Chapter Five

Dynamics of Fluid Flow

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5.1 / Introduction :

This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the forces causing flow.

5.2 / Equation of motion :

According to Newton's second law of motion , the net force F_x acting on a fluid element in the direction of x is equal to mass (m) of the fluid element multiplied by the acceleration (a) in the x – direction. Thus mathematically :

$$F_x = m\, a_x \qquad (5.1)$$

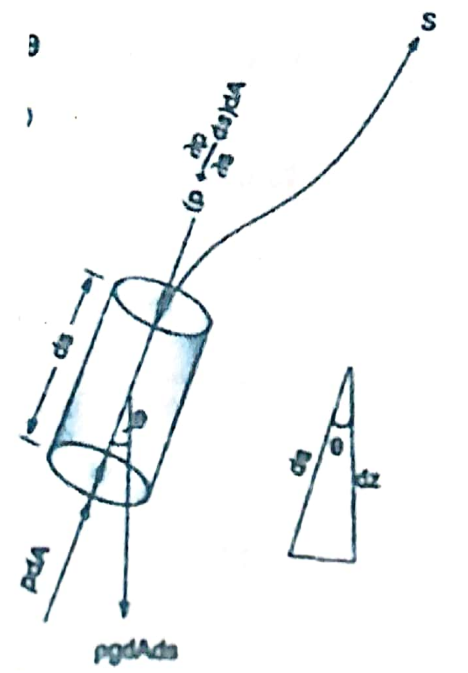
5.3 / Euler's equation of motion :

This equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of the fluid element along a stream – lines as :

Consider a stream – line in which flow is taking place in S – direction as shown in Fig.(5.1) . Consider a cylindrical element of cross – section dA and :

1. Pressure force (pdA) in the direction of flow .
2. Pressure force ($p + \frac{\partial p}{\partial s} ds$) dA opposite to the direction of flow .
3. Weight of element ($W = \gamma V = \rho g V = \rho g\, dA\, dS$) . $a_s = \frac{dv}{dt} = \frac{v \partial v}{\partial s}$

[(Note : (where v is a function of s & t , so , and ($\cos \Theta = \frac{dz}{ds}$)].



$F = m\,a$

$\sum F = m\,a$

$$p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho\, g\, dA\, dS \cos\Theta = \rho\, dA\, ds \,.\, \frac{v \partial v}{\partial s}$$

$$\frac{dp}{\rho} + g\,dz + v\,dv = 0 \qquad (5.2)$$

Equation (5.2) is known as Euler's equation of motion .

In which , p – pressure , ρ – mass density , g – gravity , z – head , v – velocity .

5.4 / Bernoulli's equation from Euler's equation :

Bernoulli's equation is obtained by integrating the Euler's equation of motion (5.2) :

$$\int \frac{dp}{\rho} + \int g\,dz + \int v dv = \text{constant}$$

If flow is compressible , ρ is constant and :

$$\frac{p}{\rho} + g\,z + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant} \qquad (5.3)$$

Equation (5.3) is a Bernoulli's equation , in which ,

$\frac{p}{\rho g}$ - pressure energy per unit weight of fluid (pressure head)

$\frac{v^2}{2g}$ - kinetic energy per unit weight of fluid (kinetic head).

Z - potential energy per unit weight of fluid (potential head).

<u>Assumptions :</u>

The following are the assumptions made in the derivation of Bernoulli's equation :

1 – The fluid is ideal (viscosity is zero).

2 – The flow is steady . ($\frac{\partial v}{\partial t} = 0$)

3 - The flow is incompressible. (ρ = constant) .

4 – The flow is irrotational .

5.5 / Bernoulli's equation for real fluid :

In real fluid , there are some losses , these losses have to be taken into consideration . Thus the Bernoulli's equation for real fluids between points 1 and 2 is given as :

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + H_L \qquad (5.4)$$

In which h_l is loss of energy (head loss) between points 1 & 2 .

5.6 / Instruments for measure the rate of flow :

1 – Venture meter .

2 – Orifice meter

3 – Pitot - tube .

1 – Venture meter :

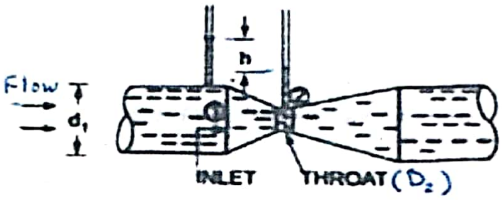


Fig.(5.2)

A venture meter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(1) short converging part , (2) Throat , (3) Diverging part .

It is based on the principle of Bernoulli's equation.

Applying Bernoulli equation at section 1 & section 2 (throat) , we get :

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As pipe is horizontal , hence $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad , \quad \frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2\,g} - \frac{V_1^2}{2\,g}$$

$\frac{P_1 - P_2}{\rho g}$ is the difference of pressure heads at section 1 & section 2

And it is equal to(h) :

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad (5.5)$$

Now , applying continuity equation at section 1 & 2 :

$$A_1 V_1 = A_2 V_2 \quad , \quad V_1 = \frac{A_2 V_2}{A_1}$$

Substituting the of V_1 in the equation (5 .5) ,

$$h = \frac{V_2^2}{2g} \left[\frac{A_1^2 - A_2^2}{A_1^2} \right]$$

$$V_2^2 = 2g\,h \frac{A_1^2}{A_1^2 - A_2^2}$$

$$V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

But , $Q = A_2 V_2$

$$Q = \frac{A_2 A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh} \qquad (5.6)$$

Equation (5.6) gives the discharge under ideal conditions and is called (theoretical discharge) , Actual discharge will be less than theoretical discharge :

$$Q = C_d \frac{A_2 A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh} \qquad (5.7)$$

Where C_d is Coefficient of discharge . The value of C_d is less than one .

The value of (h) given by differential U – tube manometer .

Note : If the liquid manometer is heaver than the liquid flowing through the pipe :

$$h = x \left[\frac{S_h}{S_o} - 1 \right]$$