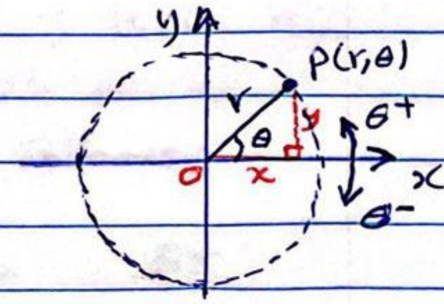
Polar Coordinates :-

The Polar Coordinates can be defined as a pair of r & θ "(r,θ)", where r is the distance of a point from the origin and θ is the angle between the line segment from the origin to the point and the x-axis.



$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\dfrac{y}{x}$

The above set of Four equations are using to Convert the polar to Cartesian (rectangular) and vice versa.

تُستخدم المعادلات الاربعة اعلاه بالتحويل من الاحداثيات القطبية للكارتيزية والعكس بالعكس.

$(r,\theta) \rightleftarrows (x,y)$

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Ex) Convert $(-1,-1)$ into polar coordinates?

Solution

$(x,y) \longrightarrow (r,\theta)$

$x=-1,\ y=-1$

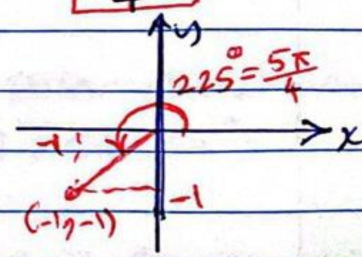
$r=\sqrt{x^2+y^2} \Longrightarrow r=\sqrt{(-1)^2+(-1)^2}=\sqrt{1+1}=\boxed{\sqrt{2}}$

$\theta=\tan^{-1}\frac{y}{x} \longrightarrow \theta=\tan^{-1}\frac{-1}{-1}=\tan^{-1}1=\boxed{45^\circ}$

since the point $(-1,-1)$ is in quadrant 3
∴ the angle 45 is not correct & need to be corrected as follows;

$\theta=\frac{\pi}{4}+\pi=45+180=\boxed{\frac{5\pi}{4}}=\boxed{225^\circ}$

$\therefore (-1,-1)=(\sqrt{2},\frac{5\pi}{4})$



Ex) Convert $(4,60^\circ)$ into rectangular coordinates?

Solution

$(r,\theta) \longrightarrow (x,y)$; $r=4,\ \theta=60^\circ$

$x=r\cos\theta \Longrightarrow x=4\cos 60=4*\frac{1}{2}=\boxed{2}$

$y=r\sin\theta \Longrightarrow y=4\sin 60=4*\frac{\sqrt{3}}{2}=\boxed{2\sqrt{3}}$

$\therefore (4,60^\circ)=(2,2\sqrt{3})$

Ex) Convert $(6, \frac{5\pi}{6})$ into Cartesian Coord.?

Solution)

$(r, \theta) \longrightarrow (x, y)$; $r = 6$, $\theta = \frac{5\pi}{6}$

لتحويل الزاوية من نظام الريديان الى الدرجات نقوم بالتالي:

$$\theta = \frac{5\cancel{\pi}}{6} * \frac{180}{\cancel{\pi}} = \frac{5 * \cancel{180}^{30}}{\cancel{6}_{1}} = \boxed{150^\circ}$$

$$x = r\cos\theta \longrightarrow x = 6 * \cos 150 = 6 * (\frac{-\sqrt{3}}{2}) = \boxed{-3\sqrt{3}}$$

$$y = r\sin\theta \longrightarrow y = 6 * \sin 150 = 6 * (\frac{1}{2}) = \boxed{3}$$

$$\therefore (6, \frac{5\pi}{6}) = (-3\sqrt{3}, 3)$$

Question For Discussion :-

- Convert $(2, -4)$ into polar coordinates (r, θ)? Ans: $(2\sqrt{5}, 296.56^\circ)$
- Convert $(-5, 5\sqrt{3})$ into (r, θ)? Ans: $(10, 120^\circ)$
- Convert $(7, 140^\circ)$ into (x, y)? Ans: $(-5.362, 4.5)$
- Convert $(1.5, 300^\circ)$ into (x, y)? Ans: $(0.75, -1.3)$

Ex) Convert $2x - 5x^3 = 1 + xy$ into polar coordinates?

