

Chapter 3

FUNDAMENTALS OF DOSIMETRY

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3.1. INTRODUCTION

Determination of the energy imparted to matter by radiation is the subject of dosimetry. The energy deposited as radiation interacts with atoms of the material, as seen in the previous chapter. The imparted energy is responsible for the effects that radiation causes in matter, for instance, a rise in temperature, or chemical or physical changes in the material properties. Several of the changes produced in matter by radiation are proportional to the absorbed dose, giving rise to the possibility of using the material as the sensitive part of a dosimeter. Also, the biological effects of radiation depend on the absorbed dose. A set of quantities related to the radiation field is also defined within the scope of dosimetry. It will be shown in this chapter that, under special conditions, there are simple relations between dosimetric and field description quantities. Thus, the framework of dosimetry is the set of physical and operational quantities that are studied in this chapter.

3.2. QUANTITIES AND UNITS USED FOR DESCRIBING THE INTERACTION OF IONIZING RADIATION WITH MATTER

Historically, measurement of the ionization produced by radiation was the first choice used to quantify the passage of radiation through matter. Indeed, the quantity exposure, or, more precisely, exposure dose, as defined by the International Commission on Radiation Units and Measurements (ICRU) in 1957, is related to the ability of a photon beam to ionize the air. In recent years, the use of this quantity has been replaced by kerma, a more general quantity that is recommended for dosimeter calibration purposes. Nevertheless, absorbed dose is the quantity that better indicates the effects of radiation on materials or on human beings, and, accordingly, all the protection related quantities are based on it. The use of dosimetric quantities is important in many aspects of the application of radiation. In diagnostic radiology, radiation protection of staff and patients is the most important application of the dosimetric quantities. This section introduces

and discusses the main dosimetric quantities, relating them, whenever possible, to the quantities that describe the radiation field.

3.2.1. Radiation fields: Fluence and energy fluence

A radiation field at a point P can be quantified by the physical non-stochastic quantity fluence, which is usually expressed in units of m^{-2} or cm^{-2} , and is given by the relation:

$$\Phi = \frac{dN}{da} \quad (3.1)$$

where dN is the differential of the expectation value of the number of particles (photons or massive particles) striking an infinitesimal sphere with a great circle area, da , surrounding point P , as shown in Fig. 3.1. It is worthwhile noting that the particles included in Φ may have any direction, but correspond to one type of radiation, so that photons and electrons are counted separately, contributing to the photon fluence and the electron fluence, respectively.

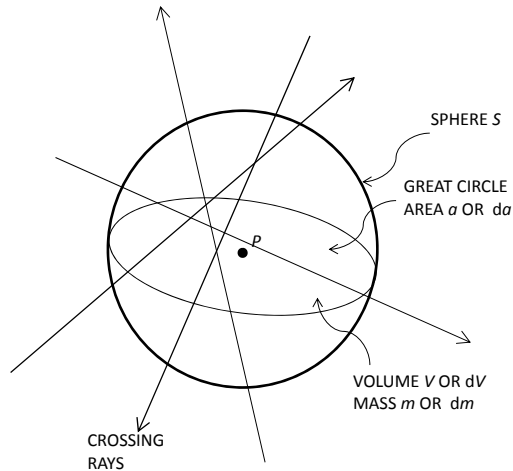


FIG. 3.1. Characterizing the radiation field at a point, P , in terms of the radiation traversing the spherical surface, S .

The concept of energy fluence follows easily, by summing the radiant energy of each particle that strikes the infinitesimal sphere:

$$\Psi = \frac{dR}{da} \quad (3.2)$$

In Eq. (3.2), dR is the differential of the radiant energy R — kinetic energy of massive particles, energy of photons — that impinges on the infinitesimal sphere. The SI unit of energy fluence is joules per square metre (J/m^2).

If the radiation field is composed of particles each with the same energy E , the energy fluence is related to the fluence (or particle fluence) through the simple expression: $\Psi = E \Phi$.

3.2.2. Energy transferred, net energy transferred, energy imparted

3.2.2.1. Energy transferred, net energy transferred

When an uncharged particle — for instance an X ray photon — interacts with matter, part of its energy is transferred in various interaction events. In a volume, V , of material, the energy transferred (ϵ_{tr}) is given by the sum of all the initial kinetic energies of charged ionizing particles liberated by the uncharged particles in the volume V . For the case where photons in the diagnostic energy range are the uncharged interacting particles, ϵ_{tr} corresponds to the sum of the kinetic energies of electrons at the moment they are set free in an incoherent scattering or photoelectric interaction in the volume V . For photon energies above the pair production threshold of 1.022 MeV (see Section 2.2.7), kinetic energy may also be transferred to positrons.

