ALMUSTAQBAL UNIVERSITY

COLLEGE OF ENGINEERING AND TECHNOLOGY

COMPUTER ENGINEERING TECHNIQUE DEPARTMENT

SUBJECT: ENGINEERING ANALYSIS

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LECTURE TWO

Laplace transform of standard functions:

(1) Prove that $L[e^{-at}] = \dfrac{1}{s+a}$ where $s+a>0$ or $s>-a$

Proof:

By definition $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$$L[e^{-at}] = \int_0^\infty e^{-st} \cdot e^{-at} dt$$

$$= \int_0^\infty e^{-t(s+a)} dt$$

$$= \left[\frac{-e^{-t(s+a)}}{s+a}\right]_0^\infty = \frac{-1}{s+a}\left[e^{-\infty} - e^0\right]$$

$$= \frac{1}{s+a}$$

Hence $L[e^{-at}] = \dfrac{1}{s+a}$

2. Prove that $L[e^{at}] = \frac{1}{s-a}$ where $s > a$

Proof :

By the defn of $L[f(t)] = \int_0^\infty e^{-st} f(t) dt$

$$L[e^{+at}] = \int_0^\infty e^{-st} \cdot e^{at} dt$$

$$= \int_0^\infty e^{-(s-a)t} dt$$

$$= \left[\frac{-e^{-(s-a)t}}{s-a} \right]_0^\infty$$

$$= \frac{-1}{s-a} \left[e^{-\infty} - e^0 \right]$$

$$= \frac{1}{s-a}$$

Hence $L[e^{at}] = \frac{1}{s-a}$

3. $L(\cos at) = \int_0^\infty e^{-st} \cos at \ dt$

$$= \left[\frac{e^{-st}}{s^2 + a^2} (-s \cos at + a \sin at) \right]_0^\infty$$

$$= 0 - \frac{1}{s^2 + a^2} (-s)$$

$$= \frac{s}{s^2 + a^2}$$

$$\because \int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} [a \sin bx - b \cos bx]$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} [a \cos bx + b \sin bx]$$

Hence $L(\cos at) = \frac{s}{s^2 + a^2}$

4. $L(\sin at) = \int_0^\infty e^{-st} \sin at \ dt$

$$= \left[\frac{e^{-st}}{s^2 + a^2} (-s \sin at - a \cos at) \right]_0^\infty$$

$$= 0 - \frac{1}{s^2 + a^2} (0 - a)$$

$$L(\sin at) = \frac{a}{s^2 + a^2}$$

5. $$L(\cos hat) = \frac{1}{2}L(e^{at} + e^{-at})$$

$$= \frac{1}{2}\left(\frac{1}{s-a} + \frac{1}{s+a}\right) = \frac{1}{2}\left(\frac{s+a+s-a}{(s+a)(s-a)}\right)$$

$$= \frac{s}{s^2 - a^2}$$

$$L(\cos hat) = \frac{s}{s^2 - a^2}$$

6. $$L(\sin hat) = \frac{1}{2}L(e^{at} - e^{-at})$$

$$= \frac{1}{2}\left(\frac{1}{s-a} + \frac{1}{s+a}\right)$$

$$= \frac{1}{2}\left(\frac{(s+a)-(s-a)}{(s-a)(s+a)}\right)$$

$$= \frac{a}{s^2 - a^2}$$

$$L(\sin hat) = \frac{a}{s^2 - a^2}$$

7. $$L(1) = \int_0^\infty e^{-st} \cdot 1 \cdot dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= \left(0 - \frac{1}{-s}\right) = \frac{1}{s}$$

$$L(1) = \frac{1}{s}$$

8.. $L(t^n) = \int_0^\infty e^{-St} t^n dt$

$$= \left[(t^n) \frac{e^{-St}}{-s} \right]_0^\infty - \int_0^\infty n t^{n-1} \left(\frac{e^{-St}}{-s} \right) dt$$

$$= (0-0) + \frac{n}{s} \int_0^\infty e^{-St} t^{n-1} dt$$

$$= \frac{n}{s} L(t^{n-1})$$

$$L(t^n) = \frac{n}{s} L(t^{n-1})$$

$$L(t^{n-1}) = \frac{n-1}{s} L(t^{n-2})$$

$$L(t^3) = \frac{3}{s} L(t^2)$$

$$L(t^2) = \frac{2}{s} L(t)$$

$$L(t^n) = \frac{n}{s} \cdot \frac{n-1}{s} \cdot \frac{n-2}{s} \cdots\cdots \frac{3}{s} \cdot \frac{2}{s} \cdot \frac{1}{s} \cdot L(1)$$

