DeMorgan's Theorems

DeMorgan's first theorem is stated as follows:

The complement of a product of variables is equal to the sum of the complements of the variables.

To apply DeMorgan's theorem, break the bar over the product of variables and change the sign from AND to OR.

The complement of two or more AND variables is equivalent to the OR of the

Complements of the individual variables.

The formula for expressing this theorem for two variables is:

$$\overline{XY} = \overline{X} + \overline{Y} \qquad \text{Equation 1}$$

DeMorgan's second theorem is stated as follows:

The complement of a sum of variables is equal to the product of the complements of the variables.

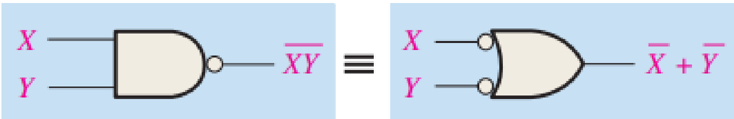
Stated another way,

The complement of two or more ORed variables is equivalent to the AND of the complements of the individual variables.

The formula for expressing this theorem for two variables is:

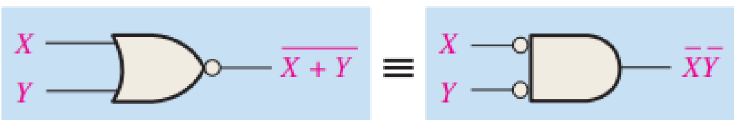
$$\overline{X + Y} = \overline{X}\,\overline{Y} \qquad \text{Equation 2}$$

Figure 1: shows the gate equivalencies and truth tables for Equations 1 and 2 above.



NAND   Negative-OR

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



NOR   Negative-AND

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

EXAMPLE 4–3

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$
$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

Related Problem

Apply DeMorgan's theorem to the expression $\overline{\overline{X} + \overline{Y} + \overline{\overline{Z}}}$.

EXAMPLE 4–4

Apply DeMorgan's theorems to the expressions $\overline{WXYZ}$ and $\overline{W + X + Y + Z}$.

Solution

$$\overline{WXYZ} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$$
$$\overline{W + X + Y + Z} = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}$$

Related Problem

Apply DeMorgan's theorem to the expression $\overline{\overline{W}\,\overline{X}\,\overline{Y}\,\overline{\overline{Z}}}$.

Applying DeMorgan's Theorems

The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression

$$\overline{\overline{A + B\overline{C}} + D(\overline{E + \overline{F}})}$$

Step 1: Identify the terms to which you can apply DeMorgan's theorems, and think of each term as a single variable. Let $\overline{A + B\overline{C}} = X$ and $D(\overline{E + \overline{F}}) = Y$.

Step 2: Since $\overline{X + Y} = \overline{X}\,\overline{Y}$,

$$\overline{(\overline{A + B\overline{C}}) + (D(\overline{E + \overline{F}}))} = (\overline{\overline{A + B\overline{C}}})(\overline{D(\overline{E + \overline{F}})})$$

Step 3: Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).

$$(\overline{\overline{A + B\overline{C}}})(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D(\overline{E + \overline{F}})})$$

Step 4: Apply DeMorgan's theorem to the second term.

$$(A + B\overline{C})(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D} + (\overline{\overline{E + \overline{F}}}))$$

Step 5: Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the $E + \overline{F}$ part of the term.

$$(A + B\overline{C})(\overline{D} + \overline{\overline{E + \overline{F}}}) = (A + B\overline{C})(\overline{D} + E + \overline{F})$$

The following three examples will further illustrate how to use DeMorgan's theorems.

