Real Time System

Third Level

Lecture Six

Digital to Analog Converters

Dr. Hussein AbdulAlmeer Alkhamees

Hussein.Alkhamees@uomus.edu.iq

Goals

Up-on completing this lecture, the student should be able to:

1- Identify the concepts behind DACs

2- Compare and contrast the different types of DACs.

Cont. to ADC

- **Conversion time** is the time required to complete a conversion of the input signal, in other words it's the time it takes for an analog-to-digital conversion.

$$F_{max} = \frac{1}{2 * \text{Conversion time}}$$

Ex:

1. An ADC has a conversion time of 100 µs. what is the maximum frequency that can be converted?
2. A 1 Khz sinusoidal signal to be digitized using 8-bit ADC. Find the conversion time that can be used?

- **Resolution** is the number of bits used for conversion (8 bits, 12 bits, …)

$$\text{resolution} = \frac{\text{full Scale Signal}}{2^n}$$

Ex:

An 8-bits ADC is used to digitize a five volt (5v) full scale signal. What is the resolution?

- **Quantization error** is defined as the difference between the actual analog input and the digital representation of that value.

$$\text{Maximum Quantization } (q_{max}) = \frac{A}{2^{n+1}}$$

$$\text{Average Quantization } (q_{av}) = \frac{A}{2^{n+2}}$$

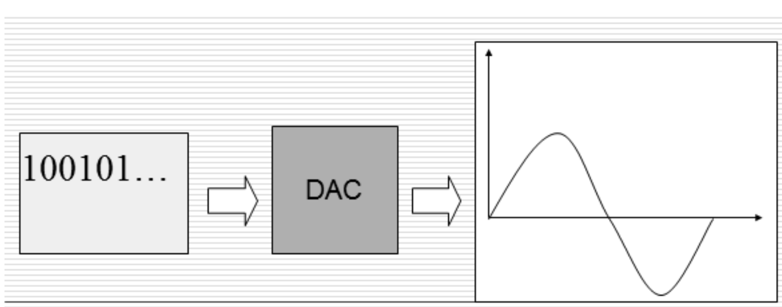
Where A is the amplitude and n is the number of bits.

Ex:

An analog signal of amplitude 12v is sampled with an 8bit ADC; calculate the maximum and average quantization error?

Digital to Analog Converters

A digital to analog converter (DAC) converts a digital signal or values to an analog voltage or current output.



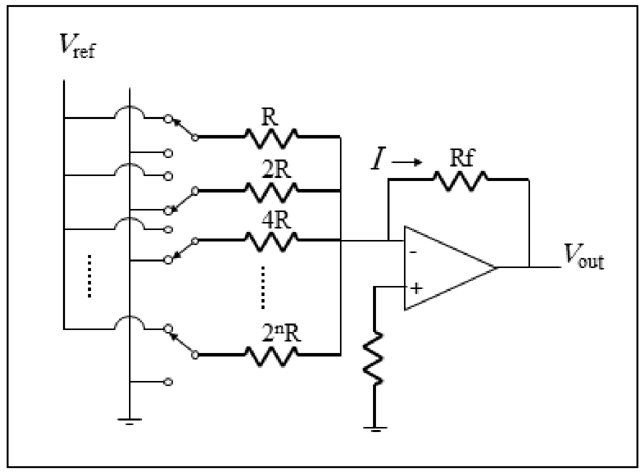
Types of DACs

There are two types of DAC and usually used switches, resistors, and op-amps to implement conversion, two types are:

- Binary Weighted Resistor.
- R-2R Ladder.

Binary Weighted Resistor:

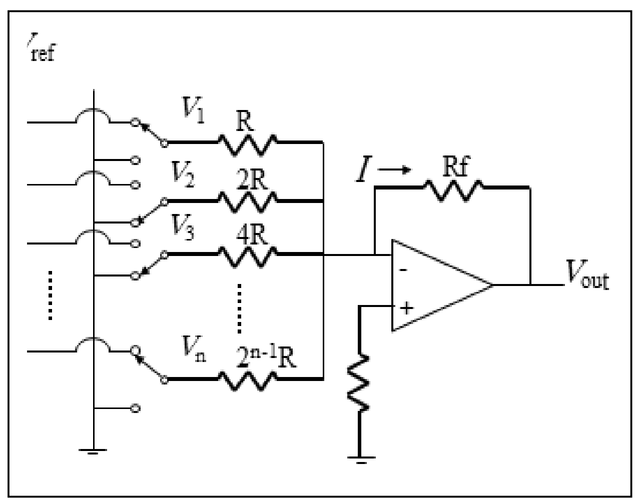
- Weighted resistors are used to distinguish each bit from the most significant to the least significant.
- Switches are used to switch between V_{ref} and ground (bit high or low).
- $V_{out} = -IR_f$
- Voltages V_1 to V_n are either V_{ref} if corresponding bit is high or ground if corresponding bit is low
- V_1 is most significant bit
- V_n is least significant bit



