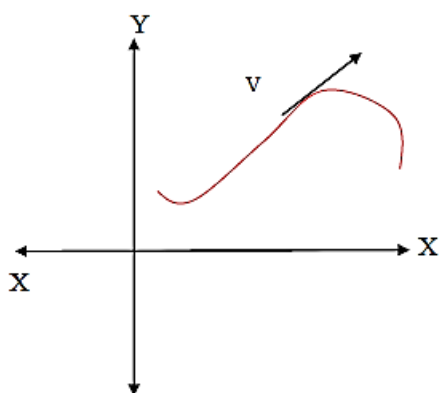


Mathematic I

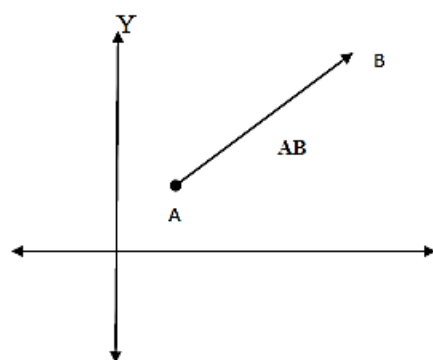
chapter two

2.1 Vector

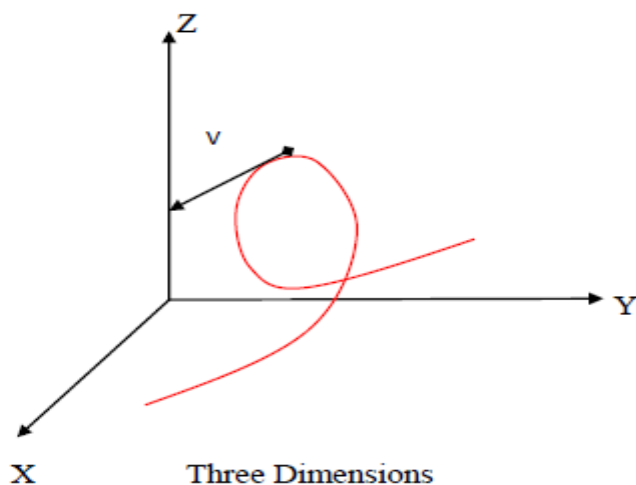
A quality such as force, displacement, or velocity is called a vector and represented by a direct line segment.



Two Dimensions



initial and terminal points.



Three Dimensions

Definition of Vector:

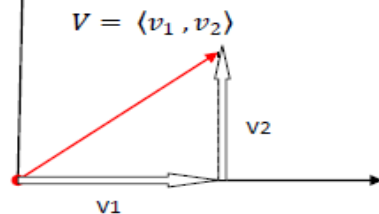
A vector is a directed line segment $\overrightarrow{AB}$ has initial point (A) and terminal point (B), it's length denoted by $|\overrightarrow{AB}|$.

If $\vec{V}$ is a two dimensional vector in plane equal to the vector with initial points at the origin and terminal point (v_1, v_2) , then the component of (V):

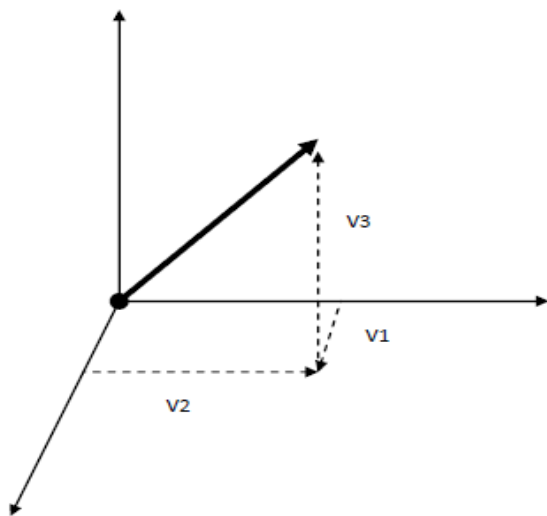
$$V = \langle v_1, v_2 \rangle$$

If $\vec{V}$ is a three-dimensional vector with initial point at the origin at the terminal point (v_1, v_2, v_3) , then the component of (V):

$$V = \langle v_1, v_2, v_3 \rangle$$



Component of two dimensional
Vector



Component of three dimensional vectors

The magnitude of length of the vector $v = \overrightarrow{PQ}$ is the non-negative number.

$$|V| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Vectors Operation:

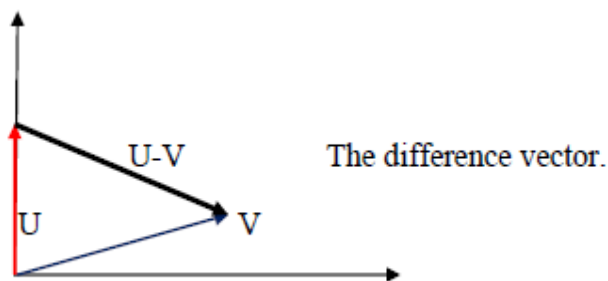
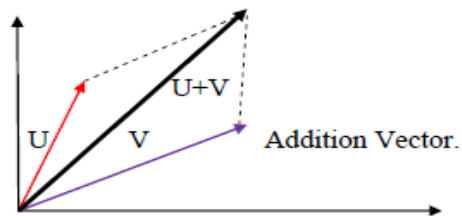
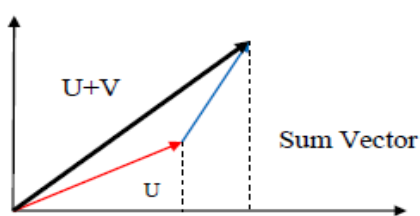
Vector Addition multiplication of a vector by scalar:

Let $U = (u_1, u_2, u_3)$, and $V = (v_1, v_2, v_3)$

K is scalar.

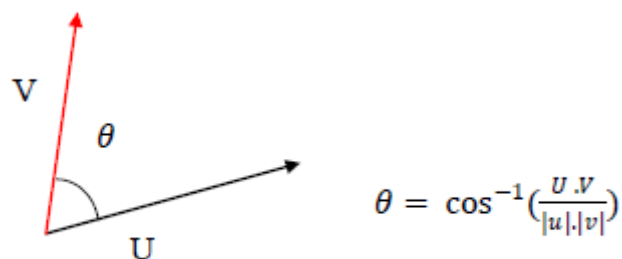
Addition: $U + V = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$

Scalar multiplication: $K \cdot U = (K \cdot u_1, K \cdot u_2, K \cdot u_3)$



Properties of Vector

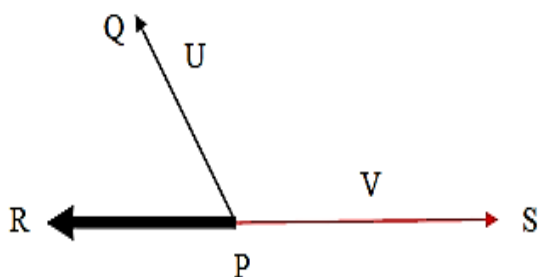
1. $U \pm V = V \pm U$
2. $a(U + V) = aU + aV$
3. $U \cdot V = V \cdot U$ (Dot product)
4. angle between two vector



5. Vector Projection.

$$U = \overrightarrow{PQ}$$

$$V = \overrightarrow{PS}$$



The vector projection of $U = \overrightarrow{PQ}$ onto non-zero vector $V = \overrightarrow{PS}$ is the vector $\overrightarrow{PR}$.

$Proj_V U$ (the vector Projection U onto V)

$$Proj_V U = (|U| \cdot \cos \theta) \cdot \frac{V}{|V|}$$

$$\therefore |U| \cdot \cos \theta = \frac{U \cdot V}{|V|}$$

$$Proj_V U = \left(\frac{U \cdot V}{|V|}\right) \cdot \frac{V}{|V|} = \left(\frac{U \cdot V}{|V|^2}\right) \cdot V \quad (\text{vector value}).$$

Example 1: Find the component form and length of vector with initial point P (-3, 4, 1) and terminal point Q (-5, 2, 2):

Solution:

a) The standard position vector V representing $\overrightarrow{PQ}$ has components:

$$v_1 = x_2 - x_1 = -5 - 3 = -8$$

$$v_2 = y_2 - y_1 = 2 - 4 = -2$$

$$v_3 = z_2 - z_1 = 2 - 1 = 1$$

The component form of $\overrightarrow{PQ}$ is:

$$V = (-8, -2, 1)$$

b) The magnitude of vector length $|V|$ or $|\overrightarrow{PQ}|$ is:

$$|V| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$|V| = \sqrt{(-8)^2 + (-2)^2 + (1)^2} = \sqrt{69} = \sqrt{69}$$

Example 1: Let $U = (-1, 3, 1)$, $V = (4, 7, 0)$, Find:

1. $2U + 3V$:

$$2U + 3V = 2(-1, 3, 1) + 3(4, 7, 0) = (-2, 6, 2) + (12, 21, 0) = (10, 27, 2)$$

2. $U - V =$

$$U - V = (-1, 3, 1) - (4, 7, 0) = (-5, -4, 1)$$

Example 5: Find the dot product of $U \cdot V$, where $U = (1, -2, -1)$, and $V = (-6, 2, -3)$.