EXAMPLE 4–5

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$

(b) $\overline{ABC + DEF}$

(c) $\overline{A\overline{B} + \overline{C}D + EF}$

Solution

(a) Let $A + B + C = X$ and $D = Y$. The expression $\overline{(A + B + C)D}$ is of the form $\overline{XY} = \overline{X} + \overline{Y}$ and can be rewritten as

$$\overline{(A + B + C)D} = \overline{A + B + C} + \overline{D}$$

Next, apply DeMorgan's theorem to the term $\overline{A + B + C}$.

$$\overline{A + B + C} + \overline{D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$$

(b) Let $ABC = X$ and $DEF = Y$. The expression $\overline{ABC + DEF}$ is of the form $\overline{X + Y} = \overline{X}\,\overline{Y}$ and can be rewritten as

$$\overline{ABC + DEF} = (\overline{ABC})(\overline{DEF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{ABC}$ and $\overline{DEF}$.

$$(\overline{ABC})(\overline{DEF}) = (\overline{A} + \overline{B} + \overline{C})(\overline{D} + \overline{E} + \overline{F})$$

(c) Let $A\overline{B} = X$, $\overline{C}D = Y$, and $EF = Z$. The expression $\overline{A\overline{B} + \overline{C}D + EF}$ is of the form $\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$ and can be rewritten as

$$\overline{A\overline{B} + \overline{C}D + EF} = (\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{A\overline{B}}$, $\overline{\overline{C}D}$, and $\overline{EF}$.

$$(\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF}) = (\overline{A} + B)(C + \overline{D})(\overline{E} + \overline{F})$$

Related Problem

Apply DeMorgan's theorems to the expression $\overline{\overline{ABC} + D + E}$.

EXAMPLE 4–6

Apply DeMorgan's theorems to each expression:

(a) $\overline{\overline{(A + B)} + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Solution

(a) $\overline{\overline{(A + B)} + \overline{C}} = (\overline{\overline{A + B}})\overline{\overline{C}} = (A + B)C$

(b) $\overline{(\overline{A} + B) + CD} = (\overline{\overline{A} + B})\overline{CD} = (\overline{\overline{A}}\,\overline{B})(\overline{C} + \overline{D}) = A\overline{B}(\overline{C} + \overline{D})$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = (\overline{(A + B)\overline{C}\,\overline{D}})(\overline{E + \overline{F}}) = (\overline{A}\,\overline{B} + C + D)\overline{E}F$

Related Problem

Apply DeMorgan's theorems to the expression $\overline{\overline{A}B(C + \overline{D}) + E}$.

Logic Simplification Using Boolean Algebra:

A logic expression can be reduced to its simplest form or changed to a more convenient form to implement the expression most efficiently using Boolean algebra.

The approach taken in this section is to use the basic laws, rules, and theorems of Boolean algebra to manipulate and simplify an expression.

This method depends on a thorough knowledge of Boolean algebra and considerable practice in its application.

- Apply the laws, rules, and theorems of Boolean algebra to simplify general expressions.

EXAMPLE 4–9

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

Solution

The following is not necessarily the only approach.

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

Step 2: Apply rule 7 ($BB = B$) to the fourth term.

$$AB + AB + AC + B + BC$$

Step 3: Apply rule 5 ($AB + AB = AB$) to the first two terms.

$$AB + AC + B + BC$$

Step 4: Apply rule 10 ($B + BC = B$) to the last two terms.

$$AB + AC + B$$

Step 5: Apply rule 10 ($AB + B = B$) to the first and third terms.

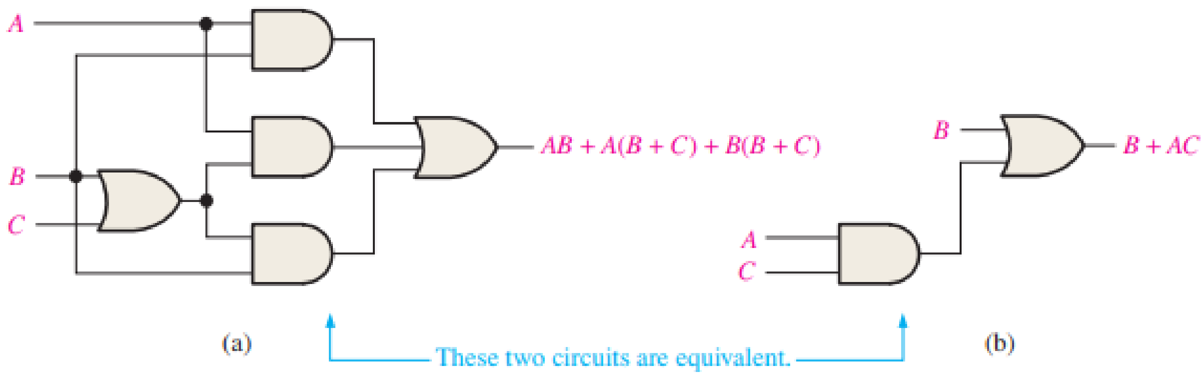
$$B + AC$$

At this point the expression is simplified as much as possible. Once you gain experience in applying Boolean algebra, you can often combine many individual steps.