$$V_{out} = -IR_f = -R_f\left(\frac{V_1}{R} + \frac{V_2}{2R} + \frac{V_3}{4R} + \cdots \frac{V_n}{2^{n-1}R}\right)$$

If $R_f = R/2$

$$V_{out} = -IR_f = -\left(\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_3}{8} + \cdots \frac{V_n}{2^n}\right)$$



For example, a 4-Bit converter yields

$$V_{out} = -V_{ref}\left(b_3\frac{1}{2} + b_2\frac{1}{4} + b_1\frac{1}{8} + b_0\frac{1}{16}\right)$$

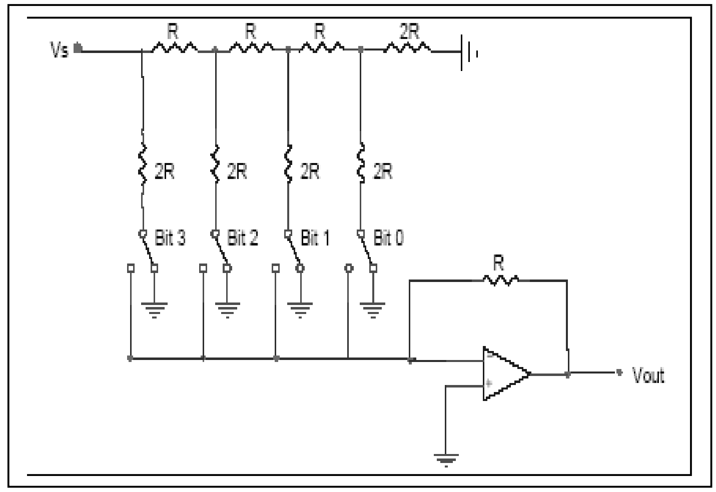
Where b_3 corresponds to Bit-3, b_2 to Bit-2, etc.

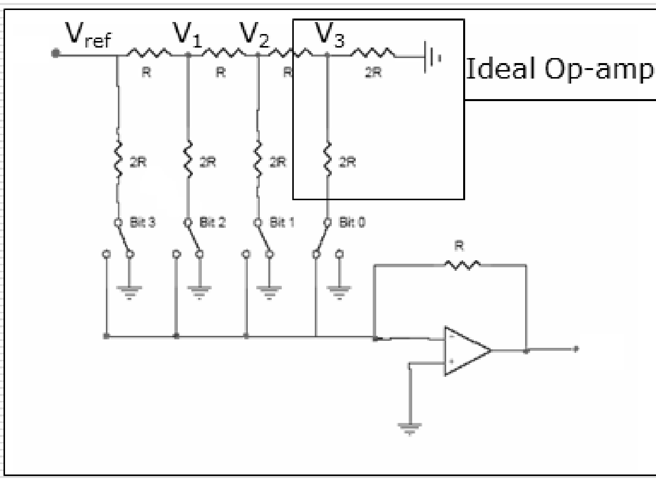
Advantages and Disadvantages

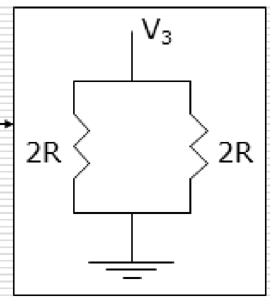
- Simple Construction, Fast Conversion.
- Limited to ~ 8 bits, large number of resistors, susceptible to noise, expensive and greater error.