As the liberated charged particles interact with matter, part of their initial kinetic energy can be irradiated as photons. There are two main processes responsible for the emission of photons:

- (i) The emission of bremsstrahlung radiation by electrons and positrons interacting with nuclei.
- (ii) The in-flight annihilation of positrons; the remaining kinetic energy of the positron (T_{ann}) at the moment of the annihilation plus the rest mass energies of the annihilated particles (1.02 MeV) being converted to photon energy.

When the energies of the bremsstrahlung photons ($h\nu_{brem}$) and the annihilation photons (T_{ann}) are subtracted from ϵ_{tr} , another quantity is defined — the net energy transferred — ϵ_{tr}^{net} :

$$\epsilon_{tr}^{net} = \epsilon_{tr} - \sum h\nu_{brem} - \sum T_{ann} \quad (3.3)$$

For both quantities (energy transferred and net energy transferred), the volume V is the volume where the initial uncharged particles interact. It does not

matter if the range of the charged particles is restricted to V or not, their initial kinetic energies are all included in ε_{tr} , and all the bremsstrahlung emissions and excess of energy of the annihilation photons are excluded from $\varepsilon_{\text{tr}}^{\text{net}}$. For photons in the diagnostic energy range, incident on low Z materials, $\varepsilon_{\text{tr}}^{\text{net}} = \varepsilon_{\text{tr}}$, as there is no pair production, and bremsstrahlung emission by the released electrons is unlikely.

3.2.2.2. Energy imparted

A very important concept regarding the deposition of energy from ionizing radiation to matter is the energy imparted. This quantity is defined for any radiation (charged or uncharged) and is related to the part of the radiant energy that can produce effects within an irradiated volume. If V is the irradiated volume, R_{in} is the radiant energy that enters the volume and R_{out} is the energy that leaves the volume, the energy imparted is defined as:

$$\varepsilon = R_{\text{in}} - R_{\text{out}} + E_{m \rightarrow R} - E_{R \rightarrow m} \quad (3.4)$$

The quantities $E_{m \rightarrow R}$ and $E_{R \rightarrow m}$ in Eq. (3.4) are the changes in energy when the rest mass of a particle is converted to radiant energy ($m \rightarrow R$) or the energy of a photon is converted to the mass of particles ($R \rightarrow m$) inside the volume V . For the energies encountered in diagnostic radiology, both of these terms are negligible and Eq. (3.4) can be rewritten as:

$$\varepsilon = R_{\text{in}} - R_{\text{out}} \quad (3.5)$$

3.2.3. Kerma and collision kerma

The physical, non-stochastic quantity kerma (K) is related to the energy transferred from uncharged particles to matter. Kerma is the acronym for **k**inetic energy **r**elaxed per unit **m**ass. It is defined as:

$$K = \frac{d\varepsilon_{\text{tr}}}{dm} \quad (3.6)$$

where the quantity $d\varepsilon_{\text{tr}}$ is the expectation value of the energy transferred from indirectly ionizing radiation to charged particles in the elemental volume dV of mass dm . The SI unit of kerma is joules per kilogram (J/kg), which is given the special name gray (Gy).

Some important remarks about kerma are:

- (i) Kerma may be defined in any material, so it is important that the material is declared when a value of kerma is presented.
- (ii) Kerma is defined for indirectly ionizing radiation — uncharged particles such as photons and neutrons — and is related to the first step of transfer of energy from these particles to matter, in which uncharged particles transmit kinetic energy to secondary charged particles.
- (iii) The kinetic energy transferred to the secondary particles is not necessarily spent in the volume (dV) where they were liberated. The kerma definition is constrained to the energy the secondary particles receive at the moment of liberation.

3.2.3.1. Components of kerma

The energy transferred from indirectly ionizing radiation to charged particles may be spent in two ways: (i) collisions resulting in ionizations and (ii) conversion to photons. Accordingly, the kerma can be divided into two parts:

$$K = K_{\text{col}} + K_{\text{rad}} \quad (3.7)$$

The collision kerma (K_{col}) is related to that part of the kinetic energy of the secondary charged particles that is spent in collisions, resulting in ionization and excitation of atoms in matter. In terms of the quantities defined before, collision kerma is obtained from the expectation value of the net energy transferred (Eq. (3.3)):

$$K_{\text{col}} = \frac{d\epsilon_{\text{tr}}^{\text{net}}}{dm} \quad (3.8)$$

The radiative kerma (K_{rad}) is related to that portion of the initial kinetic energy of the secondary charged particles that is converted into photon energy. It is simpler to define radiative kerma as the difference: $K_{\text{rad}} = K - K_{\text{col}}$. The division of kerma in those two components is more didactic than conceptual. It helps the understanding of the relationship between kerma and absorbed dose, which will be treated in the next section.

