AL-MUSTAQBAL UNIVERSITY

College of Engineering & Technology

Building and Construction Techniques Engineering Department

Design of Steel Structures

Fourth Class

Lecture No. 03 – Tensile Strength and Effective Area

Lecturer

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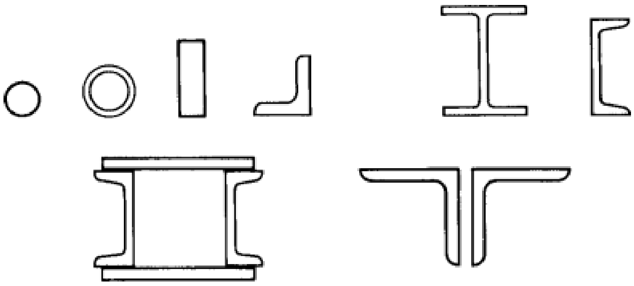
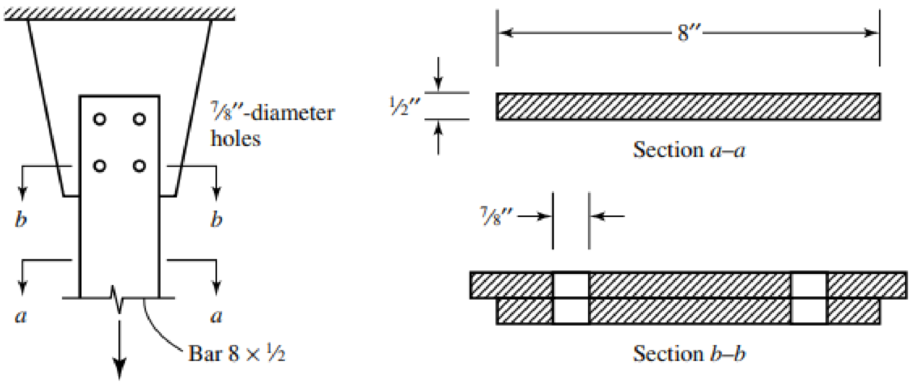
Tensile strength

- Tension members are structural elements that are subjected to (axial tensile forces).
- include truss members, bracing for buildings and bridges, cables in suspended roof systems, and cables in suspension and cable-stayed bridges.
- the only determinant of the strength of a tension member is the cross-sectional area.



Tension members are frequently connected at their ends with bolts, as illustrated in Figure below. The tension member shown, a 1⁄2 × 8 plate, is connected to a gusset plate, which is a connection element whose purpose is to transfer the load from the member to a support or to another member. The area of the bar at section a–a is (1⁄2)(8) = 4 in^2, but the area at section b–b is only 4 – (2)(1⁄2)(7⁄8) = 3.13 in^2 (net or effective area).



- A tension member can fail by reaching one of two limit states: excessive deformation or fracture.
- To prevent excessive deformation, initiated by **yielding**, the load on the gross section must be small enough that the stress on the gross section is less than the yield stress Fy.
- To prevent **fracture**, the stress on the net section must be less than the tensile strength Fu.

The nominal strength in yielding is

Pn = FyAg

and the nominal strength in fracture is

Pn = FuAe

In **LRFD**: For yielding, Ø = 0.90   Pu ≤ 0.90FyAg

For fracture, Ø = 0.75   Pu ≤ 0.75FuAe

Pu is the governing combination of factored loads.

In **ASD**: $P_a \le \dfrac{P_n}{\Omega_t}$

Where Pa is the required strength (applied load), and Pn/Ωt is the allowable strength (the factored theoretical or nominal strength.

For yielding, $\dfrac{P_n}{\Omega_t} = \dfrac{F_y A_g}{1.67} = 0.6 F_y A_g$

For fracture, $\dfrac{P_n}{\Omega_t} = \dfrac{F_u A_e}{2.00} = 0.5 F_u A_e$

(P)
Nominal/factor=design/allowable

Pu = required factored

Pa required total

ft (required)

Ft (allowable)

If we express those equations with stresses:

In yielding: $f_t = \dfrac{P_a}{A_g}$ and $F_t = \dfrac{P_n/\Omega_t}{A_g} = \dfrac{0.6 F_y A_g}{A_g} = 0.6 F_y$

In Fracture: $f_t = \dfrac{P_a}{A_e}$ and $F_t = \dfrac{P_n/\Omega_t}{A_e} = \dfrac{0.5 F_u A_e}{A_e} = 0.5 F_u$

You can find values of Fy and Fu for various structural steels in Table 2-4 in the Manual.

- **Best to view and understand in the book: Example 3.1 + 3.2 + 3.3**

Table 2-4
Applicable ASTM Specifications for Various Structural Shapes

Steel Type	ASTM Designation		F_y Yield Stress[a] (ksi)	F_u Tensile Stress[a] (ksi)	Applicable Shape Series							HSS		
					W	M	S	HP	C	MC	L	Rect.	Round	Pipe
Carbon	A36		36	58–80[b]	▒	■	■	▒	■	■	■			
	A53 Gr. B		35	60										■
	A500	Gr. B	42	58									▒	
			46	58								▒		
		Gr. C	46	62									■	
			50	62								■		
	A501	Gr. A	36	58								▒	▒	
		Gr. B	50	70								▒	▒	
	A529[c]	Gr. 50	50	65–100	▒	▒	▒	▒	▒	▒	▒			
		Gr. 55	55	70–100	▒	▒	▒	▒	▒	▒	▒			
	A709	36	36	58–80[b]	▒	▒	▒	▒	▒	▒	▒			
	A1043[d,e]	36	36–52	58	▒									
		50	50–65	65	▒									
	A1085	Gr. A	50	65								▒	▒	
High-Strength Low-Alloy	A572	Gr. 42	42	60	▒	▒	▒	▒	▒	▒	▒			
		Gr. 50	50	65	▒	▒	▒	■	▒	▒	▒			
		Gr. 55	55	70	▒	▒	▒	▒	▒	▒	▒			
		Gr. 60[e]	60	75	▒	▒	▒	▒	▒	▒	▒			
		Gr. 65[e]	65	80	▒	▒	▒	▒	▒	▒	▒			
	A618[f]	Gr. Ia[g], Ib & II	50[g]	70[g]								▒	▒	
		Gr. III	50	65								▒	▒	
	A709	50	50	65	▒	▒	▒	▒	▒	▒	▒			
		50S	50–65	65	▒	▒	▒	▒	▒	▒	▒			
		50W	50	70	▒	▒	▒	▒	▒	▒	▒			
	A913	50	50[h]	65[h]	▒	▒	▒	▒	▒	▒	▒			
		60	60	75	▒	▒	▒	▒	▒	▒	▒			
		65	65	80	▒	▒	▒	▒	▒	▒	▒			
		70	70	90	▒	▒	▒	▒	▒	▒	▒			
	A992		50[i]	65[i]	■	▒	▒	▒	▒	▒	▒			
	A1065[k]	Gr. 50[j]	50	60								▒		

■ = Preferred material specification.
▒ = Other applicable material specification, the availability of which should be confirmed prior to specification.
□ = Material specification does not apply.