$$= \frac{n!}{s^n} L[1] = \frac{n!}{s^n} \cdot \frac{1}{s}$$

$$L(t^n) = \frac{n!}{s^{n+1}} \, or \, \frac{\Gamma(n+1)}{s^{n+1}}$$

In particular $n = 1, 2, 3.....$

we get $L(t) = \frac{1}{s^2}$

$$L(t^2) = \frac{2!}{s^3}$$

$$L(t^3) = \frac{3!}{s^4}$$

2.1. Linear property of Laplace Transform

1. $L(f(t) \pm g(t)) = L(f(t) \pm L(g(t))$

2. $L(Kf(t)) = KL(f(t))$

Proof (1) : By the defn of L.T

$$L\left[f(t)\right]=\int_0^\infty e^{-st}f(t)dt$$

$$L\left[f(t)\pm g(t)\right]=\int_0^\infty e^{-st}\left[f(t)\pm g(t)\right]dt$$

$$=\int_0^\infty e^{-st}f(t)dt\pm\int_0^\infty e^{-st}g(t)dt$$

$$=L\left[f(t)\right]\pm L\left[g(t)\right]$$

Hence $L\left[f(t)\pm g(t)\right]=L\left[f(t)\right]\pm L\left[g(t)\right]$

(2) $L\left[Kf(t)\right]=KL\left[f(t)\right]$

By the defn of L.T

$$L\left[Kf(t)\right]=\int_0^\infty e^{-st}Kf(t)dt$$

$$=K\int_0^\infty e^{-st}f(t)dt$$

$$=KL\left[f(t)\right]$$

Hence $L\left[Kf(t)\right]=KL\left[f(t)\right]$

2.2. Recall

1. $2\sin A\cos B=\sin(A+B)+\sin(A-B)$
2. $2\cos A\sin B=\sin(A+B)-\sin(A-B)$
3. $2\cos A\cos B=\cos(A+B)+\cos(A-B)$
4. $2\sin A\sin B=\cos(A-B)-\cos(A+B)$
5. $\sin^2 A=\dfrac{1-\cos 2A}{2}$
6. $\cos^2 A=\dfrac{1+\cos 2A}{2}$
7. $\sin 3A=3\sin A-4\sin^3 A$
8. $\cos 3A=4\cos^3 A-3\cos A$
9. $\sin(A+B)=\sin A\cos B+\cos A\sin B$
10. $\sin(A-B)=\sin A\cos B-\cos A\sin B$
11. $\cos(A-B)=\cos A\cos B+\sin A\sin B$
12. $\cos(A+B)=\cos A\cos B-\sin A\sin B$

3.1 Problems :

1. Find Laplace Transform of $\sin^2 t$

Solution :

$$L(\sin^2 t) = L\left(\frac{1-\cos 2t}{2}\right)$$

$$= \frac{1}{2}L(1-\cos 2t)$$

$$= \frac{1}{2}\left(\frac{1}{s}-\frac{s}{s^2+4}\right)$$

2. Find $L(\cos^3 t)$

Solution :

we know that $\cos 3A = 4\cos^3 A - 3\cos A$

hence $\cos^3 A = \frac{3}{4}\cos A + \frac{1}{4}\cos 3A$

$$L(\cos^3 t) = \frac{1}{4}L(3\cos t + \cos 3t)$$

$$= \frac{1}{4}\left(\frac{3s}{s^2+1}+\frac{s}{s^2+9}\right)$$

3. Find $L(\sin 3t \cos t)$

Solution :

we know that $\sin A \cos B = \frac{1}{2}\big(\sin(A+B)+\sin(A-B)\big)$

hence $\sin 3t \cos t = \frac{1}{2}(\sin 4t + \sin 2t)$

$$L(\sin 3t \cos t) = \frac{1}{2}L(\sin 4t + \sin 2t)$$

$$= \frac{1}{2}\left(\frac{4}{s^2+16}+\frac{2}{s^2+4}\right)$$

$$= \frac{2}{s^2+16}+\frac{1}{s^2+4}$$

4. Find $L(\sin t \sin 2t \sin 3t)$

Solution :