Solution)

$x = r\cos\theta$

$y = r\sin\theta$

$\theta = \tan^{-1}\frac{y}{x}$

عوض عن كل x و y بما يساويها من العلاقات اعلاه ينتج:

$$2r\cos\theta - 5(r\cos\theta)^3 = 1 + (r\cos\theta)(r\sin\theta)$$

$$\boxed{2r\cos\theta - 5r^3\cos^3\theta = 1 + r^2\sin\theta\cos\theta} \quad \text{Ans}$$

Ex) Convert the polar equation $4r\cos\theta + r\sin\theta = 8$ into cartesian coordinates equation that express y in term of x?

Solution)

$x = r\cos\theta$ — عوض عن كل $(r\cos\theta)$ بـ (x) وعن كل

$y = r\sin\theta$ — $(r\sin\theta)$ بـ (y) لتصبح كالتالي:

$$4(r\cos\theta) + r\sin\theta = 8$$

$$4x + y = 8$$

$$\therefore \boxed{y = 8 - 4x} \quad \text{Ans}$$

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Ex/ Convert $r = -8\cos\theta$ into rectangular coordinates?

Solution/

$x = r\cos\theta \quad ; \quad y = r\sin\theta$

$r = \sqrt{x^2+y^2} \Longrightarrow r^2 = x^2+y^2$

هنا نحتاج الى ضرب طرفي المعادلة بـ r لتصبح المعادلة كالتالي:-

$$\cancelto{x^2+y^2}{r^2} = -8\,\cancelto{x}{(r\cos\theta)}$$

$$\therefore \boxed{x^2+y^2 = -8x} \quad \text{Ans}$$

كذلك يمكننا أن نعوض عن $\cos\theta$ بـ $\frac{x}{r}$ وهو طريقة ثانية للحل كما في التالي:-

$$r = -8 * \frac{x}{r} \Longrightarrow r^2 = -8x$$

هنا نعوض عن r^2 بما يساويها (x^2+y^2)

$$\therefore \boxed{x^2+y^2 = -8x} \quad \text{Ans}$$

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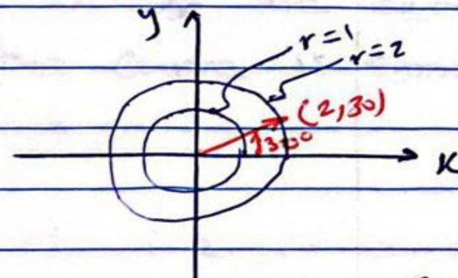
Graph of Polar Functions :-
رسم الدوال القطبية

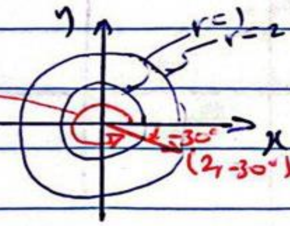
قبل البدء برسم الدوال القطبية لنتعلم كيفية تمثيل نقطة قطبية على المستوى القطبي:

Ex) Represent the point $(2, 30°)$ on to polar plane ?



Ex) Rep $(2, -30°)$ on to polar plane ?

$(2, -30°) \Longrightarrow (2, 330°)$ $360-30 = \boxed{330°}$



Ex) Rep $(-3, 60°)$ onto Polar plane?

Sol)

هنا قيمة r سالبة هنا السهم يكون بالاتجاه المعاكس والزاوية تكون زاوية مقابلة لاتجاه السهم فتصبح الزاوية فتكون النقطة $\underline{240°} = (60+180)$ الجديدة هي

$(3, 60+180) = (\underline{3, 240°})$

$(-3, 60°) \xrightarrow{\text{converted to}} (3, 240°)$

عند رسم الدوال القطبية سيظهر لنا مصطلح التناظر (symmetry) ، ويكون حول x أو y أو نقطة الأصل ، وتفصيلها في أدناه :-

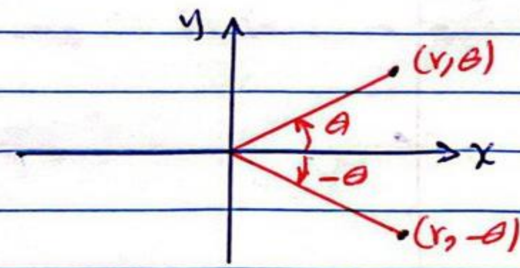
Symmetry of Polar Coordinates :-

There are three types :-

① **About x-axis (polar axis) :- (0°-axis)**

If θ is replaced by $-\theta$ & the eq= does not change, then the curve is symmetric about the x-axis.

