Al-Mustaqbal University

Department of Biomedical Engineering

Third Stage / 1st Course

"Transport Phenomena for BME"

Dr. A.Najah Saud

amir_Najah@uomus.edu.iq

Chapter 5 *Elementary Fluid Dynamics (Bernoulli equation)*

Fluid-Flow concept

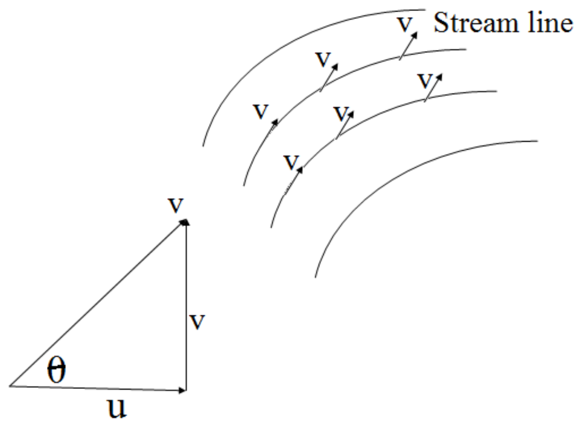
- ***Flow characteristics***
- **Ideal fluid:** is a frictionless (non-viscous (μ=0)) and incompressible ($\rho = constant$) fluid.
- **Adiabatic flow:** flow of fluid in which no heat is transferred to or from the fluid
- Steady flow: there is no change in any of the flow parameters with respect to time at a point ($\partial v \partial t$ =0)
- **Unsteady flow:** conditions at any point change with the time ($\partial v \partial t$ ≠0)
- **Uniform flow:** at every point the velocity vector is identically the same (in magnitude and direction) for any given instant
- **Non uniform flow:** the velocity vector varies from place to place at any instant

Fluid-Flow concept

- ***Flow characteristics***

- ***One-dimensional flow***: *variation in fluid properties such as velocity, pressure, …, occur only in one direction (x-direction) , and only average conditions of flow are considered at a section*
- ***Two-dimensional flow***: *variation in fluid properties occur in two directions (x and y directions)*
- ***Three-dimensional flow***: *variation in fluid properties occur in three directions (x-y and z directions)*

Streamline: *it is a continuous line drawn through the fluid so that it has the direction of the velocity vector at every point.*



RATE OF FLOW $\dot{m}$ OR DISCHARGE (Q):

- It is defined as the **quantity of fluid** flowing per second through a pipe or channel section.
- For an **incompressible fluid** (or **liquid**) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second Q.
- For **compressible fluids**, the flow rate is usually expressed as the mass of fluid flowing across the section $\dot{m}$.

Thus:-

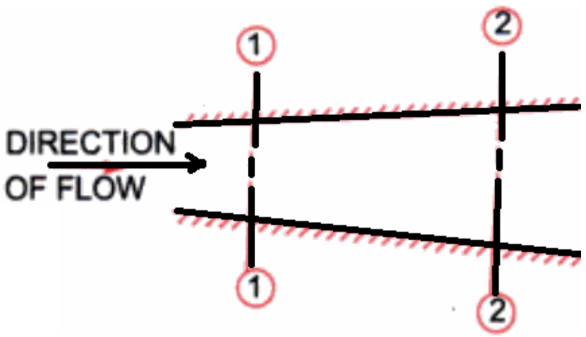
1. For liquids, the units of Q are m^3/s or liters/s
2. For gases, the units of $\dot{m}$ are kg/s

- Consider a fluid flowing through a pipe in which , A : cross-sectional area of pipe. and u = Average velocity of fluid across the section
- Then **discharge** $Q = A * u$

CONTINUITY EQUATION

- The equation based on the principle of conservation of mass is called the continuity equation.
- Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in figure.