we know that $\sin t \sin 2t \sin 3t = \sin t \frac{1}{2}(\cos t - \cos 5t)$

$$= \frac{1}{2}\sin t \cos t - \frac{1}{2}(\sin t \cos 5t)$$

$$= \frac{1}{4}\sin 2t - \frac{1}{4}(\sin 6t - \sin 4t)$$

$$L(\sin t \sin 2t \sin 3t) = \frac{1}{4}L(\sin 2t + \sin 4t - \sin 6t)$$

$$= \frac{1}{4}\left[\frac{2}{s^2+4}+\frac{4}{s^2+16}-\frac{6}{s^2+36}\right]$$

5. Find $L(1+e^{-3t}-5e^{4t})$

Solution :

$$L\left[1+e^{-3t}-5e^{4t}\right]=L[1]L\left[e^{-3t}\right]+5L\left[e^{4t}\right]$$
$$=\frac{1}{s}+\frac{1}{s+3}-\frac{5}{s-4}$$

6. Find $L(3+e^{6t}+\sin 2t-5\cos 3t)$

Solution :

$$L(3+e^{6t}+\sin 2t-5\cos 3t)=3L(1)+L(e^{6t})+L(\sin 2t)-5L(\cos 3t)$$
$$=3\cdot\frac{1}{s}+\frac{1}{s-6}+\frac{2}{s^2+4}-\frac{5s}{s^2+9}$$

7. Find $L(\sin(2t+3))$

Solution :

$$L(\sin(2t+3))=L(\sin 2t\cos 3+\sin 3\cos 2t)$$
$$=\cos 3L(\sin 2t)+\sin 3L(\cos 2t)$$
$$=\cos 3\frac{2}{s^2+4}+\sin 3\frac{s}{s^2+4}$$

8. Find $L(\sin 4t+3\sin h2t-4\cos h5t+e^{-5t})$

Solution :

$$L(\sin 4t+3\sin h2t-4\cos h5t+e^{-5t})$$
$$=L(\sin 4t)+3L(\sin h2t)-4L(\cos h5t)+L(e^{-5t})$$
$$=\frac{4}{s^2+16}+3\cdot\frac{2}{s^2-4}-4\cdot\frac{s}{s^2-25}+\frac{1}{s+5}$$
$$=\frac{4}{s^2+16}+\frac{6}{s^2-4}-\frac{4s}{s^2-25}+\frac{1}{s+5}$$

9. Find $L((1+t)^2)$

Solution :

$$L((1+t)^2)=L(1+2t+t^2)$$
$$=L(1)+2L(t)+L(t^2)$$
$$=\frac{1}{s}+2\cdot\frac{1}{s^2}+\frac{2!}{s^3}$$

10. Find the Laplace Transform of $f(t)=\begin{cases}\sin t & 0<t<\pi \\ 0 & t>\pi\end{cases}$

Solution :

By definition,

$$L(f(t))=\int_0^\infty e^{-st} f(t)dt$$

$$=\int_0^\pi e^{-st} f(t)dt+\int_\pi^\infty e^{-st} f(t)dt$$

$$=\int_0^\pi e^{-st} \sin t\ dt+\int_\pi^\infty e^{-st}(0)dt$$

$$=\int_0^\pi e^{-st} \sin t\ dt$$

$$=\left[\frac{e^{-st}}{(-s)^2+1^2}(-s\sin t-\cos t)\right]_0^\pi \qquad \because \int e^{ax}\sin bx\ dx=\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)$$

$$=\frac{e^{-s\pi}}{s^2+1}(-s\sin\pi-\cos\pi)-\frac{e^0}{s^2+1}(0-1)$$

$$=\frac{e^{-s\pi}}{s^2+1}(1)+\frac{1}{s^2+1}$$

$$=\frac{1}{s^2+1}(e^{-s\pi}+1)$$

11. Find the Laplace Transform of $f(t)=\begin{cases}e^t & 0<t<1 \\ 0 & t>1\end{cases}$

Solution :

By definition, $$L(f(t))=\int_0^\infty e^{-st} f(t)dt$$

$$=\int_0^1 e^{-st} f(t)dt+\int_1^\infty e^{-st} f(t)dt$$

$$=\int_0^1 e^{-st}e^t dt+\int_1^\infty e^{-st}0dt$$

$$=\int_0^1 e^{(-s+1)t}dt$$

$$=\left[\frac{e^{(1-s)t}}{1-s}\right]_0^1$$

$$=\frac{1}{1-s}\left(e^{1-s}-1\right)$$

3.2. Note :

1. $\Gamma(n+1)=\int_0^\infty x^n e^{-x}dx$ (By definition)