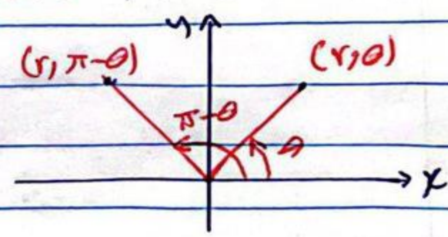
Ex) $\longrightarrow$ $\cos\theta = \cos(-\theta)$, i.e $\theta \to -\theta$



② **About y-axis (90°-axis) :-**

If θ is replaced by $(\pi-\theta)$ & the eq= does not change, then the curve is symmetric about the y-axis.

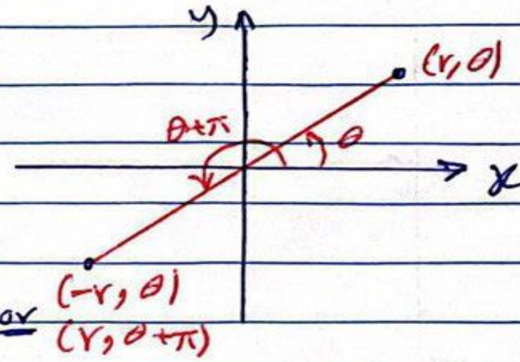
Ex) $\longrightarrow$ $\sin(\pi-\theta) = \sin\theta$, i.e $\theta \to \pi-\theta$



③ About the orsgin (Pole) :- قطب

If r is replaced by $(-r)$ & θ is replaced by $(\pi+\theta)$ & the eq= does not change, then the curve is symmetric about the orsgin

Ex) $\longrightarrow r^2 = (-r)^2$



Ex) Show that the eq=s below are equivalents?

① $r = 1 - \sin(\theta - \frac{\pi}{3})$

② $r = 1 + \cos(\theta + \frac{\pi}{6})$

③ $r = \cos(\theta + \frac{\pi}{6}) - 1$

Solution)

① $r = 1 - \sin(\theta - \frac{\pi}{3})$

$= 1 - (\sin\theta \cos\frac{\pi}{3} - \cos\theta \sin\frac{\pi}{3})$

$= 1 - (\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta)$ ✓

② $r = 1 + \cos(\theta + \frac{\pi}{6})$

$= 1 + (\cos\theta \cos\frac{\pi}{6} - \sin\theta \sin\frac{\pi}{6})$

$= 1 + (\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta)$

$= 1 - (\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta)$ ✓

③ $r = \cos\left(\theta + \frac{\pi}{6}\right) - 1$

let $r \to -r$ & $\theta = \theta + \pi$

$\therefore \; -r = \cos\left(\theta + \pi + \frac{\pi}{6}\right) - 1$

$-r = \cos\left(\theta + \frac{7\pi}{6}\right) - 1$

$-r = \cos\theta \cos\frac{7\pi}{6} - \sin\theta \sin\frac{7\pi}{6} - 1$

$\left(-r = -\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta - 1\right) \times -1$

$r = 1 - \left(\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta\right)$ ✓

$\therefore$ All the eq=s are equivalent.

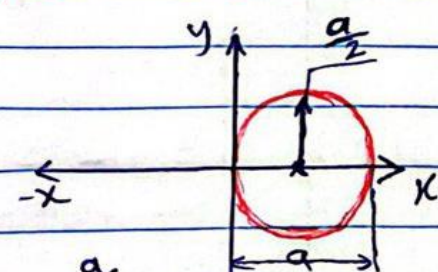
Graphing : **الرسم**

لرسم المعادلات القطبية توجد حالات (Cases) أولها
حالة رسم الدائرة (circle) وصيغتها كالتالي.