3.2.4. Kerma for photons

3.2.4.1. Kerma and fluence

Some important relationships for kerma may be obtained for the simple case of a monoenergetic photon beam irradiating matter. The first of these has already

been derived in Section 2.3.4. At a point, P , in space where there is a fluence, Φ , of photons of energy $h\nu$, kerma may be calculated as the product of the energy fluence and the mass energy transfer coefficient (μ_{tr}/ρ) of the material:

$$K = \Phi h\nu \left(\frac{\mu_{tr}}{\rho} \right) = \left(\frac{\mu_{tr}}{\rho} \right) \Psi \quad (3.9)$$

Similarly, the collision kerma for photons is related to the fluence through the use of the mass energy absorption coefficient (μ_{en}/ρ):

$$K_{col} = \Phi h\nu \left(\frac{\mu_{en}}{\rho} \right) = \Psi \left(\frac{\mu_{en}}{\rho} \right) \quad (3.10)$$

Also in Section 2.3.4, a relationship was derived between the energy absorption and energy transfer coefficients (Eq. 2.31). Using this equation, the relationship between collision and total kerma is as follows:

$$K_{col} = K(1 - g) \quad (3.11)$$

where g is the average fraction of the energy transferred to the charged particles that is lost to photons when the charged particles are slowed down in the same medium as that in which they were released.

If the photon beam has a spectrum of energies, Eqs (3.9, 3.10) may be generalized through a summation or integration over the range of energies of the discrete or continuous spectrum, respectively.

For photons in the diagnostic energy range (≤ 150 keV) interacting in low Z material, the differences between the energy absorption and energy transfer coefficients are negligible, as the fraction of the electron energy converted to bremsstrahlung X rays is very small. For these conditions, the radiative kerma is negligible and the collision kerma is numerically equal to the kerma.

3.2.4.2. Kerma and exposure

In the special situation of X ray or gamma ray photons interacting with air, another quantity, exposure, is also defined and is related to collision kerma through a simple expression. According to the ICRU, exposure (X) is defined as the ratio:

$$X = \frac{dQ}{dm} \quad (3.12)$$

The quantity dQ in Eq. (3.12) is the absolute value of the total charge of the ions of one sign produced in air when all the electrons and positrons liberated by photons in air of mass dm are stopped in air. The unit of exposure in SI is coulomb per kilogram (C/kg), even though an old non-SI unit (roentgen — R) is still in use. The conversion from R to SI units is $1 \text{ R} = 2.580 \times 10^{-4} \text{ C/kg}$.

From the above definition, the energy spent to produce the charge dQ included in the numerator of Eq. (3.12) corresponds to the expectation value of the net energy transferred to charged particles in air ($d\varepsilon_{\text{tr}}^{\text{net}}$). The relationship between these two quantities can be expressed in terms of the measurable quantity \bar{W}_{air} , the mean energy spent in dry air to form an ion pair. \bar{W}_{air} is given by the ratio:

$$\bar{W}_{\text{air}} = \frac{\sum \text{kinetic energies of electrons spent in ionization and excitation}}{\sum \text{ion pairs produced by the secondary electrons in air}} \quad (3.13)$$

It is important to emphasize that the portion of kinetic energy that is converted into photon energy and the ionizations produced by the bremsstrahlung photons are *not* included in the numerator or the denominator of Eq. (3.13).

The value of \bar{W}_{air} accepted nowadays was determined by Boutillon and Perroche-Roux in 1987 through the analysis of a set of published experimental values for this quantity:

$$\bar{W}_{\text{air}} = 33.97 \text{ eV/ion pair} = 33.97 \text{ J/C} \quad (3.14)$$

The relationship between air collision kerma and exposure X may now be obtained:

$$(K_{\text{col}})_{\text{air}} = \bar{W}_{\text{air}} X = 33.97 X \quad (\text{SI})$$

or

$$(K_{\text{col}})_{\text{air}} = 0.876 \times 10^{-2} X \quad (X \text{ in R, } K \text{ in Gy}) \quad (3.15)$$

3.2.5. Absorbed dose

Absorbed dose (D), a physical non-stochastic quantity, is defined simply as the ratio:

$$D = \frac{d\varepsilon}{dm} \quad (3.16)$$

where $d\mathcal{E}$ is the expectation value of the energy imparted by any ionizing radiation to the matter of mass, dm . Absorbed dose is expressed in the same units as kerma, i.e. joules per kilogram (J/kg) or gray (Gy).

Owing to the penetrating character of ionizing radiation, when a large volume is irradiated, energy can be imparted to the matter in a specific volume by radiation that comes from other regions, sometimes very far from the volume of interest. As the absorbed dose includes all the contributions that impart energy in the volume of interest, it is hardly possible to correlate absorbed dose and the fluence of the incident radiation. In fact, knowledge of the radiation fluence in the volume of interest, including scattered radiation, is a necessary condition for calculation of the absorbed dose. The special situation that makes this correlation possible will be dealt with in Section 3.3.