Let u_1=Average velocity at cross-section at 1-1, ρ_1 =Density at section 1-1 A_1=Area of pipe at section 1-1



And u_2, ρ_2, A_2 are corresponding values at section 2-2 Then rate of flow at section 1-1 = $u_1 \rho 1A_1$ and

Rate of flow at section 2-2 = $u_2 \rho 2A_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 \text{ ....(1)}$$

The above equation is applicable to the compressible as well as incompressible fluids is called Continuity Equation. If the fluid is **incompressible**, then $\rho_1 = \rho_2$ and continuity equation (1) reduces to

$$A_1 u_1 = A_2 u_2 \text{ ....(2)}$$

CONTINUITY EQUATION

A typical application of mass conservation is at pipe

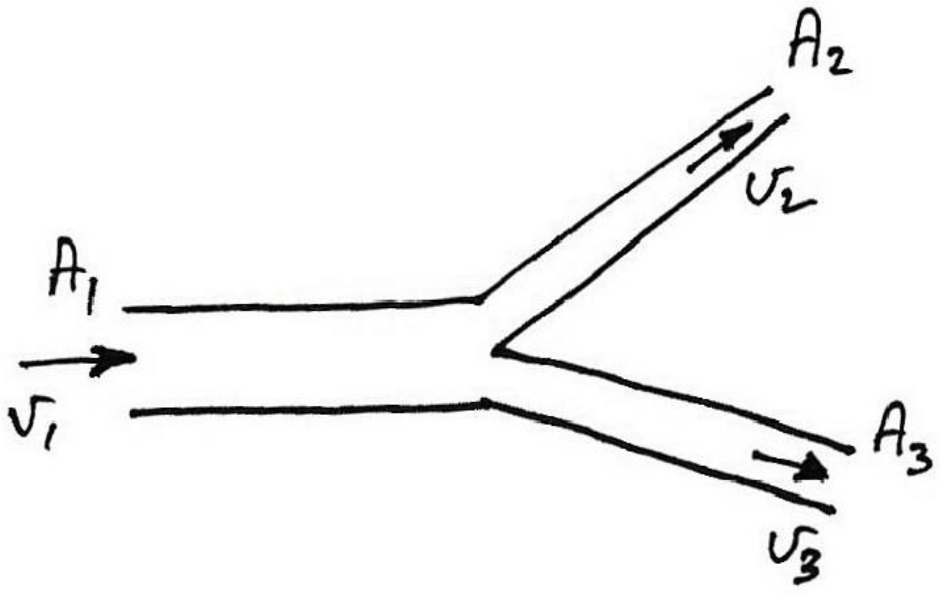
- From mass conservation we have:

$$Q_1 = Q_2 + Q_3$$

$$A_1 v_1 = A_2 v_2 + A_3 v_3$$

- If we consider inflow to be positive and outflow negative, we have:

$$\sum_{i=1}^{No.of\ Nodes} Ai\ vi = 0$$



CONTINUITY EQUATION

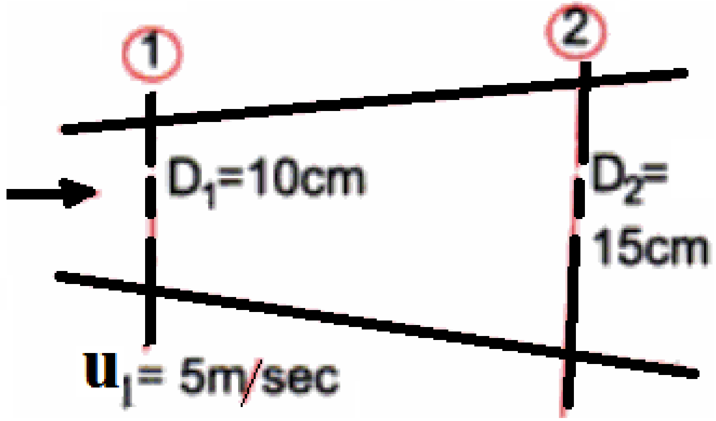
Example: The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5m/s. Determine the velocity at section 2.

Solution: $A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$ $A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$

Discharge through pipe $Q = A_1 * u_1 = .007854 \times 5 = \mathbf{0.03927 \text{ m}^3/s}$.

