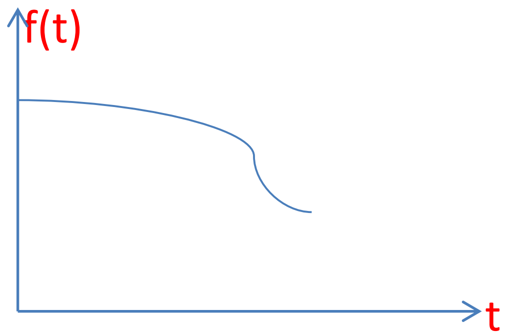
Z-TRANSFORM-PART-1 By Dr. Abdullah Jabar Hussain

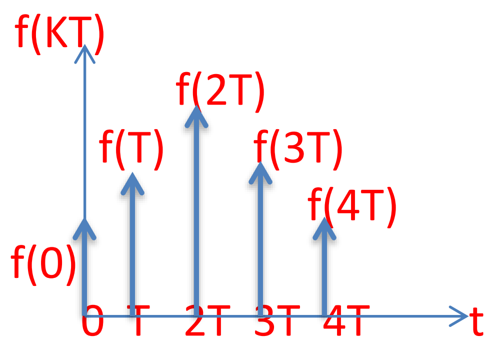
Z TRANSFORM

DEFINATION OF Z TRANSFORM

Z TRANSFORM DEALS WITH DISCRETE FUNCTION.

In mathematics and signal processing, the Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace transform.





Continuous function

discrete function

{f(KT)}={ f(0) , f(T) ,f(2T) , f(3T) ………}

For t = KT ⟹ K= 0 , 1 ,2, 3 ,4, …………….

f(t) = f(KT) = f(n) , for KT=n

The general definition of z transform that x(n) is the sequence of discrete time signal and it is used in digital system , signal analysis and stability .
Now;

$$X(n) \underset{z^{-1}}{\overset{z}{\longleftrightarrow}} \quad X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

If x(n) casual sequence i.e. for n greater than zero n> $0\ and\ x(n) = 0\ for\ n < 0$

And we have to remember that z is complex variable then according to that

X(z) = $\sum_{0}^{\infty} x(n)\ z^{-n}$ so according to this

X(z) = x(0) z^{0}+ x(1)z^{-1}+x(2)z^{-2}+x(3)z^{-3}..........

Now the most important is the region of convergence "ROC" which represent the region with x(z) become finite i.e. set of all values of z which make x(z) finite value

For any function there is x(z)

And for any function there is ROC

EX: let x(n) = u(n) find x(z)

Here the sequence is unit step function for n≥0 then its value equal unity.

Then,

X(z) = X(z) = $\sum_{0}^{\infty} x(n)\, z^{-n}$

As x(n) =1

Then,

X(z) = $\sum_{0}^{\infty} z^{-n}$ and we can write it as follows:

X(z) = $\sum_{0}^{\infty} (z^{-1})^{n}$

so,

X(z)= 1 + z^{-1} + z^{-2} + z^{-3} + ………………….

The standard for series is as follows:

1 + r + r^2 + r^3 + ……………..= $\frac{1}{1-r}$ when |r|< 1

If we compare this series with the series of x(z)

Then,

X(z) =1+ $(z^{-1})^1$ + $(z^{-1})^2$ + $(z^{-1})^3$ +…..