3.2.6. Kerma and absorbed dose

Kerma and absorbed dose are expressed with the same units, and both are related to the quantification of the interaction of radiation with the matter. Apart from the main fact that kerma is used to quantify a radiation field and absorbed dose is used to quantify the effects of radiation, there are some important points in their definitions that should be emphasized. One of the differences is the role of the volume of interest in these quantities; for kerma, it is the place where energy is transferred from uncharged to charged particles; for absorbed dose, the volume of interest is where the kinetic energy of charged particles is spent. For instance, for kerma, only the energy transfer due to interactions of uncharged particles within the volume is included; for absorbed dose, all the energy deposited in the volume is included. Thus, charged particles entering the volume of interest contribute to absorbed dose, but not to kerma. Also, charged particles liberated by a photon in the volume of interest may leave it, carrying away part of their kinetic energy. This energy is included in kerma, but it does not contribute to the absorbed dose.

The largest differences between absorbed dose and kerma appear at interfaces between different materials, as there are differences in ionization density and in scattering properties of the materials. The changes in kerma at the boundaries are stepwise (scaled by the values of the mass energy transfer coefficient), but the changes in absorbed dose are gradual, extending to a region with dimensions comparable to the secondary particle ranges. Figure 3.2 shows graphs of the ratio of the mass energy transfer coefficients of some biological tissues, showing that, for photons with energies in the diagnostic radiology range, the changes in kerma may be very pronounced at the boundary of different tissues, even with the assumption that the photon fluence is constant at the interface. Kerma at the boundaries changes according to these factors.

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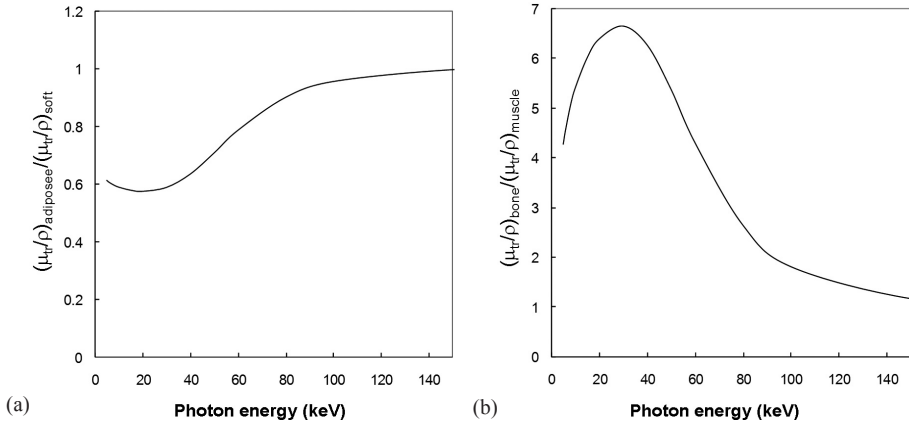


FIG. 3.2. Ratio of mass energy transfer coefficients for some tissue pairs: (a) adipose to soft tissue; (b) cortical bone to skeletal muscle.

Nonetheless, as Table 3.1 shows, the ranges of electrons set in motion by photons used in diagnostic radiology are small in biological tissues (water has been chosen in this table to simulate all the soft tissues), being less than 1 mm for most of the energies. This indicates that the changes in absorbed dose at the interface between two tissues in the body are limited to small regions. For comparison, Table 3.1 also shows the range of electrons of kinetic energy 1.0 MeV that are released by photons used for external radiotherapy. In this case, the changes in absorbed dose extend to a much greater distance — a few centimetres. This is further discussed in Section 3.3.

TABLE 3.1. RANGE OF ELECTRONS IN WATER AND IN BONE

Electron energy (keV)	Range in water ^a	Range in compact bone ^a
10	2.52 μm	1.49 μm
20	8.57 μm	5.05 μm
50	43.2 μm	25.3 μm
80	97.7 μm	57.1 μm
100	0.143 mm	0.084 mm
150	0.282 mm	0.164 mm
1000	0.437 cm	0.255 cm

^a Values of continuous slowing down approximation range obtained with the ESTAR program [3.1].

3.2.7. Diagnostic dosimeters

The experimental determination of kerma or absorbed dose and related dosimetric quantities in diagnostic radiology is described in Chapter 22 for patient dosimetry and Chapter 24 for radiation protection purposes. The measurements necessary include: determination of X ray tube output; patient dosimetry through the determination of incident or entrance air kerma, air kerma–area product (KAP) or internal organ doses; and control of doses to staff through area and individual monitoring.