Graphing Cases : حالات الرسم

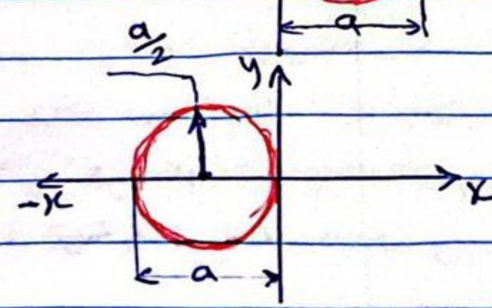
ⓐ Circles **الدوائر**

$r = a\cos\theta \longrightarrow$

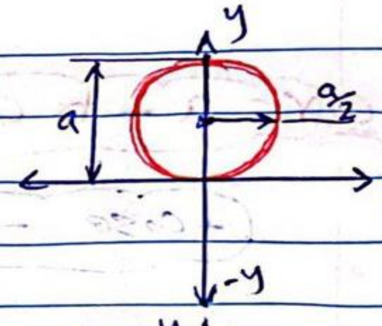


a — circle diameter

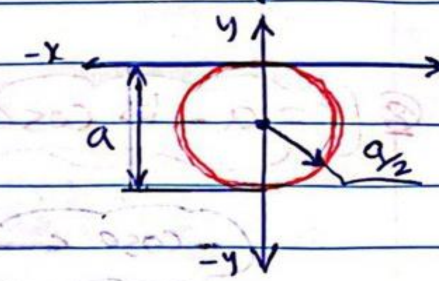
$r = -a\cos\theta \longrightarrow$



$r = a \sin\theta \Longrightarrow$



$r = -a \sin\theta \Longrightarrow$



(b) Limasons

The general equations of limasons are

$$r = a \mp b \cos\theta$$

$$r = a \mp b \sin\theta$$

* Case I: $a < b \Longrightarrow$ "Inner Loop"

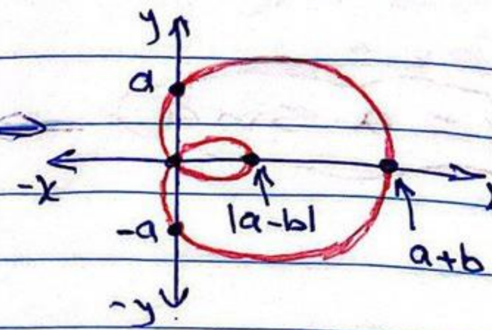
To graph this type of Limason, we want to make sure we get Four points:

Two x-intercepts & Two y-intercepts

لرسم هذا النوع من الليماسون، نحتاج لمعرفة اربع نقاط تقاطع مع الاحداثيات الافقية والعمودية، اثنان متقاطع مع x-axis واثنان مع y-axis . بالمرحلة التالية سنعرض تفاصيل الرسم.

