Al-Mustaqbal University

Department of Medical Instrumentation Techniques Engineering

Mathematics /2

second stage

LECTURE 1

vector analysis

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Vector analysis is the branch of mathematics that was developed to describe quantities that are both directional in nature and distributed over regions of space. In this chapter, we will discuss the elements of vector analysis that are directly applicable to electromagnetic phenomena. Our discussion will start by defining the concept of a physical quantity, and then identifying the properties of scalar and vector fields. The remainder of our discussion will be devoted to development of the algebra and calculus of vector fields.

- *Vector*: parameters possessing magnitude and direction which add according to the parallelogram law. Examples: displacements, velocities, accelerations.
- *Scalar*: parameters possessing magnitude but not direction. Examples: mass, volume, temperature
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- Vector classifications:
    - *Fixed* or *bound* vectors have well defined points of application that cannot be changed without affecting an analysis.
    - *Free* vectors may be freely moved in space without changing their effect on an analysis.

  Sliding vectors may be applied anywhere along their line of action without affecting an analysis

- *Equal* vectors have the same magnitude and direction.

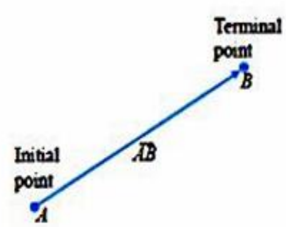
Negative vector of a given vector has the same magnitude and the opposite direction

Scalar and Vector

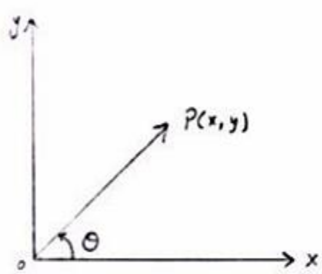
Scalar: is a quantity like volume, temperature and time, which have magnitude only. The scalar is number.

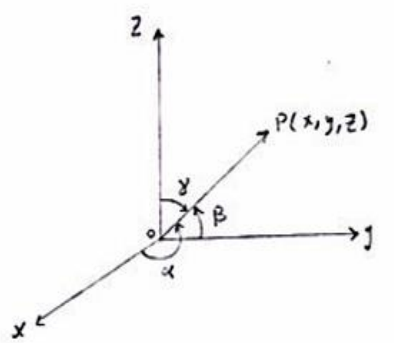
Vector: is a quantity like velocity, acceleration and force, which have magnitude and direction.

The vector is represented by a directed line segment (arrow) AB from one point A called the initial point to another point B called the terminal point, as show in figure below.



The vector $\vec{A}$ can be represented in plane and in the space as follows:





θ determines the direction of $\vec{A}$ in the plane (2D).

α, β, γ determine the direction of $\vec{A}$ in the space (3D).

Some of the important applications of vectors in real life are listed below:

1- The direction in which the force is applied to move the object can be found using vectors.

2- To understand how gravity uses a force of attraction on an object to work.

3- The motion of a body which is confined to a plane can be obtained using vectors.

4- Vectors help in defining the force applied on a body simultaneously in the three dimensions.

5 - Vectors are used in the field of Engineering, where the force is much stronger than the structure will sustain, else it will collapse.

6 - In various oscillators, vectors are used.

7- Vectors also have its applications in 'Quantum Mechanics'.

8- The velocity in a pipe can be determined in terms of the vector field—for example, fluid mechanics.

9- We may also observe them everywhere in the general relativity.

10- Vectors are used in various wave propagations such as vibration propagation, sound propagation, AC wave propagation, and so on.

<u>Operations on vectors</u>

Vector Addition

Vectors are added by adding their corresponding components. For example, if you have two vectors:

A= (A_x, A_y) *and* $B = (B_x, B_y)$

$$\text{The sum } C = A + B \text{ is given by}$$

$$C = \left(A_x + B_x\,, A_y + B_y\right)$$

Vector Subtraction

Vector subtraction is done similarly, by subtracting the corresponding components:

C= A-B ,

$$C = \left(A_x - B_x\ , A_y - B_y\right)$$

Multiplication by a Constant (Scalar Multiplication)

When you multiply a vector by a scalar (a constant), you multiply each component of the vector by that scalar. For a vector $A = \left(A_x, A_y\right)$

And a scalar k $kA = \left(kA_x\ ,\ \ k\ A_y\right)$

Example

The vector A= (2,3) and B= (4,1)

Addition:

- C=A+B= (2+4, 3+1) = (6, 4)

Subtraction:

- C=A−B= (2−4, 3−1) = (−2, 2)

Scalar Multiplication (e.g., k=3)

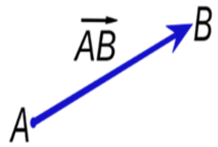
3 A = 3 (2,3) = (6,9)
3 B = 3(4,1) = (12,3)

Vectors:

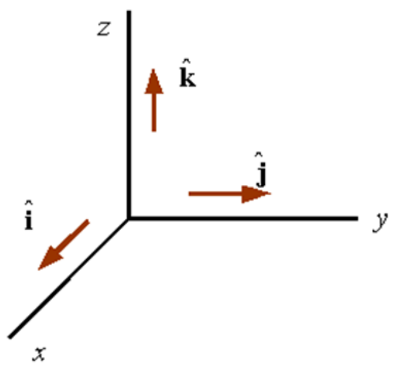
- Vector between two points
- Length of the vectors
- Unite vector
- Dot Product and Cross Product
- Angle between two vectors
- Projection
- Area between two vectors

Vector quantities are extremely useful in engineering and physics. The important characteristic of a vector quantity is that it has both a magnitude (or size) and a direction. Both of these properties must be given in order to specify a vector completely.