But $Q = A_1 u_1 = A_2 u_2$

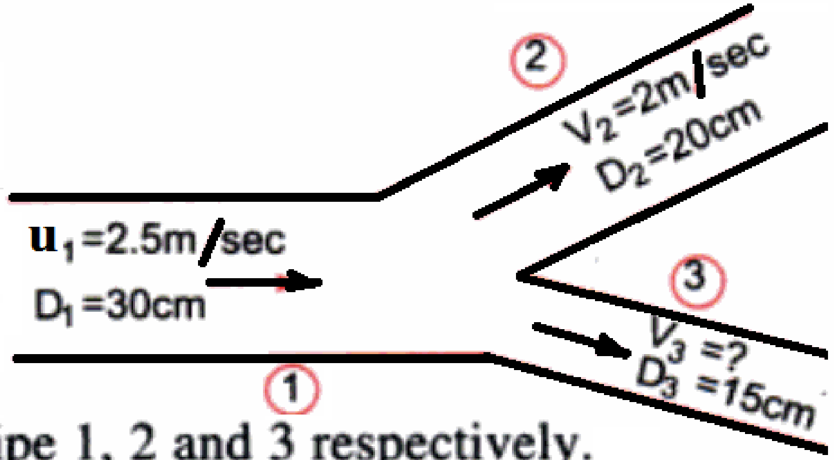
$$u_2 = \frac{A_1 u_1}{A_1} = \frac{.007854}{.01767} \times 5.0 = \mathbf{2.22 \text{ m/s}}$$



CONTINUITY EQUATION

Example: *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*

Solution:



$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} (.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2$$

$$A_3 = \frac{\pi}{4} (.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$$

Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation $Q_1 = Q_2 + Q_3$

$$Q_1 = A_1 \mathbf{u}_1 = 0.07068 \times 2.5 \text{ m}^3/\text{s} = \mathbf{0.1767 \text{ m}^3/\text{s}}$$

$$Q_2 = A_2 \mathbf{u}_2 = .0314 \times 2.0 = .0628 \text{ m}^3/\text{s}$$

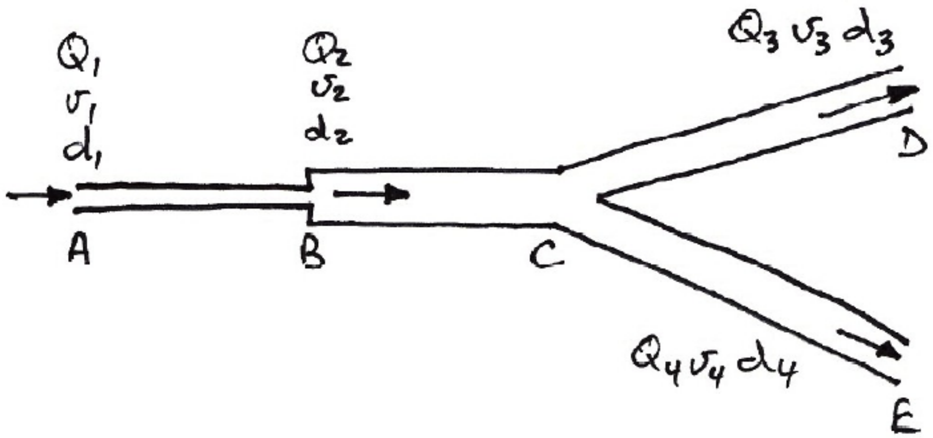
$$0.1767 = 0.0628 + Q_3 \Rightarrow \therefore Q_3 = 0.1139 \text{ m}^3/\text{s}$$

But $Q_3 = A_3 \times \mathbf{u}_3 = .01767 \times \mathbf{u}_3 = .1139$

$$\therefore \mathbf{u}_3 = \frac{.1139}{.01767} = \mathbf{6.44 \text{ m/s.}}$$

CONTINUITY EQUATION

Water flows from point *A* to points *D* and *E* as shown. Some of the flow parameters are known, as shown in the table. Determine the unknown parameters.



Section	Diameter (mm)	Flow Rate (m^3/s)	Velocity (m/s)
AB	300	?	?
BC	600	?	1.2
CD	?	$Q_3 = 2Q_4$	1.4
CE	150	$Q_4 = 0.5Q_3$	?

CONTINUITY EQUATION

Solution

From the law of mass conservation we can see:

$$Q_1 = Q_2$$

And as total inflow must equal total outflow:

$$Q_1 = Q_{out}$$
$$= Q_3 + Q_4$$
$$= Q_3 + 0.5Q_3$$
$$= 1.5\ Q3$$

We must also work out the areas of the pipes, $A_i = \frac{\pi d_i^2}{4}$ Hence:

$A_1 = 0.0707\ m^3 \quad A_2 = 0.2827\ m^3 \quad A_4 = 0.0177\ m^3$

Starting with our basic equation, $Q = Av$, we can only solve for Q_2 from the table:

$$Q_2 = (0.2827)(1.2)$$
$$= 0.3393\ m^3/s$$

We know that $Q_1 = Q_2$ and so we can now calculate Q_3 from previous:

$$Q_1 = 1.5Q_3$$

$$Q_3 = \frac{Q_1}{1.5} = \frac{0.3393}{1.5} = 0.2262\ m/s$$

CONTINUITY EQUATION

$$Q_4 = \frac{Q_3}{2} = \frac{0.2262}{2} = 0.1131\ \text{m}^3/\text{s}$$

Thus we have all the flows. The unknown velocities are:

$$v_1 = \frac{Q_1}{A_1} = \frac{0.3393}{0.0707} = 4.8\ \text{m/s}$$

$$v_4 = \frac{Q_4}{A_4} = \frac{0.1131}{0.0177} = 6.4\ \text{m/s}$$

And lastly, the diameter of pipe *CD* is:

$$A_3 = \frac{Q_3}{v_3} = \frac{0.2262}{1.4} = 0.1616\ \text{m}^2$$

$$d_3 = \sqrt{\frac{4A_3}{\pi}} = 0.454\ \text{m}$$

And so it is likely to be a 450 mm ϕ pipe.

Equation of motion along a

Work done by P_1-P_2=ΔPE+ ΔKE

W= F.X (for Solid)

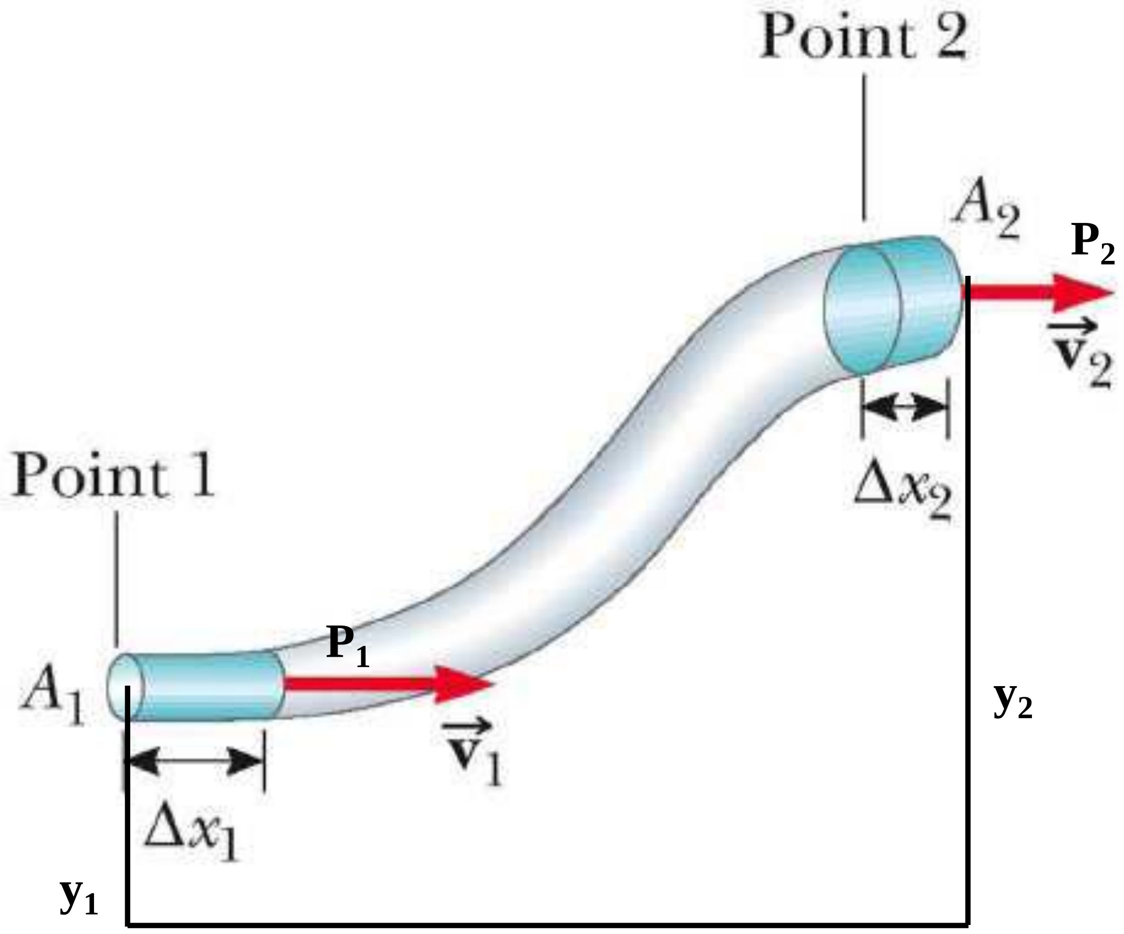
For liquids

W= P*A*X

A*X= V (volume)

