ③ $\int \coth x \, dx = \int \frac{\cosh x}{\sinh x} \, dx = \boxed{\ln|\sinh x| + C}$

④ $\int \operatorname{sech}^3 x \tanh x \, dx = \int \operatorname{sech}^2 x \operatorname{sech} x \tanh x \, dx$

هنا لدينا طريقتين لحل هذا السؤال.

① يمكن إستخدام قاعدة السلسلة باعتبار $(\operatorname{sech}^2 x)$ دالة مرفوعة لأس وهنا يجب توفير الاشتقاق السالبة لمشتقة الدالة $(\operatorname{sech} x)$ فيكون الحل هو كالتالي.

$\int \operatorname{sech}^2 x \operatorname{sech} x \tanh x \, dx = \boxed{-\frac{1}{3} \operatorname{sech}^3 + C}$ Ans

② أما الطريقة الثانية فهو باستخدام التعويض $u = \operatorname{sech} x$

نهاية محاضرة "General Review on Derivative and Intergration
مراجعة عامة عن المشتقة والتكامل"