$\Gamma(n+1)=n!, \quad n=1,2,3,.....$

$\Gamma(n+1)=n\Gamma(n), \quad n>0$

12. Find $L\left(\frac{1}{\sqrt{t}}+t^{3/2}\right)$

Solution :

$$L\left(\frac{1}{\sqrt{t}}+t^{3/2}\right)=L(t^{-1/2})+L(t^{3/2})$$

$$=\frac{\Gamma\left(-\frac{1}{2}+1\right)}{s^{-\frac{1}{2}+1}}+\frac{\Gamma\left(\frac{3}{2}+1\right)}{s^{\frac{3}{2}+1}}$$

$$=\frac{\Gamma\left(\frac{1}{2}\right)}{s^{\frac{1}{2}}}+\frac{3}{2}\cdot\frac{1}{2}\frac{\Gamma\left(\frac{1}{2}\right)}{s^{\frac{5}{2}}}$$

$$=\frac{\sqrt{\pi}}{\sqrt{s}}+\frac{3}{4}\frac{\sqrt{\pi}}{s^{\frac{5}{2}}}$$

4. First Shifting Theorem (First translation)

1. If $L(f(t))=F(s)$, then $L(e^{-at}f(t))=F(s+a)$

Proof :

By definition, $L\left[f(t)\right]=\int_0^\infty e^{-st}f(t)dt$

$$L\left[e^{-at}f(t)\right]=\int_0^\infty e^{-st}\cdot e^{-at}f(t)dt$$

$$=\int_0^\infty e^{-t(s+a)}f(t)dt$$

$$=F(s+a)$$

Hence $L\left[e^{-at}f(t)\right]=F(s+a)$

4.1. Corollary : $L(e^{at}f(t))=F(s-a)$

4.2. Note :

1. $L(e^{-at}f(t))=L\left[f(t)\right]_{s\to s+a}$

$$=\left[F(s)\right]_{s\to s+a}$$

$$=F(s+a)$$

2. $L(e^{at}f(t))=L\left[f(t)\right]_{s\to s-a}$

$$=\left[F(s)\right]_{s\to s-a}$$

$$=F(s-a)$$

4.3. Problems :

1. Find $L(te^{2t})$

Solution :

$$L(te^{2t}) = \left[L(t)\right]_{s\to s-2}$$

$$= \left(\frac{1}{s^2}\right)_{s\to s-2} = \frac{1}{(s-2)^2}$$

2. Find $L(t^5e^{-t})$

Solution :

$$L(t^5e^{-t}) = \left[L(t^5)\right]_{s\to s+1}$$

$$= \left(\frac{5!}{s^6}\right)_{s\to s+1}$$

$$= \frac{5!}{(s+1)^6}$$

3. Find $L(e^{-2t}\sin 3t)$

Solution :

$$L(e^{-2t}\sin 3t) = L(\sin 3t)_{s\to s+2}$$

$$= \left(\frac{3}{s^2+9}\right)_{s\to s+2}$$

$$= \frac{3}{(s+2)^2+9}$$

4. Find $L(e^{-t}\cos h4t)$

Solution :

$$L(e^{-t}\cos h4t) = L(\cos h4t)_{s\to s+1}$$

$$= \left(\frac{s}{s^2-16}\right)_{s\to s+1}$$

$$= \frac{s+1}{(s+1)^2-16}$$

5. Find $L(e^{3t}\sin^2 4t)$

Solution :

$$L(e^{3t}\sin^2 4t) = L(\sin^2 4t)_{s\to s-3}$$

$$= L\left(\frac{1-\cos 8t}{2}\right)_{s\to s-3}$$

$$= \frac{1}{2}\left(L(1) - L(\cos 8t)\right)_{s\to s-3}$$

$$= \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+64}\right)_{S\to S-3}$$

$$= \frac{1}{2}\left(\frac{1}{s-3} - \frac{s-3}{(s-3)^2+64}\right)$$

6. Find $L(e^{-2t}\sin 4t\cos 6t)$

Solution :