Simplification means fewer gates for the same function.

Figure below shows that the simplification process in Example 4–9 has significantly reduced the number of logic gates required to implement the expression. Part (a) shows that five gates are required to implement the expression in its original form;

however, only two gates are needed for the simplified expression, shown in part (b). It is important to realize that these two gate circuits are equivalent. That is, for any combination of levels on the A, B, and C inputs, you get the same output from either circuit.



EX: Using Boolean algebra and Demorgan's Theorems to Simplify This Expression:

$Y = (A'B'C' + A'B'C' + A'BC' + A'BC)'$

$= (A'B'C')' \, (A'B'C)' \, (A'BC')' \, (A'BC)'$

$= (A'' + B'' + C'') \, (A'' + B'' + C') \, (A'' + B' + C'') \, (A'' + B' + C')$

$= (A+B+C) \, (A+B+C') \, (A+B'+C) \, (A+B'+C')$

$= AA + AB + AC' + AB + BB + BC' + AC + BC + CC'$

$= A + AB + AC' + B + BC' + AC + BC$

$= A\,(1+B) + A\,(C'+C) + B\,(C+C') + B$

$= A + B$

$= AA + AB' + AC' + AB' + B'B' + B'C' + AC + B'C + CC'$

$= A + AB' + AC' + B' + B'C' + AC + B'C$

$= A\,(1+B') + A\,(C'+C) + B'\,(C+C') + B'$

$= (A+B)\,(A+B')$

$= A + AB + AB' + B'B$

$= A + AB'$

$= A$

Ex1: Design a logic circuit to multiply two (2-bit) numbers using minimum number of logic gates.

A1	A0	B1	B0	P3	P2	P1	P0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	1	1	0
1	1	1	1	1	0	0	1

	$\overline{B1}\,\overline{B0}$	$\overline{B1}B0$	$B1B0$	$B1\overline{B0}$
$\overline{A1}\,\overline{A0}$	0	0	0	0
$\overline{A1}A0$	0	1	1	0
$A1A0$	0	1	1	0
$A1\overline{A0}$	0	0	0	0

P0 = A0B0

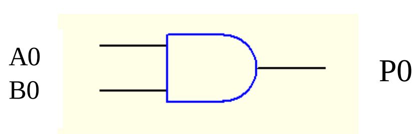
	$\overline{B1}\,\overline{B0}$	$\overline{B1}B0$	$B1B0$	$B1\overline{B0}$
$\overline{A1}\,\overline{A0}$	0	0	0	0
$\overline{A1}A0$	0	0	1	1
$A1A0$	0	1	0	1
$A1\overline{A0}$	0	1	1	0

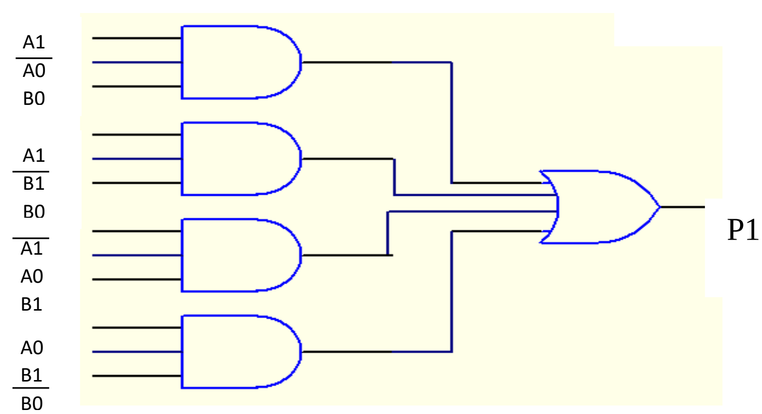
$P1 = A1\overline{A0}B0 + A1\overline{B1}B0 + \overline{A1}A0B1 + A0B1\overline{B0}$