So the work done in liquids

W= P*V …….. (1)



CONTINUITY EQUATION

$W_{P1-P2} = \Delta PE + \Delta KE$

$(P_1-P_2) * V = (m*g*y_2 - m*g*y_1) + (\frac{1}{2} m\, u_2^2 - \frac{1}{2} m\, u_1^2)$

Divided the above equation by V

$((P_1-P_2) * V)/V = ((m*g*y_2 - m*g*y_1))/V + ((\frac{1}{2} m\, u_2^2 - \frac{1}{2} m\, u_1^2))/V$

m/V= ρ (density of the fluid)

$P_1-P_2 = \rho*g*y_2 - \rho*g*y_1 + (\frac{1}{2} \rho\, u_2^2 - \frac{1}{2} \rho\, u_1^2)$

$P_1 + \rho*g*y_1 + \frac{1}{2} \rho\, u_1^2 = P_2 + \rho*g*y_2 + (1/2\, \rho\, u_2^2)$

$P + \rho*g*y + \frac{1}{2} \rho\, u^2 =$ **Constant**

Divided the above equation by ρ

$P/\rho + g*y + \frac{1}{2} u^2 =$ **Constant** (This is *Bernoulli's equation*)

Also, the equation can be expressed in the following form:

$$\frac{P}{\gamma} + \frac{V^2}{2g} + z = constant \quad (for\ liquids)$$

$$P + \frac{\rho V^2}{2} + \gamma z = constant \quad (for\ gases)$$

$$\frac{P}{\gamma} = \underbrace{pressure\ head}_{(m)} = flow\ energy\ per\ unit\ weight (N.m/N)$$

$$\frac{V^2}{2g} = \underbrace{velocity\ head}_{(m)} = Kinetic\ energy\ per\ unit\ weight (N.m/N)$$

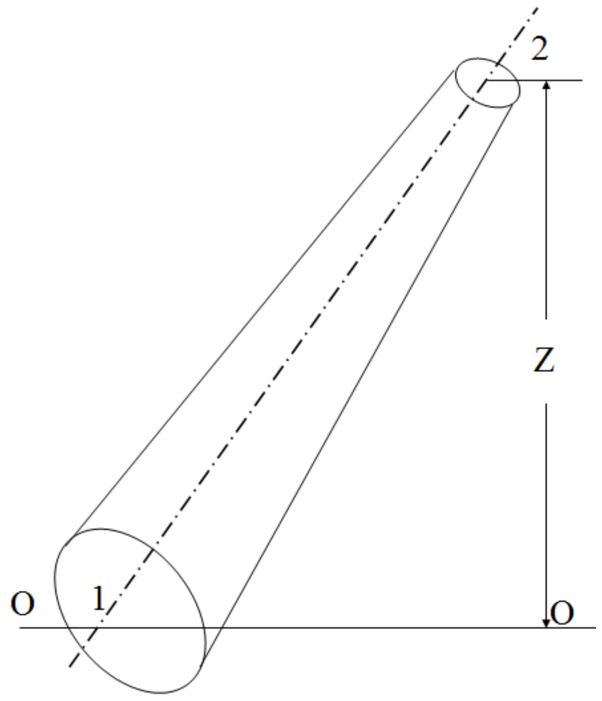
$$z = \underbrace{elevation\ head}_{(m)} = potential\ energy\ per\ unit\ weight (N.m/N)$$

Example

a pipe gradually tapers from a diameter of (0.3 m) to (0.1 m) over the length shown in the figure it conveys water at (50 L/s) the pressure at the bottom end is (196 KN/m^2) if the pressure at the upper end is (98.1 KN/m^2), find the value of (Z)?