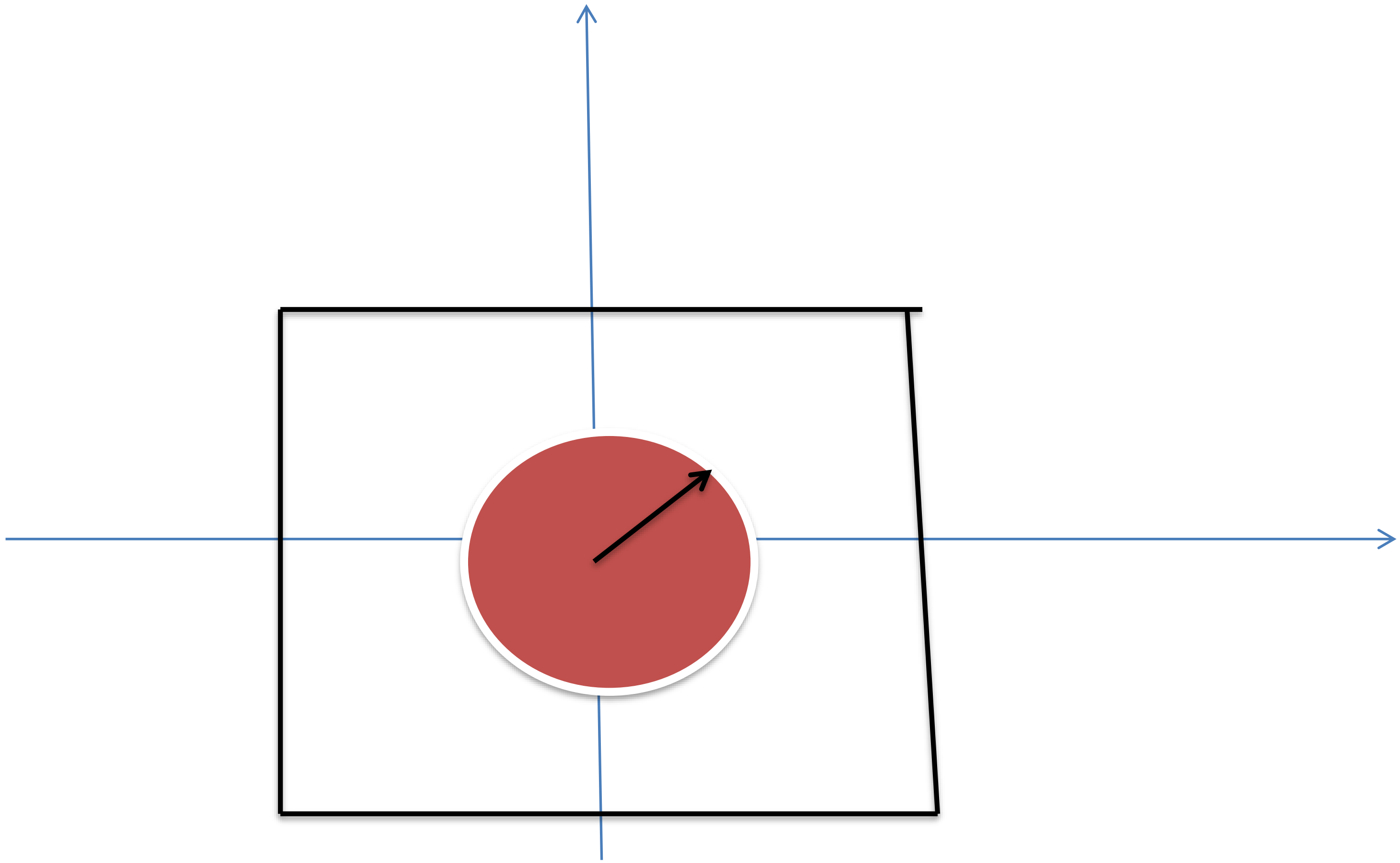
Then the second term is equivalent to r

So,

X(z) = $\frac{1}{1-z^{-1}}$ if and only if $|z^{-1}|$ < 1

In other word |z|> 1

and this is the radius of convergence ROC.



out side the radius of the circle is the “ROC” i.e. $|z| > 1$

ROC |Z| > 1

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EXAMPLE 2:

IF x(n) = a^n u(n), find x(z)

Any sequence multiply by unit step function for n grater or equal to zero and it is called casual function

Therefore,

X(z) = $\sum_0^\infty x(n)\, z^{-n}$= X(z) = $\sum_0^\infty a^n \; u(n) z^{-n}$

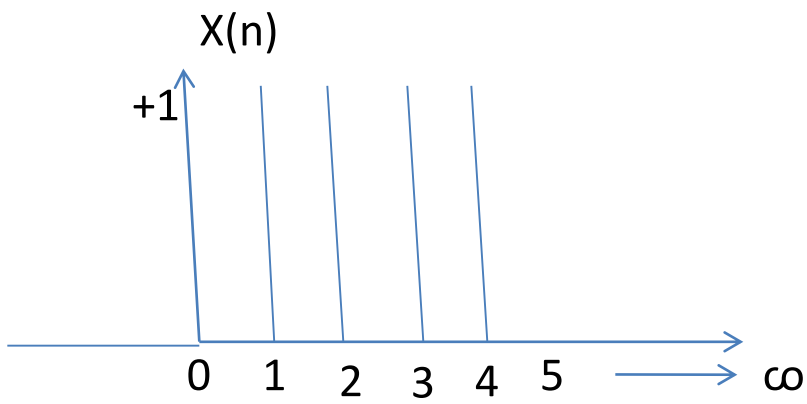
=X(z) = $\sum_0^\infty a^n z^{-n}$= X(z) =$\sum_0^\infty 1\, (az^{-1})^n$

$= \frac{1}{1-az^{-1}} = \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}$

Therefore, ROC= |Z|> a

Example:

Find the z – transform for the sequence x(n) as shown in the following figure



X(z) = $\sum_{-\infty}^{+\infty} x(n) z^{-n}$

X(n) = $\delta(n) + \ \delta(n-1) + \ \delta(n-2) + \ \delta(n-3) + \ \delta(n-4)$