Footnotes on facing page.

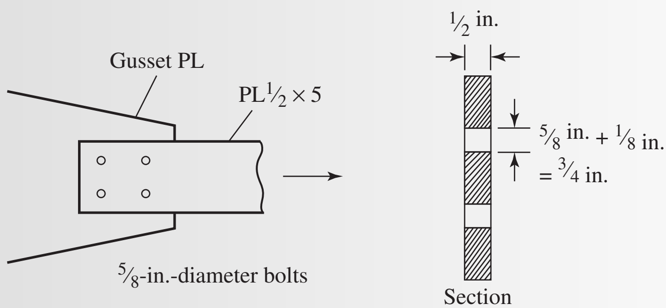
EXAMPLE 3.1

A $\frac{1}{2} \times 5$ plate of A36 steel is used as a tension member. It is connected to a gusset plate with four $\frac{5}{8}$-inch-diameter bolts as shown in Figure 3.3. Assume that the effective net area A_e equals the actual net area A_n (we cover computation of effective net area in Section 3.3).

a. What is the design strength for LRFD?

b. What is the allowable strength for ASD?

FIGURE 3.3



SOLUTION For yielding of the gross section,

$$A_g = 5(1/2) = 2.5 \text{ in.}^2$$

and the nominal strength is

$$P_n = F_y A_g = 36(2.5) = 90.0 \text{ kips}$$

For fracture of the net section,

$$A_n = A_g - A_{holes} = 2.5 - (\tfrac{1}{2})(\tfrac{3}{4}) \times 2 \text{ holes}$$
$$= 2.5 - 0.75 = 1.75 \text{ in.}^2$$

$A_e = A_n = 1.75$ in.^{2} (This is true for this example, but A_e does not always equal A_n.)

The nominal strength is

$$P_n = F_u A_e = 58(1.75) = 101.5 \text{ kips}$$

a. The design strength based on yielding is

$$\phi_t P_n = 0.90(90) = 81.0 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(101.5) = 76.1 \text{ kips}$$

ANSWER The design strength for LRFD is the smaller value: $\phi_t P_n = 76.1$ kips.

b. The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{90}{1.67} = 53.9 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{101.5}{2.00} = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

Alternative Solution Using Allowable Stress: For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 21.6(2.5) = 54.0 \text{ kips}$$

(The slight difference between this value and the one based on allowable strength is because the value of Ω in the allowable strength approach has been rounded from 5/3 to 1.67; the value based on the allowable stress is the more accurate one.)
For fracture,

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 29.0(1.75) = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

Because of the relationship given by Equation 2.8, the allowable strength will always be equal to the design strength divided by 1.5. In this book, however, we will do the complete computation of allowable strength even when the design strength is available.

The effects of stress concentrations at holes appear to have been overlooked. In reality, stresses at holes can be as high as three times the average stress on the net section, and at fillets of rolled shapes they can be more than twice the average (McGuire, 1968). Because of the ductile nature of structural steel, the usual design practice is to neglect such localized overstress. After yielding begins at a point of stress concentration, additional stress is transferred to adjacent areas of the cross section. This stress redistribution is responsible for the "forgiving" nature of structural steel. Its ductility permits the initially yielded zone to deform without fracture as the stress on the remainder of the cross section continues to increase. Under certain conditions, however, steel may lose its ductility and stress concentrations can precipitate brittle fracture. These situations include fatigue loading and extremely low temperature.

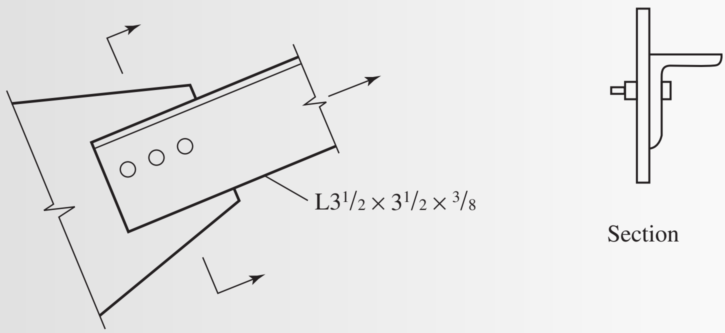
EXAMPLE 3.2

A single-angle tension member, an L3½ × 3½ × ⅜, is connected to a gusset plate with ⅞-inch-diameter bolts as shown in Figure 3.4. A36 steel is used. The service loads are 35 kips dead load and 15 kips live load. Investigate this member for compliance with the AISC Specification. Assume that the effective net area is 85% of the computed net area.

a. Use LRFD.

b. Use ASD.

FIGURE 3.4



SOLUTION First, compute the nominal strengths.

Gross section:

$$A_g = 2.50 \text{ in.}^2 \qquad \text{(from Part 1 of the } \textit{Manual}\text{)}$$

$$P_n = F_y A_g = 36(2.50) = 90 \text{ kips}$$

Net section:

$$A_n = 2.50 - \left(\frac{3}{8}\right)\left(\frac{7}{8} + \frac{1}{8}\right) = 2.125 \text{ in.}^2$$

$$A_e = 0.85A_n = 0.85(2.125) = 1.806 \text{ in.}^2 \quad \text{(in } \textit{this} \text{ example)}$$

$$P_n = F_u A_e = 58(1.806) = 104.7 \text{ kips}$$

a. The design strength based on yielding is

$$\phi_t P_n = 0.90(90) = 81 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(104.7) = 78.5 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 78.5$ kips

Factored load:

When only dead load and live load are present, the only load combinations with a chance of controlling are combinations 1 and 2.

Combination 1: $1.4D = 1.4(35) = 49$ kips

Combination 2: $1.2D + 1.6L = 1.2(35) + 1.6(15) = 66$ kips

The second combination controls; $P_u = 66$ kips.

(When only dead load and live load are present, combination 2 will always control when the dead load is less than eight times the live load. In future examples, we will not check combination 1 [$1.4D$] when it obviously does not control.)