R-2R Ladder:

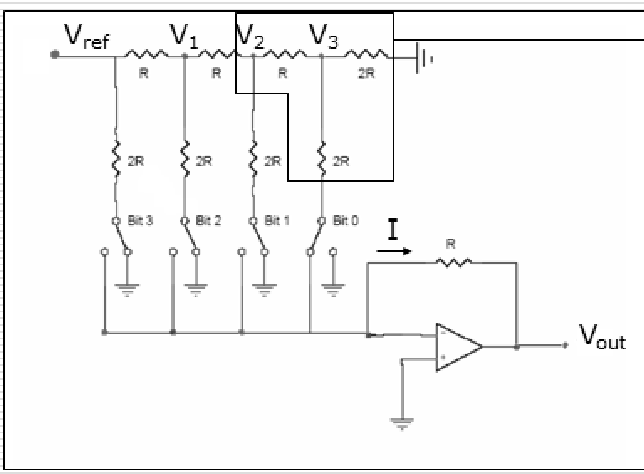
- Each bit corresponds to a switch:
- If the bit is high, the corresponding switch is connected to the inverting input of the op-amp.
- If the bit is low, the corresponding switch is connected to ground.
- All bits pass through resistance of 2R

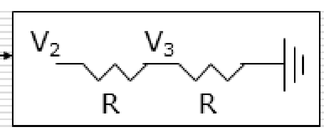






$$R_{eq} = \frac{(2R)(2R)}{(2R+2R)} = R$$





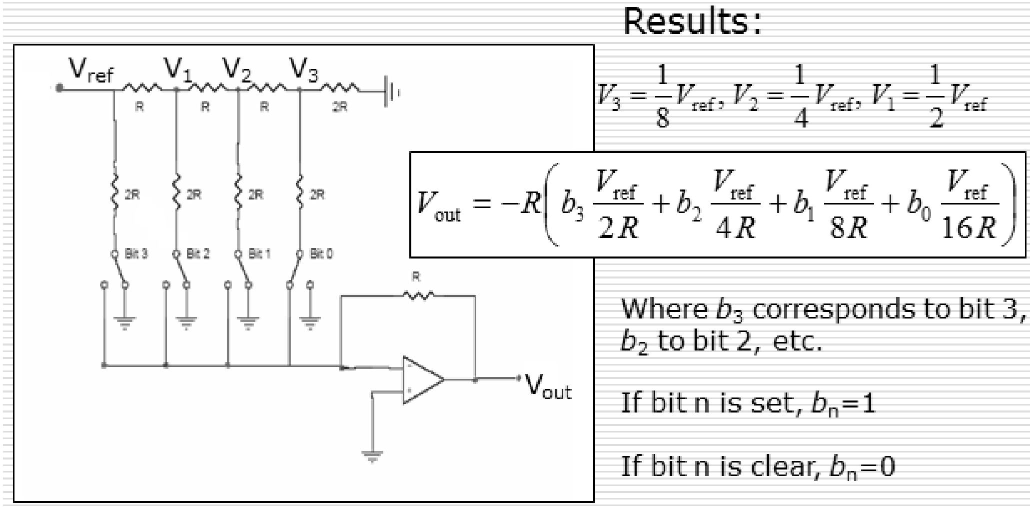
$$V_3 = \left(\frac{R}{R+R}\right)V_2 = \frac{1}{2}V_2$$

Likewise,

$$V_2 = \frac{1}{2}V_1$$

$$V_1 = \frac{1}{2}V_{ref}$$

$$V_{out} = -IR$$



For a 4-Bit R-2R Ladder

$$V_{out} = -V_{ref}\left(b_3 \frac{1}{2} + b_2 \frac{1}{4} + b_1 \frac{1}{8} + b_0 \frac{1}{16}\right)$$

For general n-Bit R-2R Ladder or Binary Weighted Resister DAC

$$V_{out} = -V_{ref} \sum_{i=1}^{n} b_{n-i} \frac{1}{2^i}$$

Advantages and Disadvantages

- Only two resistors values (R and 2R), does not require high precision resistors.
- Lower conversion speed than binary weighted DAC.

Specifications of DACs

- Resolution
- Speed
- Linearity
- Settling Time
- Reference Voltages
- Errors

Resolution is the amount of variance in output voltage for every change of the LSB in the digital input.

How closely can we approximate the desired output signal (Higher Res. = finer detail=smaller Voltage divisions)

Common DAC has an 8-16 bit resolution

$$\text{Resolution} = V_{LSB} = \frac{V_{ref}}{2^N}$$

$$\text{where } N = number\ of\ bits$$

Speed Rate of conversion of a single digital input to its analog equivalent.