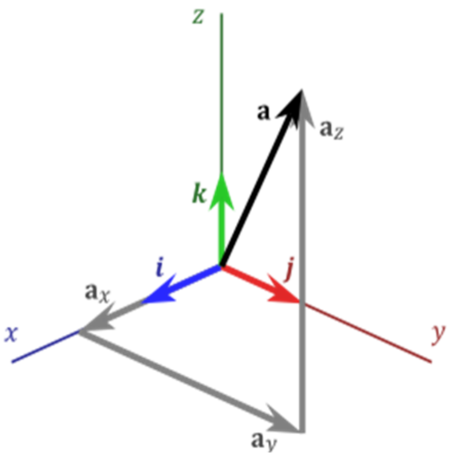
A vector has **magnitude** (Length) and **direction**:

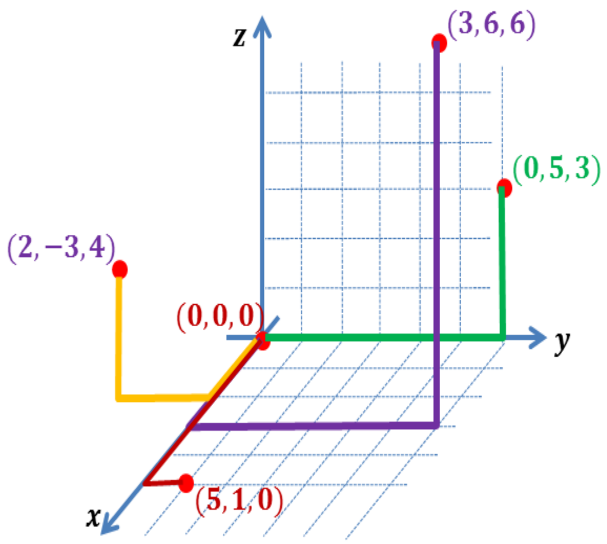


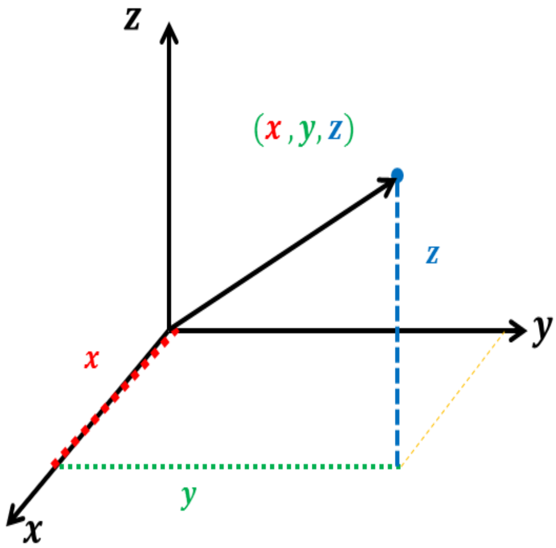
Vector components:



$$\vec{a} = a_x i + a_y j + a_z k$$

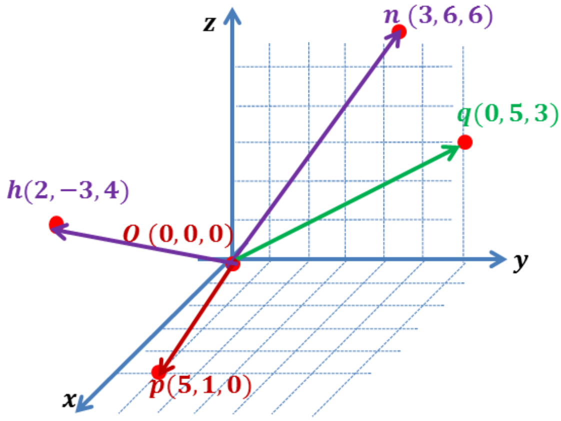






Vector and the Vector Length

Find the vector between the following points



$$\overrightarrow{O\,p} = (5-0)i + (1-0)j + (0-0)k \Rightarrow \overrightarrow{O\,p} = 5\,i + j + 0k$$

$$\overrightarrow{O\,q} = (0-0)i + (5-0)j + (3-0)k \Rightarrow \overrightarrow{O\,q} = 0\,i + 5j + 3k$$

$$\overrightarrow{O\,n} = (3-0)i + (6-0)j + (6-0)k \Rightarrow \overrightarrow{O\,n} = 3\,i + 6j + 6k$$

$$\overrightarrow{O\,h} = (2-0)i + (-3-0)j + (4-0)k \Rightarrow \overrightarrow{O\,h} = 2\,i - 3j + 4k$$

$$\overrightarrow{h\,p} = (5-2)i + (1+3)j + (0-4)k \Rightarrow \overrightarrow{h\,p} = 3\,i + 4j - 4k$$

$$\overrightarrow{q\,p} = (5-0)i + (1-5)j + (0-3)k \Rightarrow \overrightarrow{q\,p} = 5\,i - 4j - 3k$$