According to the procedure, various dosimeters can be used. Dosimeters are devices used to determine absorbed dose or kerma, or their time rates, based on the evaluation of a detector physical property, which is dose dependent. Besides the detector, which is the sensitive part of the instrument, a dosimeter includes other components that convert the detector signal to the final result of the measurement — the absorbed dose or kerma value. The desired characteristics and the details of functioning of diagnostic dosimeters are treated in Chapter 21.

3.3. CHARGED PARTICLE EQUILIBRIUM IN DOSIMETRY

When a beam of uncharged ionizing particles irradiates a homogeneous material, the ionizing radiation field is transformed to a mixture of the incident beam (attenuated by the material), scattered radiation produced by the interaction of the incident beam in the material, bremsstrahlung radiation and charged particles. Accurate description of the components of the radiation field in a volume where absorbed dose or kerma is to be determined cannot be achieved with analytical methods. Both quantities can be determined using numerical methods (e.g. Monte Carlo simulation), or by experimental means, provided that certain assumptions are fulfilled. The charged particle equilibrium (CPE) in the volume makes experimental determination possible. This section deals with this concept for external photon irradiation.

3.3.1. CPE

An idealized irradiation geometry is shown in Fig. 3.3(a). The radiation field, composed of photons with energy $h\nu$, and normally incident on the block of material under consideration, comes from the left, from vacuum.

As the interactions of photons occur inside the material, electrons are liberated. Assume, for simplicity, that all these charged particles have the same direction in the material, the same energy and a straight track. These tracks are shown in the upper portion of Fig. 3.3(b), where a small volume, dV , inside the

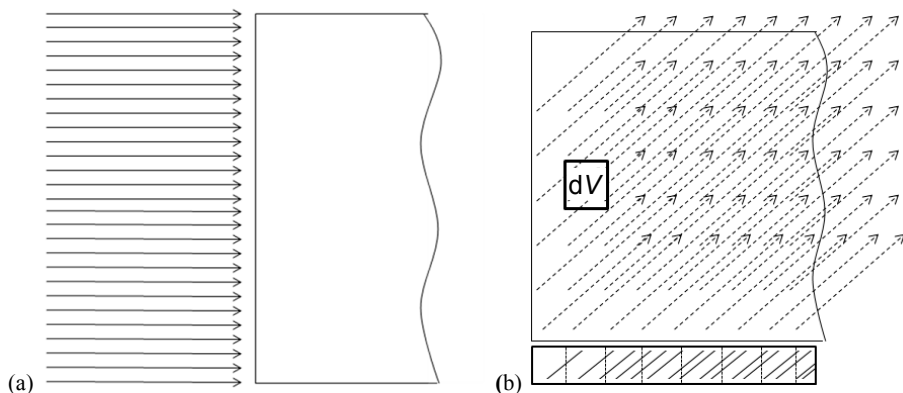


FIG. 3.3. (a) Geometry of a material irradiated from the left with a monoenergetic beam of photons; (b) tracks of the charged particles liberated in the material. The bottom section of the figure shows the path lengths of the charged particles.

material is also shown. Consider what happens as the position of the volume dV moves in a direction parallel to the incoming beam. The number of electron tracks that cross dV is small near the surface of the material, but increases as the volume moves to a greater depth, because more electrons are liberated by photon interactions. This is depicted in the diagram at the bottom of Fig. 3.3(b), where each cell represents the volume dV at a different depth inside the material. As the electron paths have finite lengths in the material (given by their ranges), the number of tracks reaches a maximum at a particular position of dV , and eventually begins to decrease, as the beam is attenuated for greater depths. The total path length of charged particles in each volume can be considered as representing the number of ionizations that occur in the volume. In this idealized situation, the expectation value of the ionization produced in volume dV will vary with depth, as shown in Fig. 3.4(a) for the simplified case where the photon fluence does not vary with depth.

The state of constant ionization shown in Fig. 3.4(a) is termed CPE because in this situation the charged particles that are liberated in the volume dV and leave the volume are balanced, in number and energy, by particles that were liberated elsewhere, and that enter volume dV .

Figure 3.4(b) shows the more realistic situation where the attenuation of the photon beam cannot be neglected and the photon fluence decreases with depth. As a consequence, the expectation value of the total ionization in volume dV increases initially but then decreases slowly with increasing depth in the medium. This state, at depths beyond the maximum of ionization, is called transient charged particle equilibrium. The depth at which the CPE is attained, or the maximum ionization is reached, is of the order of the charged particle range in the material.

Both graphs in Fig. 3.4 assume that the ionization at the surface of the material is zero. In fact, in a more realistic situation, even at the surface, the contribution of scattered radiation has to be taken into account, giving rise to non-zero ionization at the surface. Also, if the photon energies are restricted to the radiology energy range, the maximum ionization is found to be very near to the surface of the irradiated material. There are two main reasons for this: (i) the secondary electrons have a small range in biological tissues (examples are given in Table 3.1) and in solid materials are almost isotropically emitted and (ii) the incoherent scattering of low energy photons produces few photons that are scattered close to the forward direction (see Eqs (2.7, 2.21) and Fig. 2.6).