ANSWER Since $P_u < \phi_t P_n$, (66 kips < 78.5 kips), the member is satisfactory.

b. For the gross section, The allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{90}{1.67} = 53.9 \text{ kips}$$

For the net section, the allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{104.7}{2.00} = 52.4 \text{ kips}$$

The smaller value controls; the allowable strength is 52.4 kips. When the only loads are dead load and live load, ASD load combination 2 will always control:

$$P_a = D + L = 35 + 15 = 50 \text{ kips}$$

ANSWER Since 50 kips < 52.4 kips, the member is satisfactory.

Alternative Solution Using Allowable Stress

For the gross area, the applied stress is

$$f_t = \frac{P_a}{A_g} = \frac{50}{2.50} = 20 \text{ ksi}$$

and the allowable stress is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

For this limit state, $f_t < F_t$ (OK)

For the net section,

$$f_t = \frac{P_a}{A_e} = \frac{50}{1.806} = 27.7 \text{ ksi}$$

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi} > 27.7 \text{ ksi} \quad \text{(OK)}$$

ANSWER Since $f_t < F_t$ for both limit states, the member is satisfactory.

What is the difference in computational effort for the two different approaches? Regardless of the method used, the two nominal strengths must be computed (if a stress approach is used with ASD, an equivalent computation must be made). With LRFD, the nominal strengths are multiplied by resistance factors. With ASD, the nominal strengths are divided by load factors. Up to this point, the number of steps is the same. The difference in effort between the two methods involves the load side of the relationships. In LRFD, the loads are factored before adding. In ASD, in most cases the loads are simply added. Therefore, for tension members LRFD requires slightly more computation.

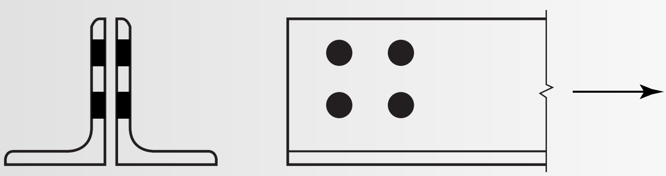
EXAMPLE 3.3

A double-angle shape is shown in Figure 3.5. The steel is A36, and the holes are for ½-inch-diameter bolts. Assume that $A_e = 0.75A_n$.

a. Determine the design tensile strength for LRFD.

b. Determine the allowable strength for ASD.

FIGURE 3.5



SOLUTION

Figure 3.5 illustrates the notation for unequal-leg double-angle shapes. The notation LLBB means "long-legs back-to-back," and SLBB indicates "short-legs back-to-back."

When a double-shape section is used, two approaches are possible: (1) consider a single shape and double everything, or (2) consider two shapes from the outset. (Properties of the double-angle shape are given in Part 1 of the *Manual*.) In this example, we consider one angle and double the result. For one angle, the nominal strength based on the gross area is

$$P_n = F_y A_g = 36(2.41) = 86.76 \text{ kips}$$

There are two holes in each angle, so the net area of one angle is

$$A_n = 2.41 - \left(\frac{5}{16}\right)\left(\frac{1}{2} + \frac{1}{8}\right) \times 2 = 2.019 \text{ in.}^2$$

The effective net area is

$$A_e = 0.75(2.019) = 1.514 \text{ in.}^2$$

The nominal strength based on the net area is

$$P_n = F_u A_e = 58(1.514) = 87.81 \text{ kips}$$

a. The design strength based on yielding of the gross area is

$$\phi_t P_n = 0.90(86.76) = 78.08 \text{ kips}$$

The design strength based on fracture of the net area is

$$\phi_t P_n = 0.75(87.81) = 65.86 \text{ kips}$$

ANSWER Because 65.86 kips < 78.08 kips, fracture of the net section controls, and the design strength for the two angles is $2 \times 65.86 = 132$ kips.

b. The allowable stress approach will be used. For the gross section,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

The corresponding allowable load is

$$F_t A_g = 21.6(2.41) = 52.06 \text{ kips}$$

For the net section,

$$F_t = 0.5F_u = 0.5(58) = 29 \text{ ksi}$$

The corresponding allowable load is

$$F_t A_e = 29(1.514) = 43.91 \text{ kips}$$

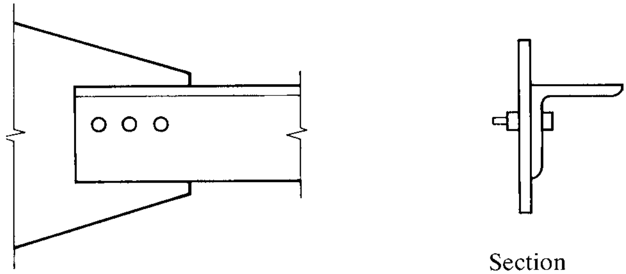
ANSWER Because 43.91 kips < 52.06 kips, fracture of the net section controls, and the allowable strength for the two angles is $2 \times 43.91 = 87.8$ kips.

3.3 EFFECTIVE AREA

Of the several factors influencing the performance of a tension member, the manner in which it is connected is the most important. A connection almost always weakens the member, and the measure of its influence is called the *joint efficiency*. This factor is a function of the ductility of the material, fastener spacing, stress concentrations at holes, fabrication procedure, and a phenomenon known as *shear lag*. All contribute to reducing the effectiveness of the member, but shear lag is the most important.

Shear lag occurs when some elements of the cross section are not connected, as when only one leg of an angle is bolted to a gusset plate, as shown in Figure 3.6. The consequence of this partial connection is that the connected element becomes overloaded and the unconnected part is not fully stressed. Lengthening the connected region will reduce this effect. Research reported by Munse and Chesson (1963)

FIGURE 3.6



suggests that shear lag be accounted for by using a reduced, or effective, net area. Because shear lag affects both bolted and welded connections, the effective net area concept applies to both types of connections.