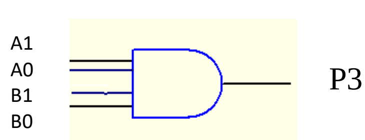
	$\overline{B1}\,\overline{B0}$	$\overline{B1}B0$	$B1B0$	$B1\overline{B0}$
$\overline{A1}\,\overline{A0}$	0	0	0	0
$\overline{A1}A0$	0	0	0	0
$A1A0$	0	0	0	1
$A1\overline{A0}$	0	0	1	1

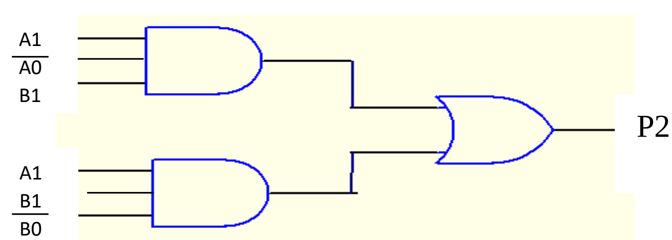
$P2 = A1\overline{A0}B1 + A1B1\overline{B0}$

$P3 = A1A0B1B0$









Ex2: Design a logic circuit to compare between two (2-bit) numbers.

A1	A0	B1	B0	A=B	A>B	A<B
0	0	0	0	1	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	0	1	0
0	1	0	1	1	0	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	1	0	0
1	0	1	1	0	0	1
1	1	0	0	0	1	0
1	1	0	1	0	1	0
1	1	1	0	0	1	0
1	1	1	1	1	0	0

	$\bar{B}1\bar{B}0$	$\bar{B}1B0$	$B1B0$	$B1\bar{B}0$
$\bar{A}1\bar{A}0$	1	0	0	0
$\bar{A}1A0$	0	1	0	0
$A1A0$	0	0	1	0
$A1\bar{A}0$	0	0	0	1

A = B : $\bar{A}1\bar{A}0\bar{B}1\bar{B}0 + \bar{A}1A0\bar{B}1B0 + A1A0B1B0 + A1\bar{A}0B1\bar{B}0$

$= \bar{A}1\bar{B}1\,(\bar{A}0\bar{B}0 + A0B0) + A1B1\,(A0B0 + \bar{A}0\bar{B}0)$

$= \bar{A}1\bar{B}1\,(\overline{A0 \oplus B0}) + A1B1\,(\overline{A0 \oplus B0})$

Let $\overline{A0 \oplus B0} = X$

$= \overline{A1}\,\overline{B1}X + A1B1X$

$= X\,(\overline{A1 \oplus B1})$

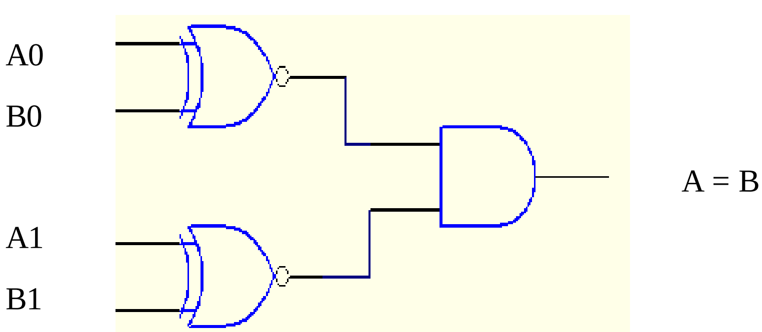
$= (\overline{A0 \oplus B0}) \cdot (\overline{A1 \oplus B1})$