ولغرض المقارنه لدينا صيغه معروفه كما يلي:

$$\sum_{k=0}^{n} r^k = \frac{r^{n+1} - 1}{r - 1}$$

From equation of x(z) then,

X(z) = $1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \frac{1}{z^4}$

Then ,

$\sum_{n=0}^{n=4}(z^{-1})$**n

So, according to the following:

$$\sum_{k=0}^{n} r^k = \frac{r^{n+1}-1}{r-1}$$

Then,

X(z)=$\frac{(z^{-1})^5 \ -1}{z^{-1}-1}$ = $\frac{z}{z-1}$ (1-z^{-5})

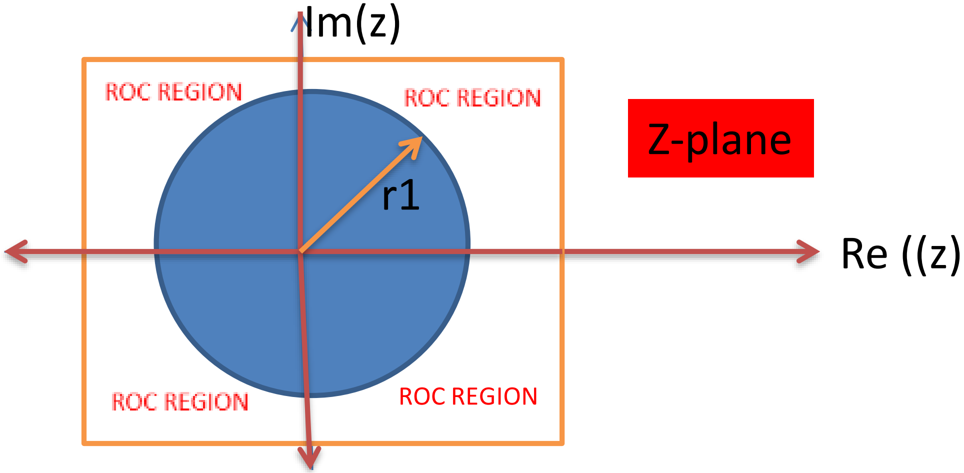
Then how to find ROC

لغرض ايجاد ROC

اتجنب قيم z التي تجعل x(z) مالانهايه

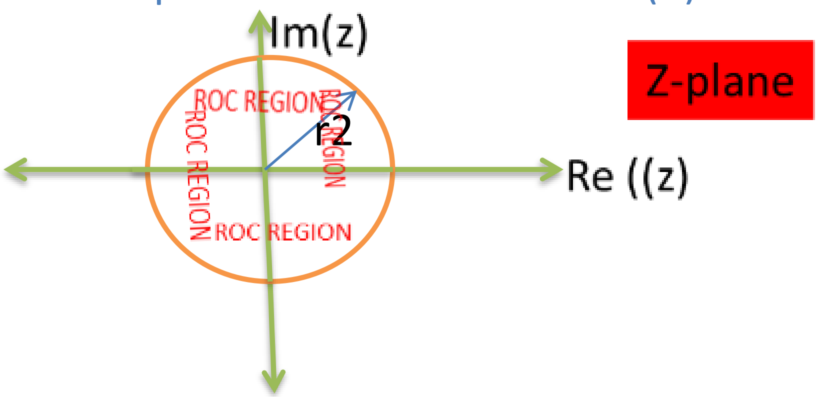
NOTE:

1-It is important to mention that if it is casual , x(n) =0 for n less than zero.