For bolted connections, the effective net area is

$$A_e = A_n U \qquad \text{(AISC Equation D3-1)}$$

For welded connections, we refer to this reduced area as the *effective area* (rather than the effective *net* area), and it is given by

$$A_e = A_g U$$

where the reduction factor U is given in AISC D3, Table D3.1. The table gives a general equation that will cover most situations as well as alternative numerical values for specific cases. These definitions of U will be presented here in a different format from that in the Specification. The rules for determining U fall into five categories:

1. A general category for any type of tension member except plates and round HSS with $\ell \geq 1.3D$ (See Figure 3.7e.)
2. Plates
3. Round HSS with $\ell \geq 1.3\,D$
4. Alternative values for single and double angles
5. Alternative values for W, M, S, and HP shapes

1. For any type of tension member except plates and round HSS with $\ell \geq 1.3D$

$$U = 1 - \frac{\bar{x}}{\ell} \qquad (3.1)$$

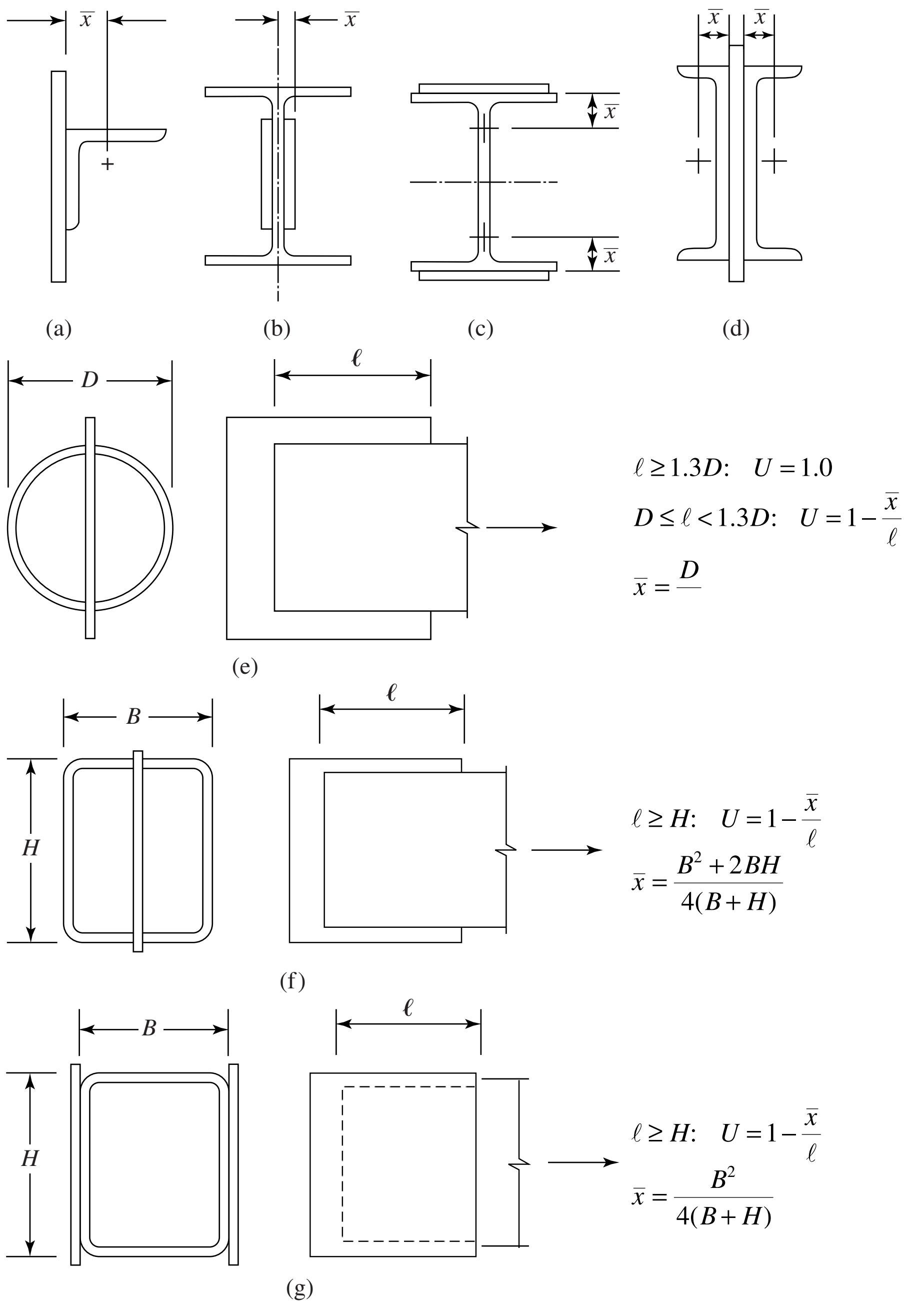
where

$\bar{x}$ = distance from centroid of connected area to the plane of the connection
ℓ = length of the connection

This definition of $\bar{x}$ was formulated by Munse and Chesson (1963). If a member has two symmetrically located planes of connection, $\bar{x}$ is measured from the centroid of the nearest one-half of the area. Figure 3.7 illustrates $\bar{x}$ for various types of connections.

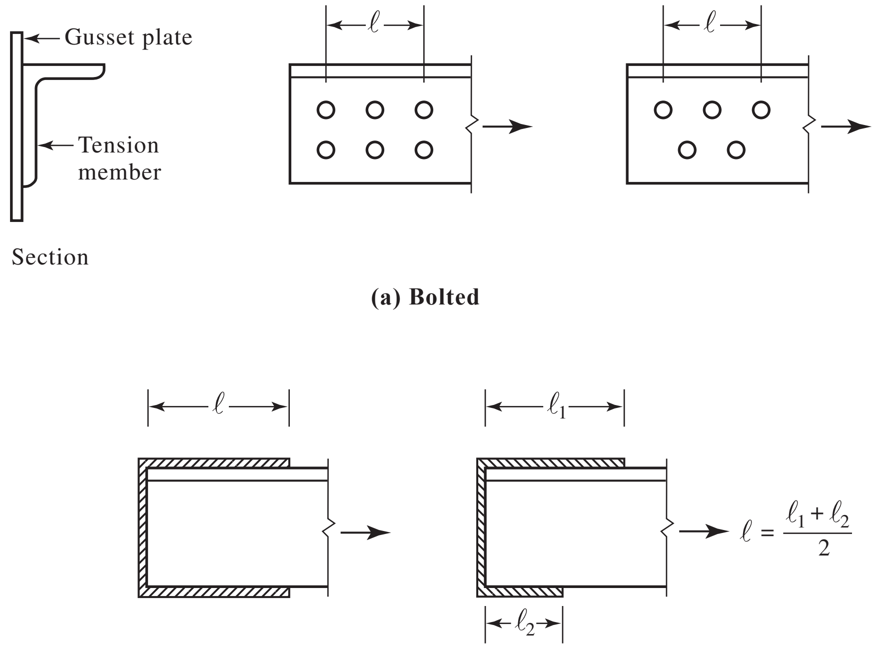
The length ℓ in Equation 3.1 is the length of the connection in the direction of the load, as shown in Figure 3.8. For bolted connections, it is measured from the center of the bolt at one end of the connection to the center of the bolt at the other end. For welds, it is measured from one end of the weld to the other. If there are weld segments of different lengths in the direction of the load, use the average length.