Conversion rate depends on:

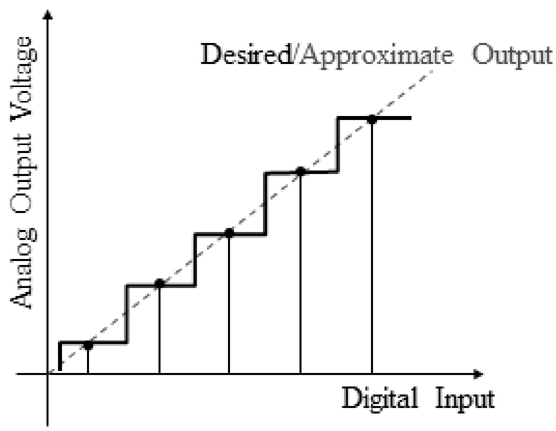
- clock speed of input signal
- settling time of converter

When the input changes rapidly, the DAC conversion speed must be high.

Linearity is the difference between the desired analog output and the actual output over the full range of expected values.

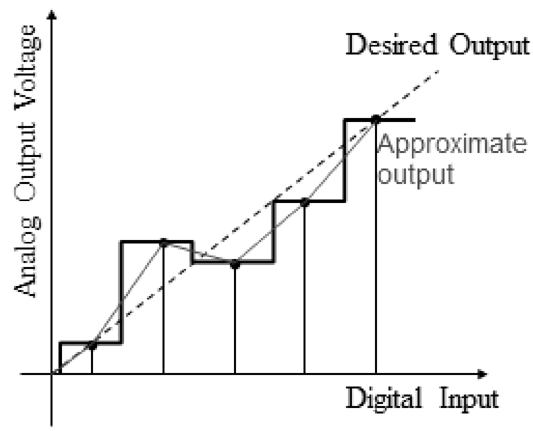
Ideally, a DAC should produce a linear relationship between a digital input and the analog output; this is not always the case.

Linearity(Ideal Case)



Perfect Agreement

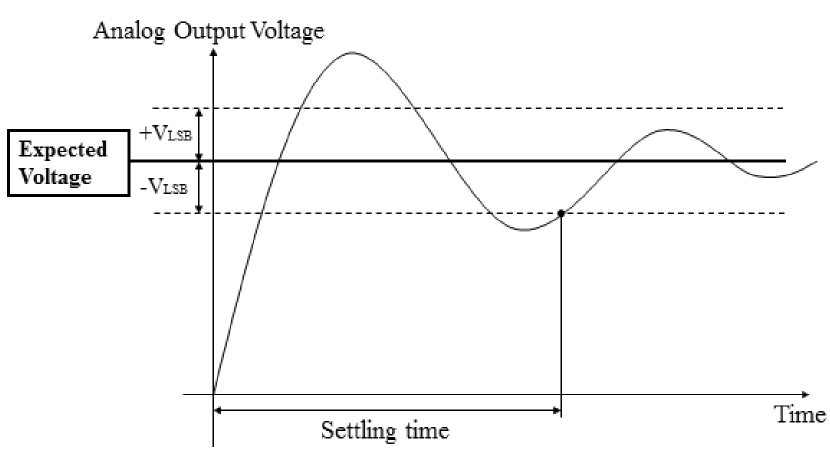
NON-Linearity(Real World)



Miss-alignment

Settling Time The time required for the input signal voltage to settle to the expected output voltage (within +/- VLSB).

Any change in the input state will not be reflected in the output state immediately. There is a time lag, between the two events.



Reference Voltages used to determine how each digital input will be assigned to each voltage division.

Types:

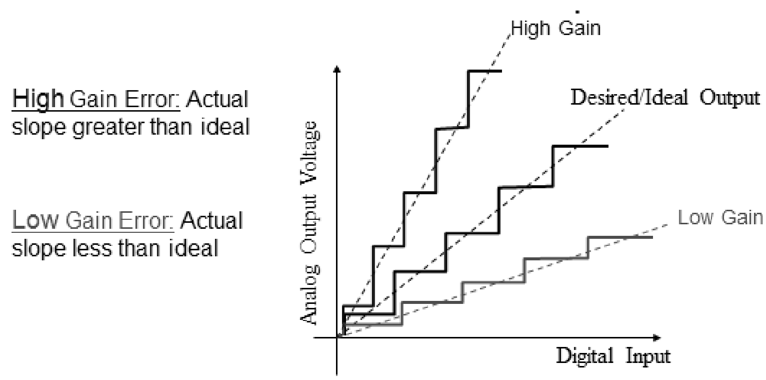
- Non-multiplier DAC: V_{ref} is fixed (internal, fixed, and defined by manufacturer).
- Multiplier DAC: V_{ref} provided by external source (external, variable, user specified).