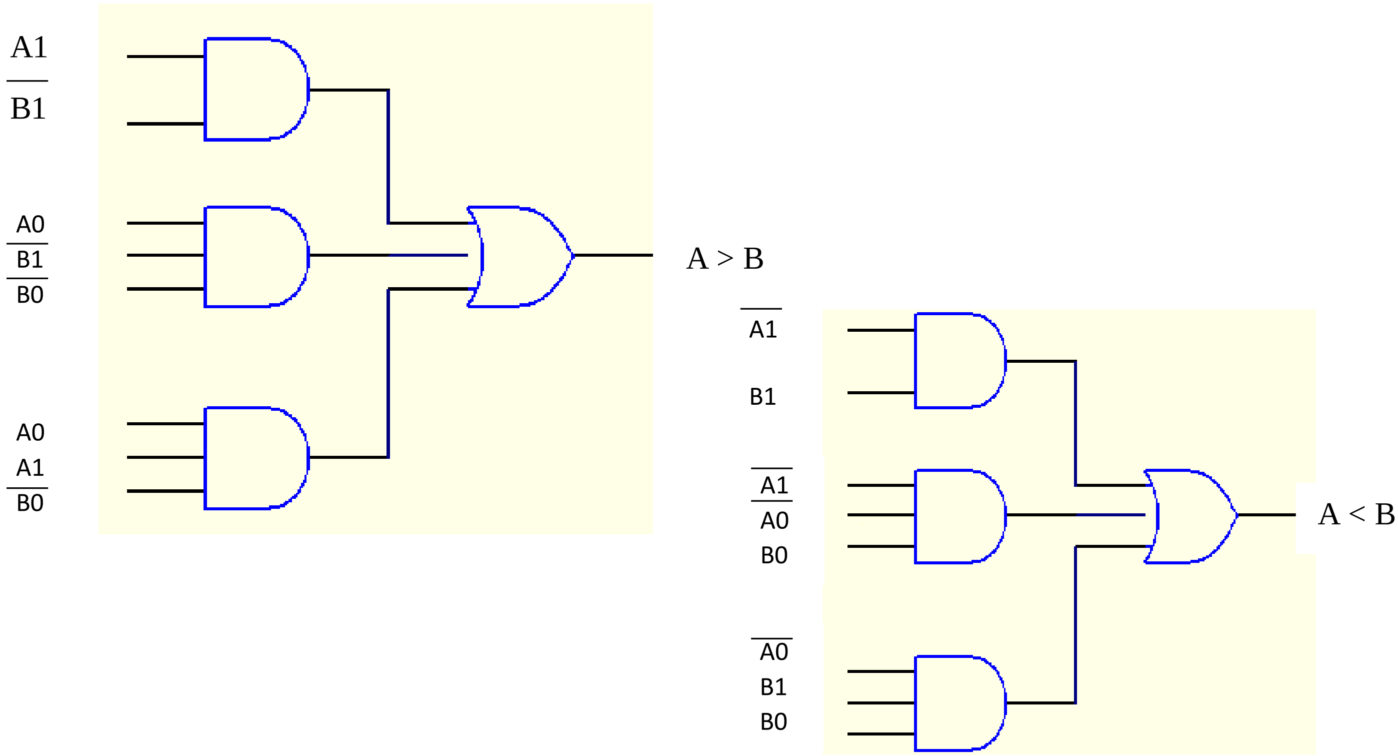
	$\overline{B1}\,\overline{B0}$	$\overline{B1}B0$	$B1B0$	$B1\overline{B0}$
$\overline{A1}\,\overline{A0}$	0	0	0	0
$\overline{A1}A0$	1	0	0	0
$A1A0$	1	1	0	1
$A1\overline{A0}$	1	1	0	0

$A > B = A1\overline{B1} + A0\overline{B1}\,\overline{B0} + A1A0\overline{B0}$

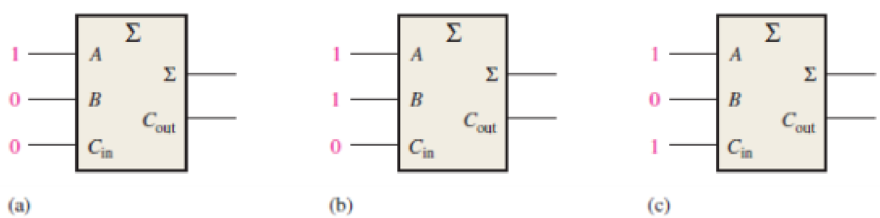
	$\overline{B1}\,\overline{B0}$	$\overline{B1}B0$	$B1B0$	$B1\overline{B0}$
$\overline{A1}\,\overline{A0}$	0	1	1	1
$\overline{A1}A0$	0	0	1	1
$A1A0$	0	0	0	0
$A1\overline{A0}$	0	0	1	0

$A < B = \overline{A1}B1 + \overline{A1}\,\overline{A0}B0 + \overline{A0}B1B0$





EX3: For each of the three full-adders in Figure below, determine the outputs for the inputs shown.