Vector Length $|\overline{p_1p_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$$|\overline{op}| = \sqrt{(5-0)^2 + (1-0)^2 + (0-0)^2} \Rightarrow |\overline{op}| = \sqrt{26}\ \textit{unit length}$$

$$|\overline{O\,q}| = \sqrt{(0-0)^2 + (5-0)^2 + (3-0)^2} \Rightarrow |\overline{O\,q}| = \sqrt{34}\ \textit{unit length}$$

$$|\overline{O\,n}| = \sqrt{(3)^2 + (6)^2 + (6)^2} \Rightarrow |\overline{O\,n}| = 9\ \textit{unit length}$$

Unit Vector The vector of the length 1 is called a unit vector. If the vector $\vec{V} = ai + bj + ck$, The unit vector of the vector $\vec{V}$ is

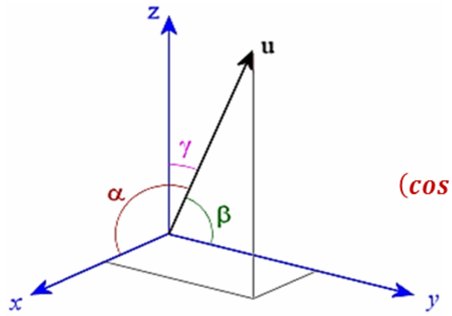
$$\vec{u}_v = \frac{\vec{V}}{|\vec{V}|} \Rightarrow \overrightarrow{U_v} = \frac{ai + bj + ck}{\sqrt{a^2 + b^2 + c^2}} \qquad \vec{u}_v = \frac{a}{|V|}i + \frac{b}{|V|}j + \frac{c}{|V|}k \qquad \vec{u}_v = \frac{Vx}{|V|}i + \frac{Vy}{|V|}j + \frac{Vz}{|V|}k$$

Example: find the unit vector of $\vec{V}$ where $\vec{V} = 2i + 3j + 4k$ unit vector $\vec{u}_v = \frac{\vec{V}}{|\vec{V}|} \Rightarrow \overrightarrow{U_v} = \frac{2i + 3j + 4k}{\sqrt{2^2 + 3^2 + 4^2}}$

$$\cos\alpha = \frac{2}{\sqrt{2^2 + 3^2 + 4^2}}$$

$$\cos\beta = \frac{3}{\sqrt{2^2 + 3^2 + 4^2}}$$

$$\cos\gamma = \frac{4}{\sqrt{2^2 + 3^2 + 4^2}}$$



$$(\cos\alpha)^2 + (\cos\beta)^2 + (\cos\gamma)^2 = 1$$

Example: Find the components of the force $(F = 50)$ in the direction from

1. $B\,(-1, 2, -1)\ to\ C(3, 4, 5)$
2. $H\,(1, 0, -1)\ to\ M(-2, 1, -1)$

$$\textit{The componnent of the force } \vec{F} = F\ \overrightarrow{U_v} = \frac{ai + bj + ck}{\sqrt{a^2 + b^2 + c^2}}$$

1. $B\,(-1, 2, -1)\ to\ C(3, 4, 5)$, $\quad \vec{F} = F\ \overrightarrow{U_v} = 50\left(\frac{(3+1)i + (4-2)j + (5+1)k}{\sqrt{(3+1)^2 + (4-2)^2 + (5+1)^2}}\right)$,

$$\vec{F} = F\ \overrightarrow{U_v} = 50\ \frac{4i + 2j + 6k}{\sqrt{4^2 + 2^2 + 6^2}}$$

$$\vec{F} = F\ \overrightarrow{U_v} = \frac{50}{\sqrt{56}}(4i + 2j + 6k)$$

2. $H\,(1, 0, -1)\ to\ M(-2, 1, -1)$, $\quad \vec{F} = F\ \overrightarrow{U_v} = 50\left(\frac{(-2-1)i + (1-0)j + (-1+1)k}{\sqrt{(-2-1)^2 + (1-0)^2 + (-1+1)^2}}\right)$,

$$\vec{F} = F\ \overrightarrow{U_v} = 50\ \frac{-3i + j + 0k}{\sqrt{9 + 1 + 0}}$$

$$\vec{F} = F\ \overrightarrow{U_v} = \frac{50}{\sqrt{10}}(-3i + j)$$