Errors

Types of Errors Associated with DACs

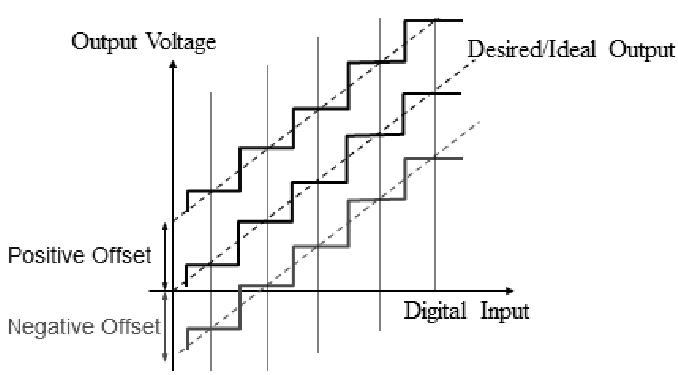
- **Gain**
- **Offset**
- **Full Scale**
- **Resolution**
- **Non-Linearity**
- **Non-Monotonic**

<u>**Gain Error:**</u> Difference in slope of the ideal curve and the actual DAC output**.**

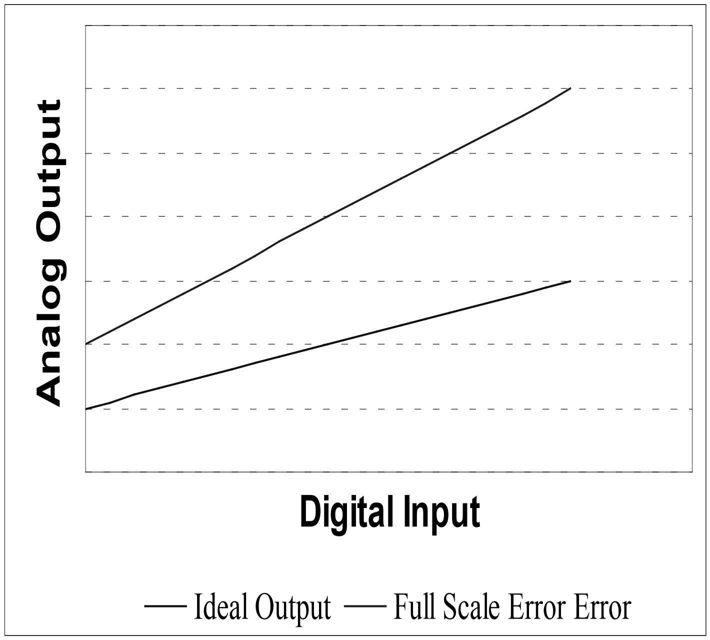


<u>**Offset Error:**</u> A constant voltage difference between the ideal DAC output and the actual.

- The voltage axis intercept of the DAC output curve is different than the ideal.

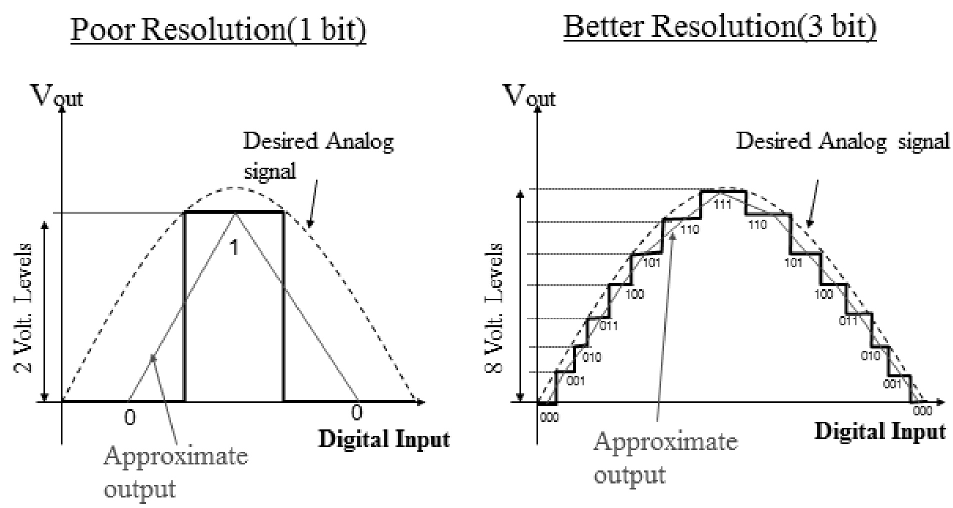


Full Scale Error: Occurs when the actual signal has both gain and offset errors.