$$L(e^{-2t}\sin 4t\cos 6t)=L(\sin 4t\cos 6t)_{s\to s+2}$$
$$=\frac{1}{2}\left(L(2\sin 4t\cos 6t)\right)_{s\to s+2}$$
$$=\frac{1}{2}\left(L(\sin(4t+6t)+(\sin 4t-6t)\right)_{s\to s+2}$$
$$=\frac{1}{2}\left(L(\sin 10t-\sin 2t)\right)_{s\to s+2}$$
$$=\frac{1}{2}\left(\frac{10}{s^2+100}-\frac{2}{s^2+4}\right)_{s\to s+2}$$
$$=\frac{1}{2}\left(\frac{10}{(s+2)^2+100}-\frac{2}{(s+2)^2+4}\right)$$

7. Find $L\left(e^{4t}(\sin^3 3t+\cosh^3 3t)\right)$

Solution:

$$L\left(e^{4t}(\sin^3 3t+\cosh^3 3t)\right)=L\left(\sin^3 3t+\cosh^3 3t\right)_{s\to s-4}$$
$$=L\left(\frac{3\sin 3t-\sin 9t}{4}+\frac{3\cosh 3t+\cosh 9t}{4}\right)_{s\to s-4}$$
$$\because \sin^3\theta=\frac{3\sin\theta-\sin 3\theta}{4},\cosh^3\theta=\frac{3\cosh\theta+\cosh 3\theta}{4}$$
$$=\left[\frac{3}{4}L(\sin 3t)-\frac{1}{4}L(\sin 9t)+\frac{3}{4}L(\cosh 3t)+\frac{1}{4}(\cosh 9t)\right]_{s\to s-4}$$
$$=\left(\frac{3}{4}\cdot\frac{3}{s^2+9}-\frac{1}{4}\frac{9}{s^2+81}+\frac{3}{4}\cdot\frac{s}{s^2-9}+\frac{1}{4}\frac{s}{s^2-81}\right)_{s\to s-4}$$
$$=\frac{3}{4}\cdot\frac{3}{(s-4)^2+9}-\frac{1}{4}\frac{9}{(s-4)^2+81}+\frac{3}{4}\cdot\frac{s-4}{(s-4)^2-9}+\frac{1}{4}\frac{s-4}{(s-4)^2-81}$$

8. Find $L(\cosh t\cos 2t)$

Solution :

$$L(\cos ht\cos 2t)=L\left(\left(\frac{e^t+e^{-t}}{2}\right)\cos 2t\right)$$
$$=\frac{1}{2}L(e^t\cos 2t+e^{-t}\cos 2t)$$
$$=\frac{1}{2}\left[L(\cos 2t)_{s\to s-1}+L(\cos 2t)_{s\to s+1}\right]$$
$$=\frac{1}{2}\left[\left(\frac{s}{s^2+4}\right)_{s\to s-1}+\left(\frac{s}{s^2+4}\right)_{s\to s+1}\right]$$
$$=\frac{1}{2}\left(\frac{s-1}{(s-1)^2+4}+\frac{s+1}{(s+1)^2+4}\right)$$

5. Theorem

If $L(f(t)) = F(s)$, then $L(tf(t)) = \frac{-d}{ds}(F(s))$

Proof :

Given $F(s) = L(f(t))$

differentiate both sides, w.r. to 's'

$$\frac{d}{ds}(F(s)) = \frac{d}{ds}(L(f(t)))$$

$$= \frac{d}{ds}\left(\int_0^\infty e^{-st} f(t)dt\right)$$

$$= \int_0^\infty \frac{\partial}{\partial s}(e^{-st} f(t))dt$$

$$= \int_0^\infty (-t)e^{-st} f(t)dt$$

$$= -\int_0^\infty tf(t)e^{-st}dt$$

$$\frac{d}{ds}(F(s)) = -L(tf(t))$$

$$\therefore L(tf(t)) = \frac{-d}{ds}F(s)$$

(or) $L(tf(t)) = -F'(s)$ where $F(s) = L(f(t))$

similarly we can show that,

$$L(t^2 f(t)) = (-1)^2 \frac{d^2}{ds^2}F(s)$$

$$L(t^3 f(t)) = (-1)^3 \frac{d^3}{ds^3}F(s)$$

In general, $L(t^n f(t)) = (-1)^n \frac{d^n}{ds^n}F(s)$

5.1. Problems :

1. Find $L(te^{3t})$

Solution :