FIGURE 3.7



$\bar{x}$
(a)
(b)
(c)
(d)
D
ℓ
$\ell \geq 1.3D$: $U = 1.0$
$D \leq \ell < 1.3D$: $U = 1 - \frac{\bar{x}}{\ell}$
$\bar{x} = \frac{D}{\ }$
(e)
B
H
$\ell \geq H$: $U = 1 - \frac{\bar{x}}{\ell}$
$\bar{x} = \frac{B^2 + 2BH}{4(B+H)}$
(f)
$\ell \geq H$: $U = 1 - \frac{\bar{x}}{\ell}$
$\bar{x} = \frac{B^2}{4(B+H)}$
(g)

FIGURE 3.8



(a) Bolted

(b) Welded

The Commentary of the AISC Specification further illustrates $\bar{x}$ and ℓ. Figure C-D3.2 shows some special cases for $\bar{x}$, including channels and I-shaped members connected through their webs. To compute $\bar{x}$ for these cases, the Commentary uses the concept of the plastic neutral axis to explain the procedure. Since this concept is not covered until Chapter 5 of this book, we will use $\bar{x}$ for channels as shown in Case 2 of Specification Table D3.1 and in Figure 3.7b of this book. For I-shaped members and tees connected through the web, we can use Case 2 or Case 7 of Specification Table D3.1.

2. Plates

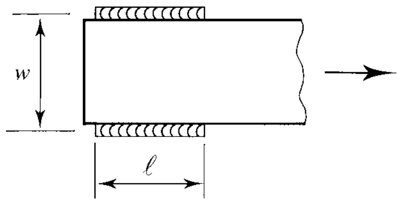
In general, $U = 1.0$ for plates, since the cross section has only one element and it is connected. There is one exception for welded plates, however. If the member is connected with longitudinal welds on each side with no transverse weld (as in Figure 3.9), the following values apply:

- For $\ell \geq 2w$ $U = 1.0$
- For $1.5w \leq \ell < 2w$, $U = 0.87$
- For $w \leq \ell < 1.5w$, $U = 0.75$

3. Round HSS with $\ell \geq 1.3D$ (see Figure 3.7e):

$U = 1.0$

FIGURE 3.9



4. Alternatives to Equation 3.1 for Single and Double Angles:

The following values may be used in lieu of Equation 3.1.

- For four or more fasteners in the direction of loading, $U = 0.80$.
- For three fasteners in the direction of loading, $U = 0.60$.

5. Alternatives to Equation 3.1 for W, M, S, HP, or Tees Cut from These Shapes:

If the following conditions are satisfied, the corresponding values may be used in lieu of Equation 3.1.

- Connected through the flange with three or more fasteners in the direction of loading, with a width at least ⅔ of the depth: $U = 0.90$.
- Connected through the flange with three or more fasteners in the direction of loading, with a width less than ⅔ of the depth: $U = 0.85$.
- Connected through the web with four or more fasteners in the direction of loading: $U = 0.70$.

Figure 3.10 illustrates the alternative values of U for various connections.

If a tension member is connected with only transverse welds, $U = 1.0$, and A_n is the area of the connected element. Figure 3.11 illustrates the difference between transverse and longtitudinal welds. Connections by transverse welds alone are not common.

There are some limiting values for the effective area:

- For bolted *splice plates,* $A_e = A_n \leq 0.85A_g$. This limit is given in a user note and is from a requirement in Chapter J of the Specification "Design of Connections."
- For open cross-sectional shapes (such as W, M, S, C, HP, WT, and ST) and (angles), the value of U need not be less than the ratio of the connected element gross area to the total gross area.

EXAMPLE 3.4

Determine the effective net area for the tension member shown in Figure 3.12.

SOLUTION

$$A_n = A_g - A_{\text{holes}}$$
$$= 5.77 - \frac{1}{2}\left(\frac{5}{8} + \frac{1}{8}\right)(2) = 5.02 \text{ in.}^2$$

Only one element (one leg) of the cross section is connected, so the net area must be reduced. From the properties tables in Part 1 of the *Manual,* the distance from the centroid to the outside face of the leg of an L6 × 6 × ½ is