- **Sol//**

$$Q = 50\ L/s = 50 \times 10^{-3} = 0.05\ m^3/s$$



$$Q = A_1V_1 = A_2V_2$$

$$0.05 = \left(\frac{\pi}{4}(0.3)^2\right)V_1$$

$$V_1 = 0.707\ m/s$$

$$0.05 = \left(\frac{\pi}{4}(0.1)^2\right)V_2$$

$$V_2 = 6.365\ m/s$$

Applying Bernoulli's eq. along the axis of pipe between (1 and 2) taking (OO) as datum:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\frac{196000}{9806} + \frac{(0.707)^2}{2 \times 9.81} + 0 = \frac{98100}{9806} + \frac{(6.365)^2}{2 \times 9.81} + z$$

$$z = 7.944\ m$$

Example

Water flows steadily up the vertical *0.1 m* diameter pipe and out the nozzle, which is *0.05 m* in diameter, discharging to atmospheric pressure. The stream velocity at the nozzle exit must be *20 m/s*. Calculate the minimum gage pressure required at section①.

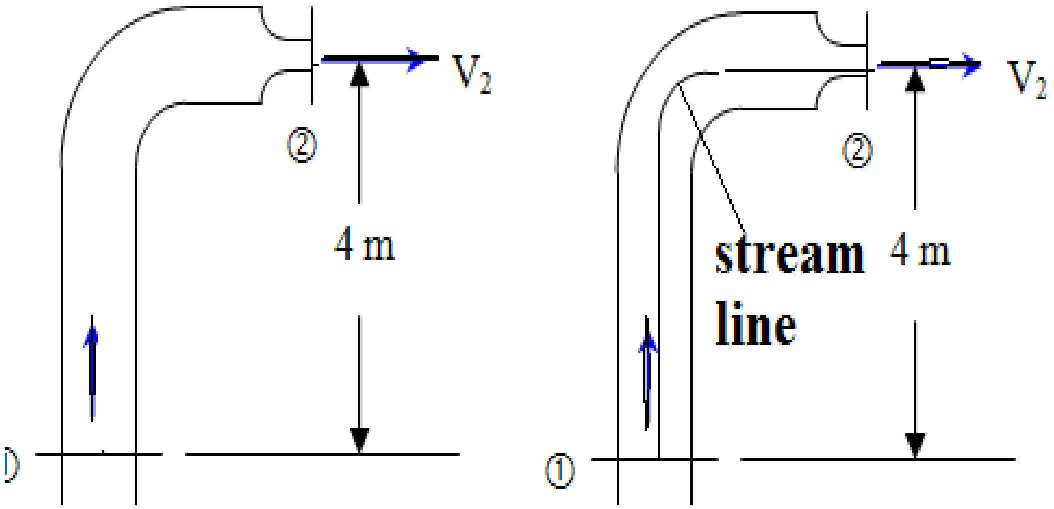
Solution: applying continuity equation between 1 and 2 we have $u_1 A_1 = u_2 A_2$ and $u_1 = u_2 A_2 / A_1$

The Bernoulli's equation can be applied between any two points on a streamline provided that the all assumptions are satisfied. The result is

$$\frac{p_1}{\rho} + \frac{u_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{u_2^2}{2} + gz_2 \text{ and } \frac{p_1 - p_2}{\rho} = \frac{1}{2}\left[u_2^2 - u_1^2\right] + g(z_2 - z_1) \text{ also}$$

$$\frac{p_1 - p_2}{\rho} = \frac{1}{2}\left[u_2^2 - \left(\frac{A_2}{A_1}u_2\right)^2\right] + g(z_2 - z_1) \text{ and finally } p_{1gage} - p_{2gage} = \rho\left\{\frac{1}{2}u_2^2\left[1 - \left(\frac{D_2^2}{D_1^2}\right)^2\right] + g(z_2 - z_1)\right\}$$

$$p_{1gage} - (0) = (1000)\left\{\frac{1}{2}(20)^2\left[1 - \left(\frac{(0.05)^2}{(0.1)^2}\right)^2\right] + (9.81)[(4m) - (0)]\right\} = 226500Pa = 226.5kPa$$



H.W

Water is discharged through the drain pipe at *B* from the large basin حوض at 0.03 m^3/s. If the diameter of the drain pipe is $d = 60$ mm, determine the pressure at *B* just inside the drain when the depth of the water is $h = 2$ m.