We know that $L(tf(t)) = \frac{-d}{ds}L(f(t))$

Here $f(t) = e^{3t}$

$$L(te^{3t}) = \frac{-d}{ds} L(e^{3t})$$

$$= \frac{-d}{ds}\left(\frac{1}{s-3}\right)$$

$$= -\left(\frac{(s-3)(0)-(1)}{(s-3)^2}\right)$$

$$= \frac{1}{(s-3)^2}$$

2. Find $L(t\sin 3t)$

Solution :

$$L(tf(t)) = \frac{-d}{ds} L(f(t))$$

$$L(tf(t)) = \frac{-d}{ds} L(\sin 3t)$$

$$= \frac{-d}{ds}\left(\frac{3}{s^2+9}\right)$$

$$= \left(\frac{-(s^2+9)(0)+3(2s)}{(s^2+9)^2}\right)$$

$$= \frac{6s}{(s^2+9)^2}$$

3. Find $L(t\cos^2 3t)$

Solution :

$$L(t\cos^2 3t) = \frac{-d}{ds} L(\cos^2 3t)$$

$$= \frac{-d}{ds} L\left(\frac{1+\cos 6t}{2}\right)$$

$$= \frac{-1}{2}\frac{d}{ds}\left(L(1)+L(\cos 6t)\right)$$

$$= \frac{-1}{2}\frac{d}{ds}\left(\frac{1}{s}+\frac{s}{s^2+16}\right)$$

$$= \frac{-1}{2}\left(\frac{-1}{s^2}+\frac{(s^2+16)\cdot 1-s(2s)}{(s^2+16)^2}\right)$$

$$= \frac{-1}{2}\left(\frac{-1}{s^2}+\frac{16-s^2}{(s^2+16)^2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{s^2}+\frac{s^2-16}{(s^2+16)^2}\right)$$

4. Find $L(te^{-2t}\sin 3t)$

Solution :

$$L(e^{-2t}(t\sin 3t)) = L(t\sin 3t)_{s\to s+2}$$
$$= \left\{\frac{-d}{ds}(L(\sin 3t)\right\}_{s\to s+2}$$
$$= \left\{\frac{-d}{ds}\left(\frac{3}{s^2+9}\right)\right\}_{s\to s+2}$$
$$= \left\{\frac{(s^2+9)0-3(2s)}{(s^2+9)^2}\right\}_{s\to s+2}$$
$$= \frac{6(s+2)}{((s+2)^2+9)^2}$$

5. Find $L(te^{-2t}\sin 2t\sin 3t)$

Solution :

$$L(te^{-2t}\sin 2t\sin 3t)$$
$$= L(t\sin 2t\sin 3t)_{s\to s+2}$$
$$= \left[\frac{1}{2}\times L(t\cdot 2\sin 2t\sin 3t)\right]_{s\to s+2}$$
$$= \frac{1}{2}L\big(t(\cos(2t-3t)-\cos(2t+3t))\big)_{s\to s+2}$$
$$= \frac{1}{2}L\big(t\cos t - t\cos 5t\big)_{s\to s+2}$$
$$= \frac{1}{2}\left[\frac{-d}{ds}L(\cos t)+\frac{d}{ds}L(\cos 5t)\right]_{s\to s+2}$$
$$= \frac{1}{2}\left[\frac{-d}{ds}\left(\frac{s}{s^2+1}\right)+\frac{d}{ds}\left(\frac{s}{s^2+25}\right)\right]_{s\to s+2}$$
$$= \frac{1}{2}\left[-\left(\frac{(s^2+1)\cdot 1-s(2s)}{(s^2+1)^2}\right)+\frac{d}{ds}\left(\frac{(s^2+25)\cdot 1-s(2s)}{(s^2+25)^2}\right)\right]_{s\to s+2}$$
$$= \frac{1}{2}\left[-\left(\frac{1-s^2}{(s^2+1)^2}\right)+\left(\frac{25-s^2}{(s^2+25)^2}\right)\right]_{s\to s+2}$$
$$= \frac{1}{2}\left[\frac{s^2-1}{(s^2+1)^2}+\frac{25-s^2}{(s^2+25)^2}\right]_{s\to s+2}$$
$$= \frac{1}{2}\left[\frac{(s+2)^2-1}{\left((s+2)^2+1\right)^2}+\frac{25-(s+2)^2}{\left((s+2)^2+25\right)^2}\right]$$