$$\bar{x} = 1.67 \text{ in.}$$

FIGURE 3.10

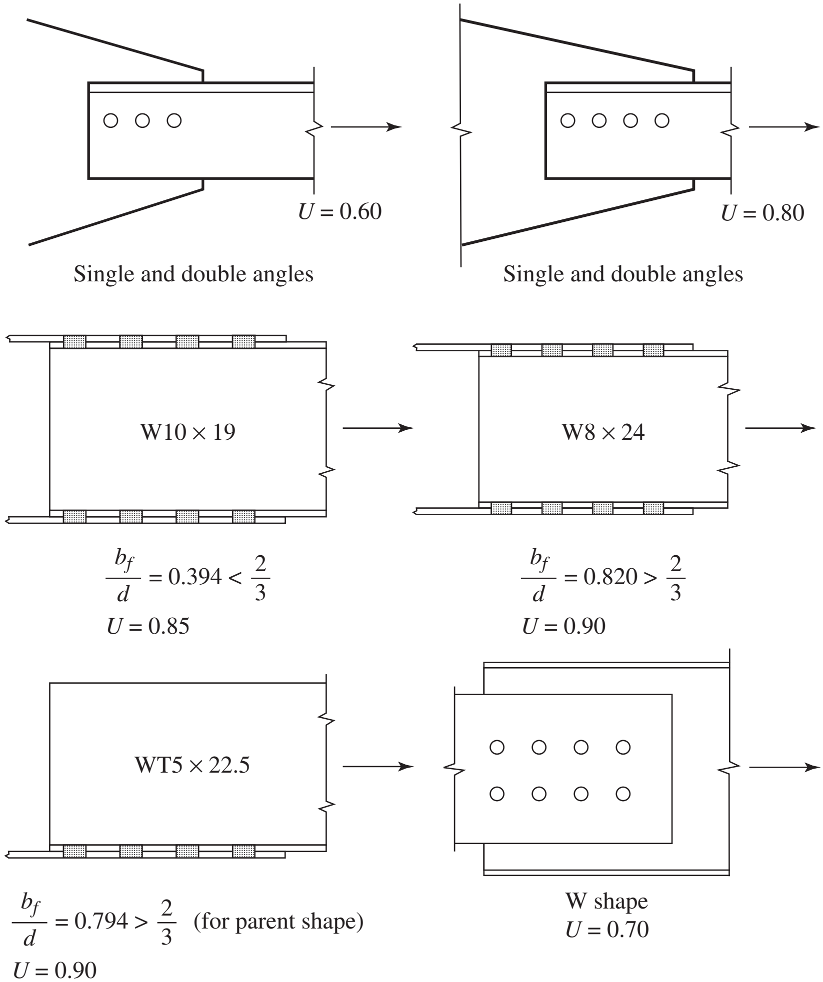


FIGURE 3.11

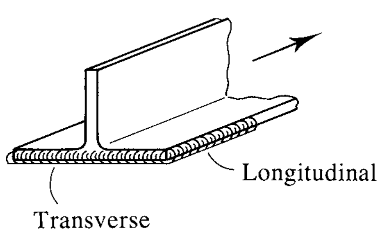
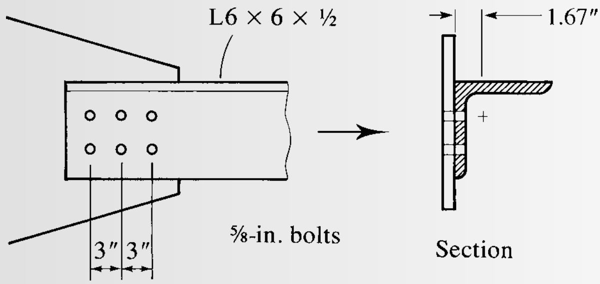


FIGURE 3.12



The length of the connection is

$$\ell = 3 + 3 = 6 \text{ in.}$$

$$\therefore U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.67}{6}\right) = 0.7217$$

$$A_e = A_n U = 5.02(0.7217) = 3.623 \text{ in.}^2$$

The alternative value of U could also be used. Because this angle has three bolts in the direction of the load, the reduction factor U can be taken as 0.60, and

$$A_e = A_n U = 5.02(0.60) = 3.012 \text{ in.}^2$$

Either U value is acceptable, and the Specification permits the larger one to be used. However, the value obtained from Equation 3.1 is more accurate. The alternative values of U can be useful during preliminary design, when actual section properties and connection details are not known.

EXAMPLE 3.5

If the tension member of Example 3.4 is welded as shown in Figure 3.13, determine the effective area.

SOLUTION

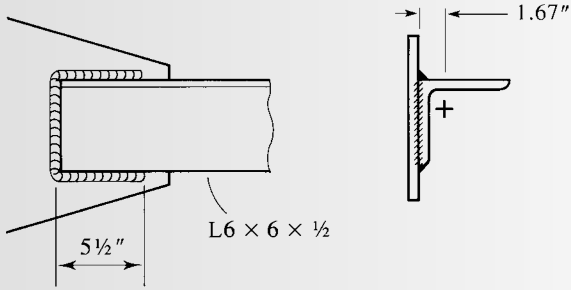
As in Example 3.4, only part of the cross section is connected and a reduced effective area must be used.

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.67}{5.5}\right) = 0.6964$$

ANSWER

$$A_e = A_g U = 5.77(0.6964) = 4.02 \text{ in.}^2$$

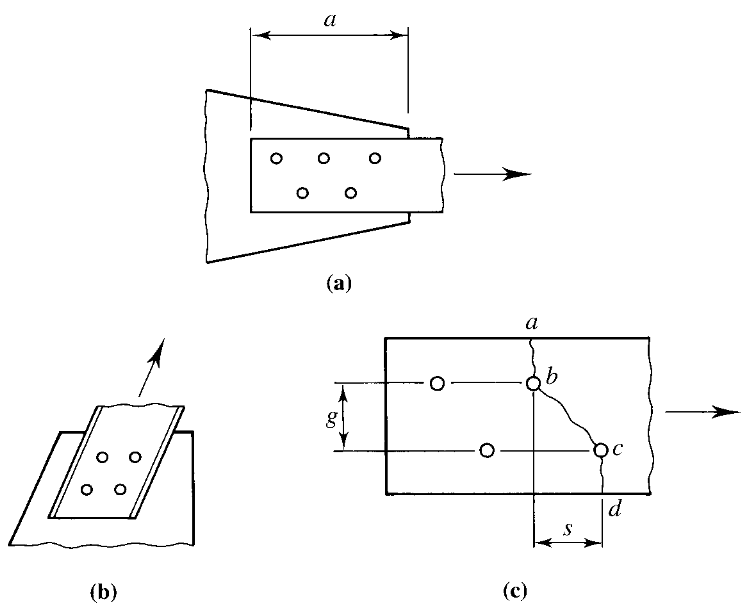
FIGURE 3.13



3.4 STAGGERED FASTENERS

If a tension member connection is made with bolts, the net area will be maximized if the fasteners are placed in a single line. Sometimes space limitations, such as a limit on dimension *a* in Figure 3.14a, necessitate using more than one line. If so, the reduction in cross-sectional area is minimized if the fasteners are arranged in a staggered pattern, as shown. Sometimes staggered fasteners are required by the geometry of a connection, such as the one shown in Figure 3.14b. In either case, any cross section passing through holes will pass through fewer holes than if the fasteners are not staggered.

FIGURE 3.14



If the amount of stagger is small enough, the influence of an offset hole may be felt by a nearby cross section, and fracture along an inclined path such as *abcd* in Figure 3.14c is possible. In such a case, the relationship $f = P/A$ does not apply, and stresses on the inclined portion b–c are a combination of tensile and shearing stresses. Several approximate methods have been proposed to account for the effects of staggered holes. Cochrane (1922) proposed that when deducting the area corresponding to a staggered hole, use a reduced diameter, given by

$$d' = d - \frac{s^2}{4g} \tag{3.2}$$

where d is the hole diameter, s is the stagger, or pitch, of the bolts (spacing in the direction of the load), and g is the gage (transverse spacing). This means that in a failure pattern consisting of both staggered and unstaggered holes, use d for holes at the end of a transverse line between holes ($s = 0$) and use d' for holes at the end of an inclined line between holes.

The AISC Specification, in Section B4.3b, uses this approach, but in a modified form. If the net area is treated as the product of a thickness times a net width, and the diameter from Equation 3.2 is used for all holes (since $d' = d$ when the stagger $s = 0$), the net width in a failure line consisting of both staggered and unstaggered holes is

$$\begin{aligned} w_n &= w_g - \Sigma d' \\ &= w_g - \Sigma\left(d - \frac{s^2}{4g}\right) \\ &= w_g - \Sigma d + \Sigma \frac{s^2}{4g} \end{aligned}$$

where w_n is the net width and w_g is the gross width. The second term is the sum of all hole diameters, and the third term is the sum of $s^2/4g$ for all inclined lines in the failure pattern.