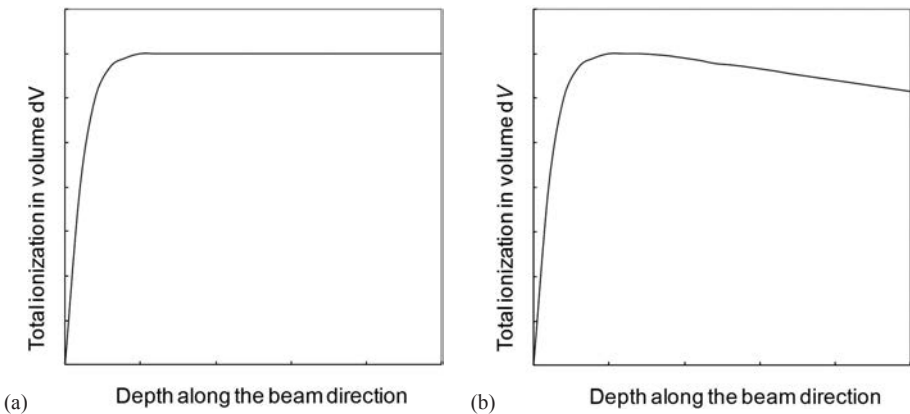


FIG. 3.4. Total ionization inside a volume dV as a function of the depth of the volume in the material, with the following assumptions: (a) the photon fluence is constant; (b) the photon beam is attenuated as it traverses the material.

3.3.2. Relationships between absorbed dose, collision kerma and exposure under CPE

For the photon beam shown in Fig. 3.3(a), the kerma and collision kerma at the entrance of the material are readily obtained by using Eqs (3.8, 3.9). As the material is homogeneous, the collision kerma values within the material vary in proportion to the incident fluence. When the number of interactions is so small that the fluence may be considered constant inside the medium, the variation of K_{col} with depth will be in accordance with Fig. 3.5(a). Usually, however, it is considered that the fluence decreases exponentially with depth in the material, with similar behaviour for K_{col} , as shown in Fig. 3.5(b).

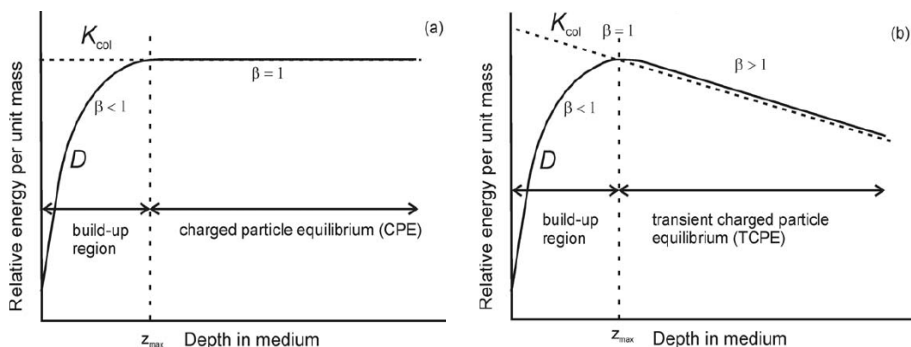


FIG. 3.5. Collision kerma and absorbed dose as a function of depth in a medium, irradiated by a high energy photon beam (see Ref. [3.2]).

As far as absorbed dose is concerned, the calculations are not so simple. First of all, because the absorbed dose, D , depends on the deposition of energy by charged particles, it is smaller at the surface of the material than inside it. As the photon interactions occur, electrons are liberated, contributing to the growth of the energy imparted, according to the growth of the ionization in the material, as shown in Fig. 3.4. Thus, there is a buildup region for the dose, at small depths in the medium, as shown in Fig. 3.5. The buildup region has dimensions (z_{\max}) similar to the range of the charged particles in the medium. For high energy photons, this region can extend to 1 or 2 cm, and this effect is responsible for the known effect of skin sparing in external radiotherapy. For diagnostic photon beams, the energies are lower, and the electron ranges are too small to produce this effect: the maximum dose is reached within the skin.