Solution

(a) The input bits are A = 1, B = 0, and Cin = 0.

1 + 0 + 0 = 1 with no carry Therefore, Σ = 1 and Cout = 0.

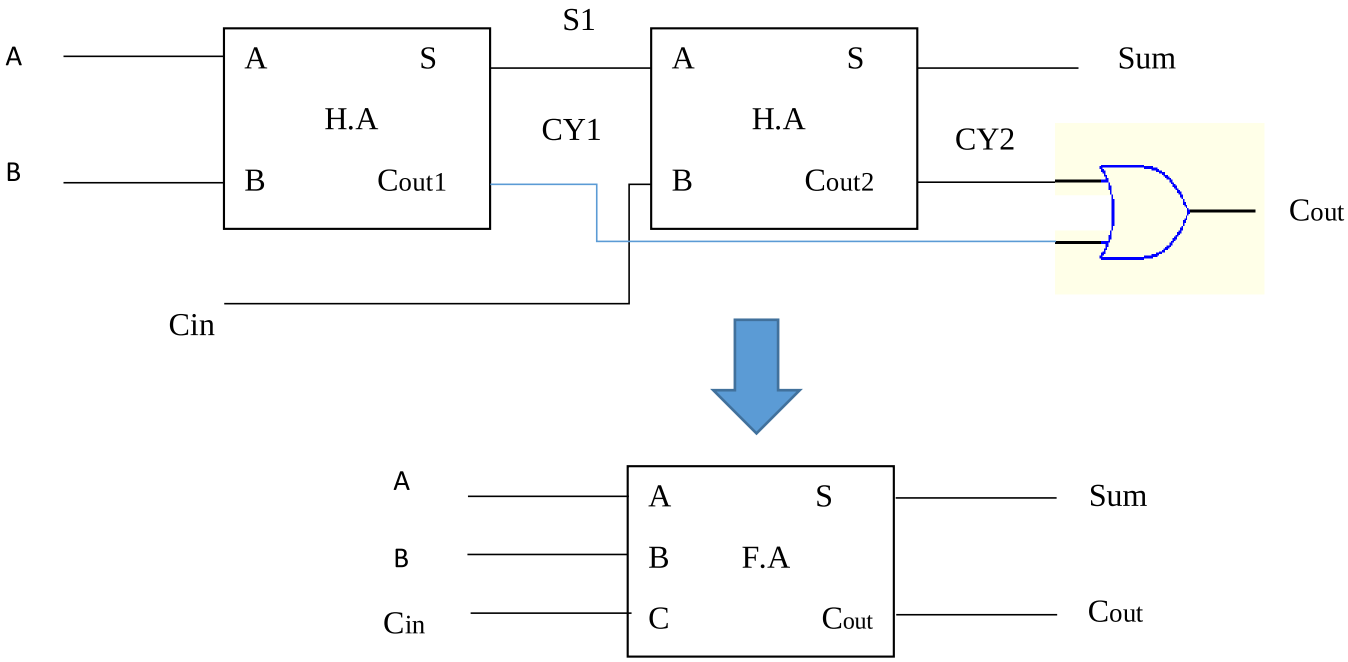
(b) The input bits are A = 1, B = 1, and Cin = 0.

1 + 1 + 0 = 0 with a carry of 1 Therefore, Σ = 0 and Cout = 1.

(c) The input bits are A = 1, B = 0, and Cin = 1.

1 + 0 + 1 = 0 with a carry of 1 Therefore, Σ = 0 and Cout = 1.

Ex4: Design a Full.Adder by using Half Adders and OR gate.



Ex5: Design a logic circuit to add two (2-bit) numbers using adders.