When more than one failure pattern is conceivable, all possibilities should be investigated, and the one corresponding to the smallest load capacity should be used. Note that this method will not accommodate failure patterns with lines parallel to the applied load.

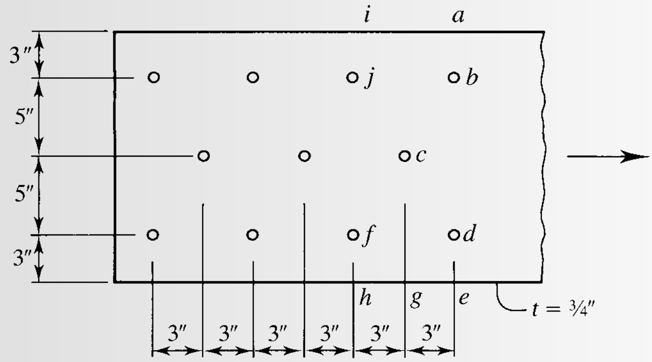
EXAMPLE 3.6

Compute the smallest net area for the plate shown in Figure 3.15. The holes are for 1-inch-diameter bolts.

SOLUTION The effective hole diameter is $1 + \frac{1}{8} = 1\frac{1}{8}$ in. For line *abde,*

$$w_n = 16 - 2(1.125) = 13.75 \text{ in.}$$

FIGURE 3.15



For line *abcde,*

$$w_n = 16 - 3(1.125) + \frac{2(3)^2}{4(5)} = 13.52 \text{ in.}$$

The second condition will give the smallest net area:

ANSWER $A_n = tw_n = 0.75(13.52) = 10.1 \text{ in.}^2$

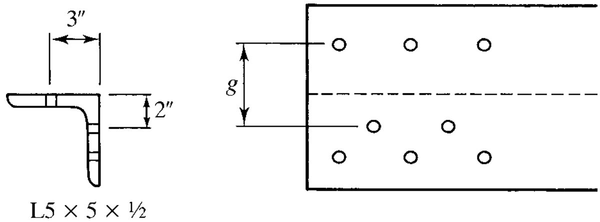
Equation 3.2 can be used directly when staggered holes are present. In the computation of the net area for line *abcde* in Example 3.6,

$$\begin{aligned} A_n &= A_g - \sum t \times (d \text{ or } d') \\ &= 0.75(16) - 0.75(1.125) - 0.75\left[1.125 - \frac{(3)^2}{4(5)}\right] \times 2 = 10.1 \text{ in.}^2 \end{aligned}$$

As each fastener resists an equal share of the load (an assumption used in the design of simple connections; see Chapter 7), different potential failure lines may be subjected to different loads. For example, line *abcde* in Figure 3.15 must resist the full load, whereas *ijfh* will be subjected to $8/11$ of the applied load. The reason is that $3/11$ of the load will have been transferred from the member before *ijfh* receives any load.

When lines of bolts are present in more than one element of the cross section of a rolled shape, and the bolts in these lines are staggered with respect to one another, the use of areas and Equation 3.2 is preferable to the net-width approach of the AISC Specification. If the shape is an angle, it can be visualized as a plate formed by "unfolding" the legs to more clearly identify the pitch and gage distances. AISC B4.3b specifies that any gage line crossing the heel of the angle be reduced by an amount that equals the angle thickness. Thus, the distance g in Figure 3.16, to be used in the $s^2/4g$ term, would be $3 + 2 - ½ = 4½$ inches.

FIGURE 3.16



EXAMPLE 3.7

An angle with staggered fasteners in each leg is shown in Figure 3.17. A36 steel is used, and holes are for ⅞-inch-diameter bolts.

a. Determine the design strength for LRFD.
b. Determine the allowable strength for ASD.

SOLUTION From the dimensions and properties tables, the gross area is A_g = 6.80 in.^{2}. The effective hole diameter is ⅞ + ⅛ = 1 in.

For line *abdf*, the net area is

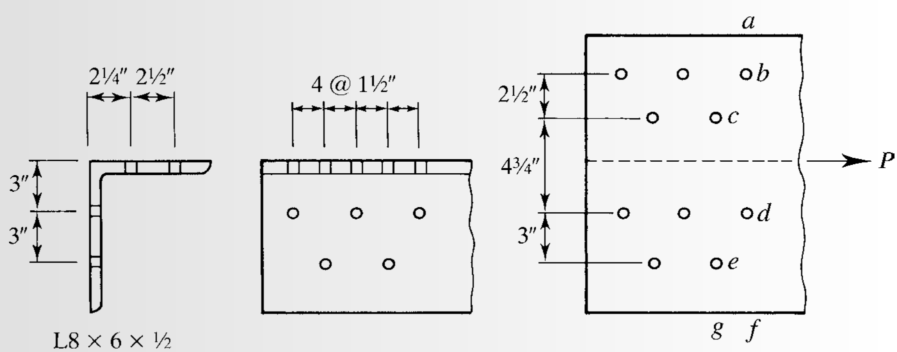
$$\begin{aligned} A_n &= A_g - \Sigma t_w \times (d \text{ or } d') \\ &= 6.80 - 0.5(1.0) \times 2 = 5.80 \text{ in.}^2 \end{aligned}$$

For line *abceg*,

$$A_n = 6.80 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5(1.0) = 5.413 \text{ in.}^2$$

Because 1⁄10 of the load has been transferred from the member by the fastener at *d*, this potential failure line must resist only 9⁄10 of the load. Therefore, the net area

FIGURE 3.17



of 5.413 in.^{2} should be multiplied by $^{10}/_9$ to obtain a net area that can be compared with those lines that resist the full load. Use $A_n = 5.413(^{10}/_9) = 6.014$ in.^{2} For line *abcdeg,*

$$g_{cd} = 3 + 2.25 - 0.5 = 4.75 \text{ in.}$$

$$A_n = 6.80 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(4.75)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(3)}\right]$$

$$= 5.065 \text{ in.}^2$$

The last case controls; use

$$A_n = 5.065 \text{ in.}^2$$

Both legs of the angle are connected, so

$$A_e = A_n = 5.065 \text{ in.}^2$$

The nominal strength based on fracture is

$$P_n = F_u A_e = 58(5.065) = 293.8 \text{ kips}$$

The nominal strength based on yielding is

$$P_n = F_y A_g = 36(6.80) = 244.8 \text{ kips}$$

a. The design strength based on fracture is

$$\phi_t P_n = 0.75(293.8) = 220 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(244.8) = 220 \text{ kips}$$

ANSWER Design strength = 220 kips.

b. For the limit state of fracture, the allowable stress is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable strength is

$$F_t A_e = 29.0(5.065) = 147 \text{ kips}$$

For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

$$F_t A_g = 21.6(6.80) = 147 \text{ kips}$$

ANSWER Allowable strength = 147 kips.