When the absorbed dose reaches the maximum value, assuming that all the radiative photons produced in the medium escape from the volume, there is a coincidence of its value and the collision kerma, as true CPE is achieved. Beyond the buildup region, assuming that the changes in photon fluence are small, as in Fig. 3.5(a), and the volume of interest has small dimensions compared with the electron range, giving rise to true CPE, the relation between absorbed dose and collision kerma remains:

$$D^{\text{CPE}} = K_{\text{col}} = \Phi h\nu \left(\frac{\mu_{\text{en}}}{\rho} \right) \quad (3.17)$$

For conditions such as those shown in Fig. 3.5(b), in transient charged particle equilibrium, where the attenuation of the photon beam is not negligible, there is no numerical coincidence between the values of the collision kerma and absorbed dose. Beyond the maximum, the absorbed dose is larger than the collision kerma, as the energy imparted is a result of charges liberated by photon

fluences slightly greater than the fluence in the volume of interest. Because there is practically a constant ratio between these quantities, it is usual to write:

$$D = \beta K_{\text{col}} \quad (3.18)$$

Usually, the approximation $\beta \approx 1$ can be used for diagnostic radiology and low Z materials.

3.3.3. Conditions that enable CPE or cause its failure

The simplification of the overall situation shown above should not be interpreted as the only condition that can give rise to a true equilibrium of charged particles in the medium. The necessary and sufficient conditions that guarantee the CPE are:

- (a) The medium is homogeneous in both atomic composition and mass density.
- (b) The photon field is homogeneous in the volume considered.

The first condition avoids changes in the charged particle distribution in the material owing to changes in scattering and absorption properties. The homogeneity in density is not as important as the constancy in composition, according to the Fano theorem (see Section 3.4.2). The second condition requires that the dimensions of the volume of interest are not very large, compared with the mean free path ($1/\mu$) of the photons.

Some examples of practical situations where there is a failure in the conditions, meaning that the CPE cannot be accomplished, are:

- (a) Large beam divergence, as with irradiations close to the radiation source;
- (b) Proximity of boundaries of the material and any other medium.

3.4. CAVITY THEORY

Determination of absorbed dose in an extended medium generally requires the use of a detector inside the medium. As the sensitive volume of the dosimeter is, in general, not made of the same material as the medium, its presence is a discontinuity. It is called a 'cavity' because the early basis of dosimetry was developed for gaseous detectors. Nowadays, the concept has been extended to liquid and solid detectors, but the traditional use of the letters g and w is retained to symbolize the cavity and the medium, respectively, as they come from the

gas and the wall that constitute an ionization chamber. The main interests of the cavity theory are to study the modifications of charge and radiation distribution produced in the medium by the cavity, and to establish relations between the dose in the sensitive volume of the dosimeter and the dose in the medium. A brief description of the foundations and the main results of the cavity theory for external irradiation with photons and applications to diagnostic radiology are given next.

3.4.1. Bragg–Gray cavity theory

A cavity can be of small, large or intermediate size compared with the range of the charged particles in the cavity. The Bragg–Gray theory deals with small cavities. W.H. Bragg began the development of the theory, in 1910, but it was L.H. Gray, during his PhD work, co-supervised by Bragg, who formalized the theory. The two main assumptions of this theory are:

- (i) The cavity dimensions are so small compared with the range of charged particles within it that the fluence of charged particles inside the cavity is not perturbed by the presence of the cavity.
- (ii) There are no interactions of uncharged particles in the cavity, so that the absorbed dose deposited in the cavity is due to the charged particles that cross the cavity.

The first assumption is equivalent to the requirement that the fluence of charged particles in the cavity is equal to that in the medium, w , that surrounds it, and the second assumption means that no charged particle starts or finishes its range in the cavity.

Under these conditions, the ratio of absorbed dose in the medium, w , surrounding the cavity, D_w , to that in the cavity, g (D_g), is given by the ratio of the integrals of the collision stopping powers of electrons in both media, weighted by the same electron fluence — the fluence energy distribution of the electrons in the medium $(d\Phi/dT)_w$:

$$\frac{D_w}{D_g} = \frac{\int_{T_{\min}}^{T_{\max}} \left(\frac{d\Phi}{dT} \right)_w \left(\frac{dT}{\rho dx} \right)_{c,w} dT}{\int_{T_{\min}}^{T_{\max}} \left(\frac{d\Phi}{dT} \right)_w \left(\frac{dT}{\rho dx} \right)_{c,g} dT} = \bar{\bar{S}}_g^w \quad (3.19)$$

The numerator and denominator in Eq. (3.19) are the generalizations of Eq. (3.20), which relates absorbed dose, D , to collision stopping power, $(dT/\rho dx)_{c,T}$, for irradiation with a fluence, Φ , of monoenergetic electrons:

$$D = \left(\frac{dT}{\rho dx} \right)_{c,T} \Phi \quad (3.20)$$

For the medium and the cavity materials, Eq. (3.19) assumes that there is a distribution of energies of electrons inside the cavity, which is equal to that outside. The symbol $\bar{\bar{S}}_g^w$ has the double bar to indicate that this ratio of average stopping powers considers both the average over the photon generated electron spectrum and the changes in this spectrum that are due to the continuous loss of kinetic energy in the materials.