Resolution Error: Poor representation of ideal output due to poor resolution.

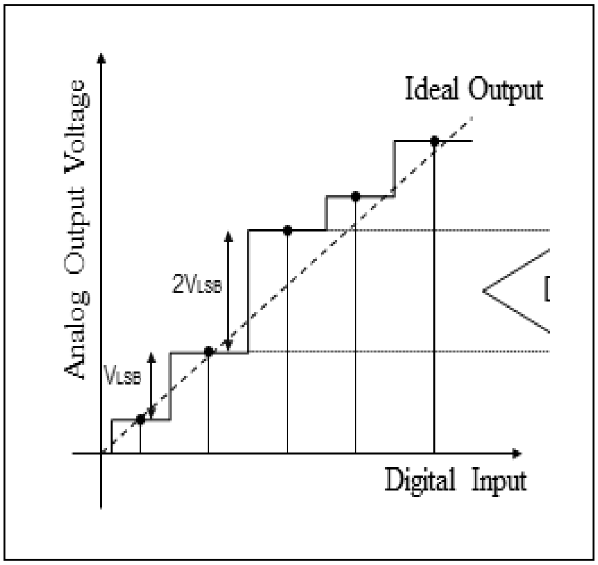
Size of voltage divisions affects the resolution

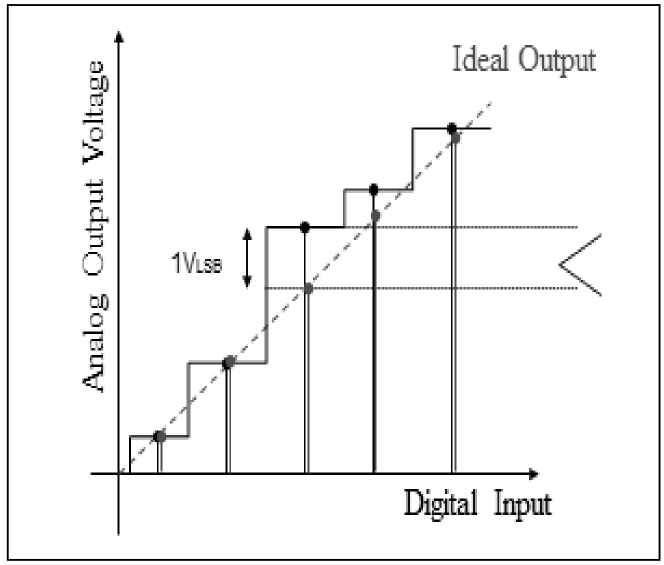


Non-Linearity Error: Occurs when analog output of signal is non-linear.

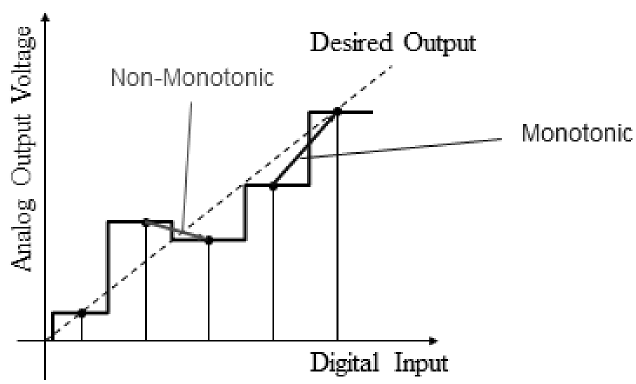
Differential Non-Linearity: Difference in voltage step size from the previous DAC output.

Integral Non-Linearity: Deviation of the actual DAC output from the ideal.





<u>Non-Monotonic:</u> A decrease in output voltage with an increase in the digital input.



Applications of DACs:

- Function Generators/Oscilloscopes.
- Digital Motor Control.
- Computer Printers.
- Sound Equipment (e.g. CD/MP3 Players, etc.).
- Electronic Cruise Control.
- Digital Thermostat.

Summary:
1- DACs rely heavily on digital switches and binary weights to yield analog values.
2- Different DACs have different specs, all of which should be accounted for in the selection process.

Questions:
1- Derive the equation for R-2R ladder.
2- How does binary-weighted-resistor A/D work ?