EXAMPLE 3.8

Determine the smallest net area for the American Standard Channel shown in Figure 3.18. The holes are for ⅝-inch-diameter bolts.

SOLUTION

$$A_n = A_g - \sum t_w \times (d \text{ or } d')$$

$$d = \text{bolt diameter} + \frac{1}{8} = \frac{5}{8} + \frac{1}{8} = \frac{3}{4} \text{ in.}$$

Line *abe:*

$$A_n = A_g - t_w d = 3.82 - 0.437\left(\frac{3}{4}\right) = 3.49 \text{ in.}^2$$

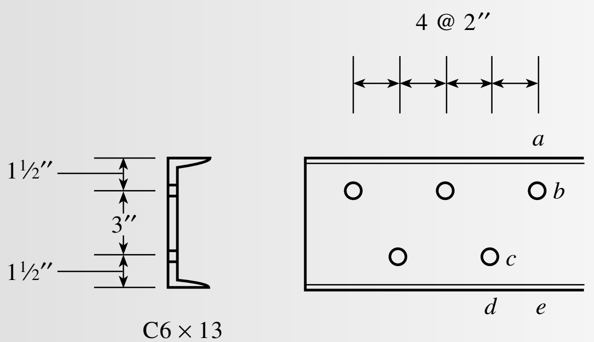
Line *abcd:*

$$A_n = A_g - t_w(d \text{ for hole at } b) - t_w(d' \text{ for hole at } c)$$

$$= 3.82 - 0.437\left(\frac{3}{4}\right) - 0.437\left[\frac{3}{4} - \frac{(2)^2}{4(3)}\right] = 3.31 \text{ in.}^2$$

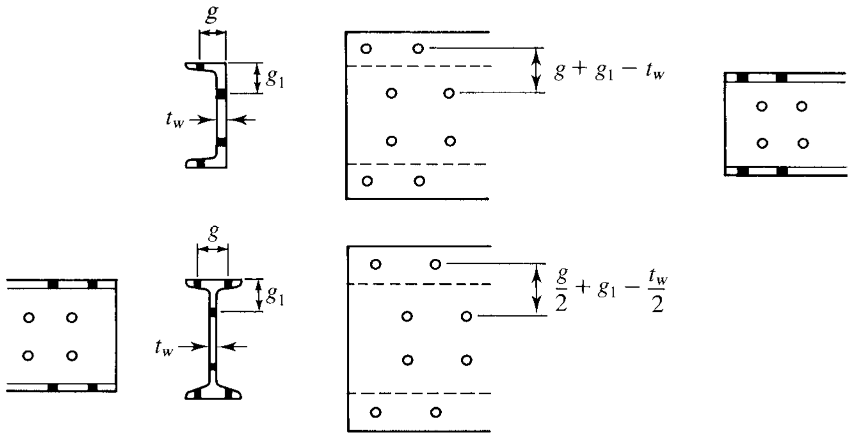
ANSWER Smallest net area = 3.31 in.^{2}

FIGURE 3.18



When staggered holes are present in shapes other than angles, and the holes are in different elements of the cross section, the shape can still be visualized as a plate, even if it is an I-shape. The AISC Specification furnishes no guidance for gage lines crossing a "fold" when the different elements have different thicknesses. A method for handling this case is illustrated in Figure 3.19. In Example 3.8, all of the holes are in one element of the cross section, so this difficulty does not arise. Example 3.9 illustrates the case of staggered holes in different elements of an S-shape.

FIGURE 3.19



EXAMPLE 3.9

Find the available strength of the S-shape shown in Figure 3.20. The holes are for ¾-inch-diameter bolts. Use A36 steel.

SOLUTION Compute the net area:

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$\text{Effective hole diameter} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

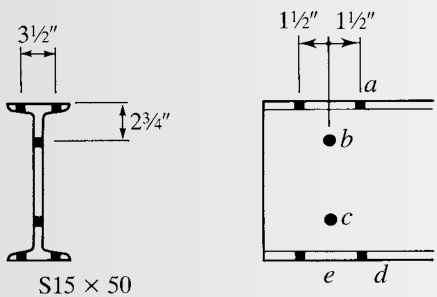
For line *ad*,

$$A_n = 14.7 - 4\left(\frac{7}{8}\right)(0.622) = 12.52 \text{ in.}^2$$

For line *abcd*, the gage distance for use in the $s^2/4g$ term is

$$\frac{g}{2} + g_1 - \frac{t_w}{2} = \frac{3.5}{2} + 2.75 - \frac{0.550}{2} = 4.225 \text{ in.}$$

FIGURE 3.20



Starting at a and treating the holes at b and d as the staggered holes gives

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$= 14.7 - 2(0.622)\left(\frac{7}{8}\right) - (0.550)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right]$$

$$- (0.550)\left(\frac{7}{8}\right) - 2(0.622)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right] = 11.73 \text{ in.}^2$$

Line *abcd* controls. As all elements of the cross section are connected,

$$A_e = A_n = 11.73 \text{ in.}^2$$

For the net section, the nominal strength is

$$P_n = F_u A_e = 58(11.73) = 680.3 \text{ kips}$$

For the gross section,

$$P_n = F_y A_g = 36(14.7) = 529.2 \text{ kips}$$

LRFD SOLUTION

The design strength based on fracture is

$$\phi_t P_n = 0.75(680.3) = 510 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(529.2) = 476 \text{ kips}$$

Yielding of the gross section controls.

ANSWER

Design strength = 476 kips.

ASD SOLUTION

The allowable stress based on fracture is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_e = 29.0(11.73) = 340$ kips.

The allowable stress based on yielding is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_g = 21.6(14.7) = 318$ kips.

Yielding of the gross section controls.

ANSWER

Allowable strength = 318 kips.

3.5 BLOCK SHEAR

For certain connection configurations, a segment or "block" of material at the end of the member can tear out. For example, the connection of the single-angle tension member shown in Figure 3.21 is susceptible to this phenomenon, called *block shear*.