3.4.2. The Fano theorem

The conditions required by the Bragg–Gray theory are better accomplished if the composition of the cavity is similar to that of the medium, i.e. if both cavity and medium have similar atomic numbers. This was observed in experiments with cavities filled with different gas compositions, and in 1954, U. Fano proved the following theorem:

“in a medium of given composition exposed to a uniform field of primary radiation the field of secondary radiation is also uniform and independent of the density of the medium, as well as of the density variations from point to point.”

The Fano theorem is important because it relaxes the requirements on the size of the cavity, which are very hard to meet, for instance, when the photon beam is of low energy. It should be noted, however, that the theorem is valid only for infinite media and in conditions where the stopping power is independent of density.

3.4.3. Other cavity sizes

Consider now the material surrounding the cavity, w , which is large enough to guarantee CPE in almost its entire volume, excluding a very small portion near the boundaries, but which does not disturb the photon fluence in the medium where it is immersed. The region of the medium m that surrounds w is also under

CPE conditions. Under these conditions, the dose to material w and the dose to the medium m are related by the expression:

$$\frac{D_m}{D_w} = \frac{\left(\frac{\mu_{\text{en}}}{\rho}\right)_m}{\left(\frac{\mu_{\text{en}}}{\rho}\right)_w} \quad (3.21)$$

Equation (3.21) is a simple ratio of mass energy absorption coefficients, and three conditions are implicit:

- (i) There is CPE in both media.
- (ii) The photon beam is monoenergetic.
- (iii) The photon fluence is the same for both media.

If the elemental compositions of w and m are not similar, the backscattering of photons at the boundary can change the photon fluence significantly, regardless of the dimensions of w .

For a spectrum of photons irradiating both materials, Eq. (3.21) may be integrated over the photon energy spectrum, giving:

$$\frac{D_m}{D_w} = \frac{\int_0^{h\nu_{\text{max}}} \left(\frac{d\Phi}{dh\nu}\right)_m \left(\frac{\mu_{\text{en}}}{\rho}\right)_m h\nu d(h\nu)}{\int_0^{h\nu_{\text{max}}} \left(\frac{d\Phi}{dh\nu}\right)_m \left(\frac{\mu_{\text{en}}}{\rho}\right)_w h\nu d(h\nu)} \equiv \left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_w^m \quad (3.22)$$

In this equation, $\left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_w^m$ is an average ratio of mass absorption energy coefficients, which takes into account the photon spectrum that irradiates equally both material, w , considered a large cavity, and m .

Cavities with intermediate sizes are usually treated by Burlin cavity theory, assuming that the cavity and the medium are in CPE and that the elemental compositions of both are similar. The theory will not be developed here, but Burlin's expression is given in Eq. (3.23), where the parameter d assumes values between 0 and 1, according to the cavity dimensions: $d \rightarrow 1$ for small cavities and $d \rightarrow 0$ for large ones.

$$\frac{D_g}{D_w} = d \bar{S}_w^g + (1-d) \left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_w^g \quad (3.23)$$

3.5. PRACTICAL DOSIMETRY WITH ION CHAMBERS

Ionization chambers are frequently used in diagnostic radiology. They are usually built with a wall that functions as a large cavity; besides containing the gas, it has a thickness that is enough to guarantee CPE in the wall. If the elemental composition of this wall w is similar to the composition of the medium m , where the dose is to be measured, and there is also CPE in the medium, it is possible to relate the dose in the medium to the dose in the wall using Eq. (3.21) or Eq. (3.22). On the other hand, the gas inside the ion chamber is irradiated mainly by the charged particles released in the wall that cross the gas volume according to the Bragg–Gray conditions. In this way, the dose to the material where the chamber is inserted can be obtained through Eq. (3.24), which combines Eqs (3.19, 3.22).

$$D_m = D_g \bar{S}_g^w \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_w^m \quad (3.24)$$

If, instead of the dose to the gas, the charge produced in the gas (Q) and the mass of the gas (m_g) are known, Eq. (3.23) reduces to:

$$D_m = \frac{Q}{m_g} \bar{W}_g \bar{S}_g^w \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_w^m \quad (3.25)$$

where \bar{W}_g is the mean energy spent in the gas to form an ion pair, as already described for air in Eq. (3.13).

A particularly useful (and common) situation occurs when the wall of the chamber is made of a material with the same atomic composition as the cavity, so that the dose to the cavity and the dose to the wall are considered equal. Under these circumstances, Eqs (3.24, 3.25) are simplified, reducing to:

$$D_m = D_g \left(\frac{\mu_{\text{en}}}{\rho} \right)_g^m \quad \text{for chambers with gas equivalent wall} \quad (3.26)$$

$$D_m = \frac{Q}{m_g} \bar{W}_g \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_g^m \quad \text{for chambers with gas equivalent wall} \quad (3.27)$$