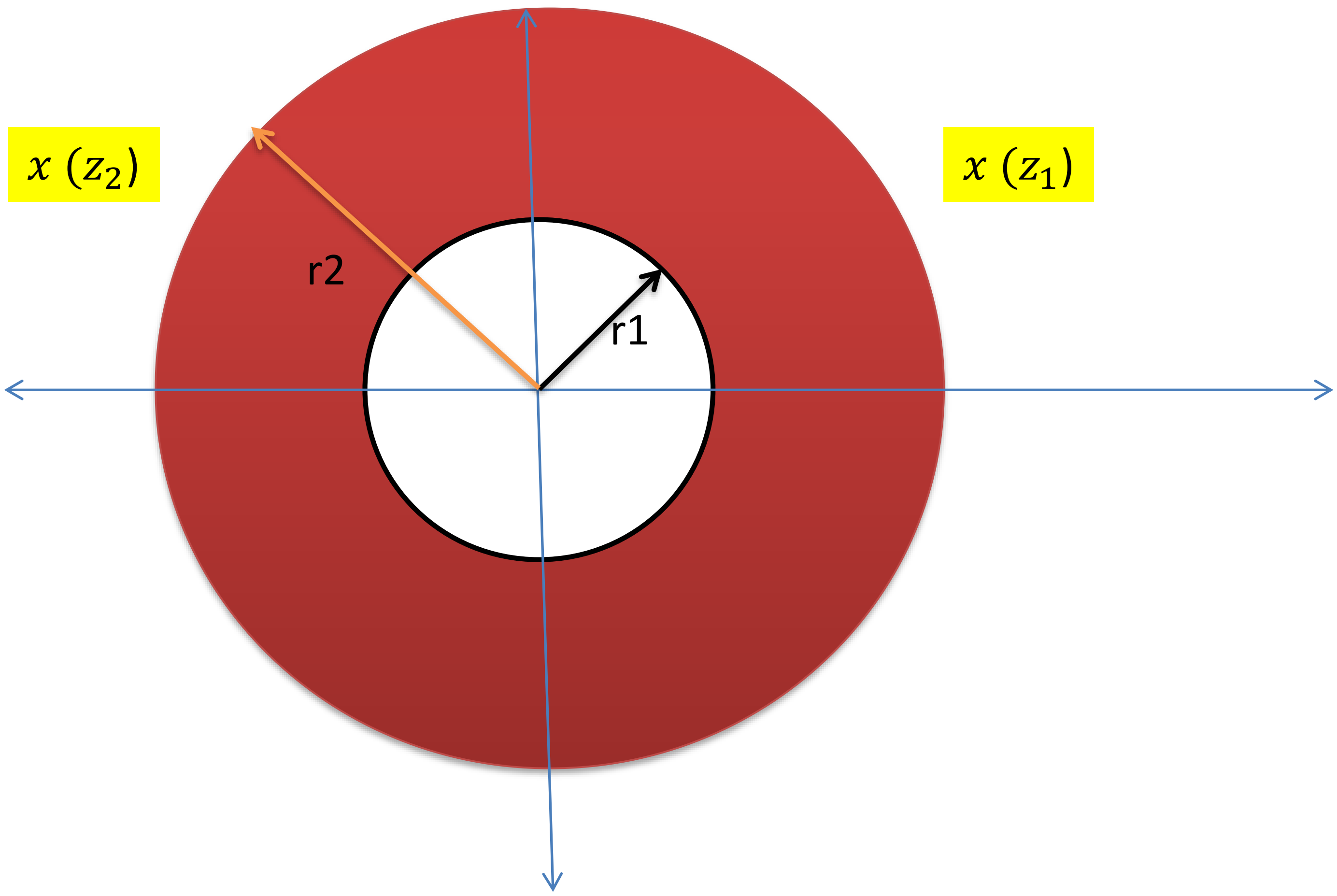


ROC i.e. |z| greater than r1

2. If the sequence is anti casual i.e. x(n) =0 for n greater than zero.



3. Two sides sequences i.e. Casual + anti casual



ROC for , r2 > |z | > r1 i.e. the red color region

No.	X(n) , n ≥0	Z(x)	ROC
1	δ (n)	1	|z|>0
2	u(n)	$\frac{1}{1-z^{-1}}=\frac{z}{z-1}$	|z|>1
3	a u(n) a= constant	$\frac{az}{z-1}$	|z|>1
4	n u(n), n =variable	$\frac{z}{(z-1)^2}$	|z|>1
5	n^2 u(n)	$\frac{z\,(z+1)}{(z-1)^3}$	|z|>1
6	a^n u (n)	$\frac{z}{z-a}$	|z|>a
7	e^{-na} u(n)	$\frac{z}{z-e^{-a}}$	|z|>e^{-a}
8	Sin(an) u(n)	$\frac{z\sin a}{z^2-2z\,cosa+1}$	|z|>1
9	cos (an) u(n)	$\frac{z\,\{z-\cos(a)\}}{z^2-2zcosa+1}$	|z|>|a|
10	a^n sin(b n) u(n)	$\frac{(a\,sinb)z}{z^2-(2a\,cosb)z+b^2}$	|z|>|a|

Examples:

1.

$$\boldsymbol{Z\{a^n\} = \frac{z}{z-a}}$$

$$Z\{a^n\} = \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= 1 + \frac{a}{z} + \frac{a^2}{z^2} + \frac{a^3}{z^3} + \cdots + \frac{a^n}{z^n} + \cdots$$

$$= \frac{1}{1-\frac{a}{z}}, \quad \left|\frac{a}{z}\right| < 1$$

$$\therefore Z\{a^n\} = \frac{z}{z-a}, \quad \left|\frac{a}{z}\right| < 1$$

2.

$$\boldsymbol{Z\{1\} = \frac{z}{z-1}}$$

$Z\{1\} = \frac{z}{z-1}$ Putting $a = 1$ in Result **1**