Solution:

$$\begin{aligned}U \cdot V &= (1 \times -6) + (-2 \times 2) + (-1 \times -3) \\&= -6 - 4 + 3 = -7\end{aligned}$$

Example 6: Find the angle θ between $U = i - 2j - 2k$, and $V = 6i + 3j + 2k$.

Solution:

$$\begin{aligned}U \cdot V &= (1 \times 6) + (-2 \times 3) + (-2 \times 2) \\&= 6 - 6 - 4 = -4\end{aligned}$$

$$|U| = \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{9} = 3$$

$$|V| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = 7$$

$$\theta = \cos^{-1} \frac{U \cdot V}{|U| \cdot |V|}$$

$$\theta = \cos^{-1} \frac{(-4)}{3 \times 7} \cong 1.76 \text{ rad.}$$

Example Find the vector projection of $U = 6i + 3j + 2k$ onto $V = i - 2j - 2k$. and the scalar component of U in the direction of V .

Solution:

1) Find $\text{Proj}_V u$:

$$\text{Proj}_V u = \frac{U \cdot V}{|V| \cdot |V|} \cdot V$$

$$\text{Proj}_V u = \frac{(6 \times 1) + (3 \times -2) + (2 \times -2)}{(1^2 + (-2)^2 + (-2)^2)} \cdot (i - 2j - 2k)$$

$$\text{Proj}_V u = \frac{-4}{9} \cdot (i - 2j - 2k) = \frac{-4}{9} i + \frac{8}{9} j + \frac{8}{9} k$$

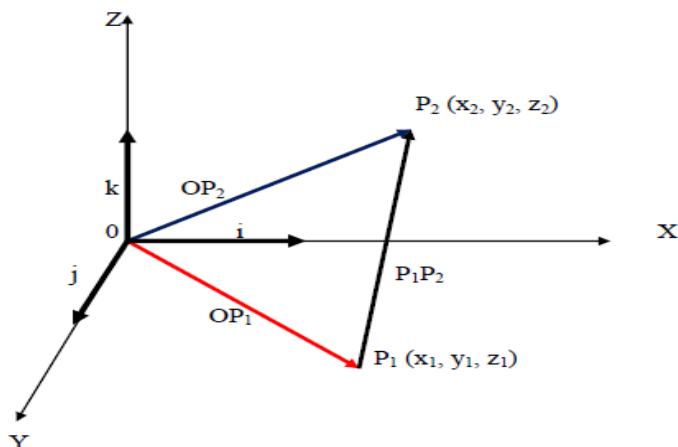
2) The scalar component of U in the direction of V .

$$|U| \cdot \cos \theta = \frac{U \cdot V}{|V|} = \frac{(6 \times 6) + (3 \times -2) + (2 \times -2)}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}} = \frac{6 - 6 + 4}{\sqrt{9}} = \frac{4}{3}$$

Unit Vector

The length of vector is called **unit vector**. The standard of units vectors are:

$$i = (1, 0, 0), \quad j = (0, 1, 0), \quad k = (0, 0, 1)$$



$$V = v_1 i + v_2 j + v_3 k$$

i, j , and k as scalar vector.

i = component of vector v_1 , j = component of vector v_2

k = component of vector v_3

$$V_1=(X_2-X_1) , V_2=(Y_2-Y_1) , V_3=(Z_2-Z_1)$$

$$\text{Unit vector} = \frac{v}{|v|} = \frac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|}$$

Example 2: Find a unit vector in the direction of the vector $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.

Solution:

$$\begin{aligned}\overrightarrow{P_1P_2} &= V_1 i + v_2 j + v_3 k \\ &= (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k \\ &= (3-1)i + (2-0)j + (0-1)k = 2i + 2j - k\end{aligned}$$

$$\begin{aligned}2. |\overrightarrow{P_1P_2}| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{(3 - 1)^2 + (2 - 0)^2 + (0 - 1)^2} \\ &= \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3\end{aligned}$$

3. Unit vector:

$$\begin{aligned}U &= \frac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|} = \frac{2i + 2j - k}{3} \\ U &= \frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k\end{aligned}$$