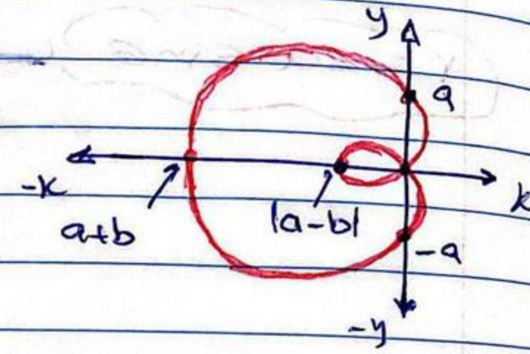
① $r = a + b\cos\theta \Rightarrow$

$+\cos\theta \longrightarrow$



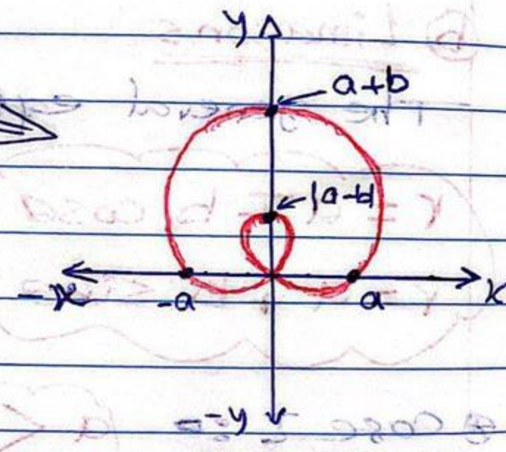
② $r = a - b\cos\theta \Rightarrow$

$-\cos\theta \longleftarrow$



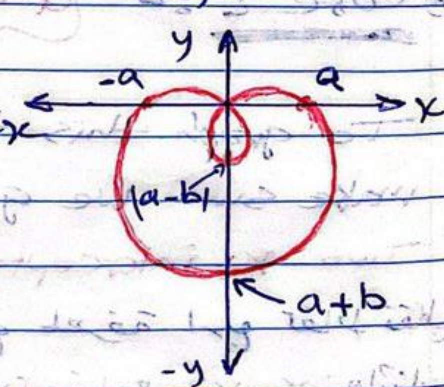
③ $r = a + b\sin\theta \Rightarrow$

$+\sin\theta \uparrow$



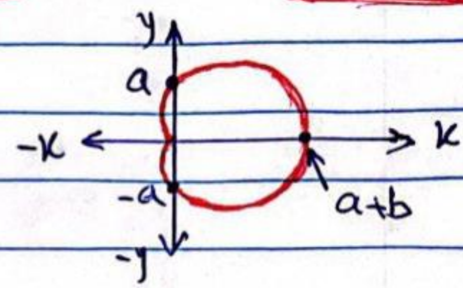
④ $r = a - b\sin\theta$

$-\sin\theta \downarrow$

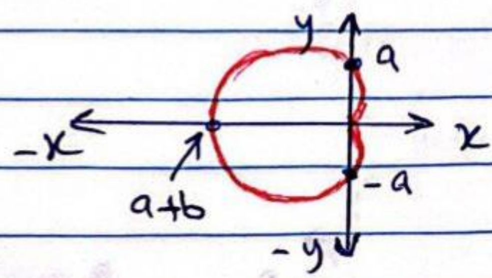


* **Case II :** $(a = b) \Longrightarrow$ "Cardioid"

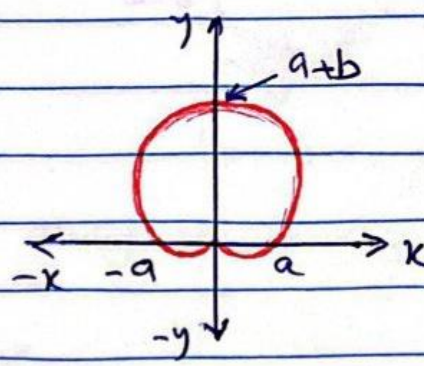
① $+\cos\theta \Longrightarrow$



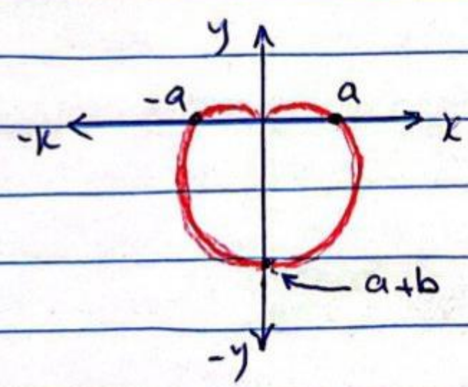
② $-\cos\theta \Longrightarrow$



③ $+\sin\theta \Longrightarrow$



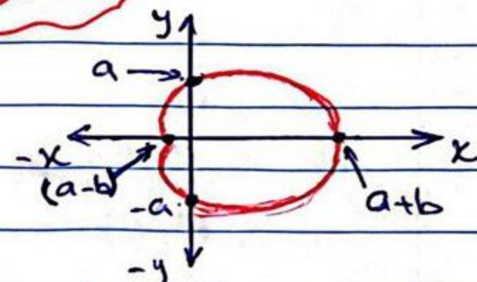
④ $-\sin\theta \Longrightarrow$



Case III :- $a > b$

a- $b < a < 2b$

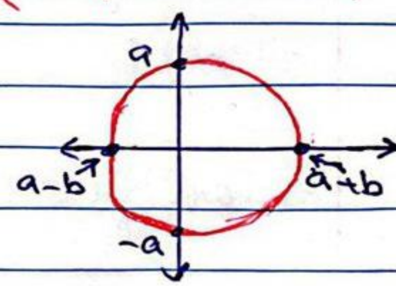
* يتأخذ تقعر حالة $\boxed{\cos\theta}$ الموجبة
مثال :



(Dimple Cardioid)

b- $a \geq 2b$

* كذلك يتأثر حالة $\boxed{\cos\theta}$ الموجبة
مثال :



(No Dimple)

Examples :-

① Graph $r = 4\cos\theta$

Sol.

The shap is circle, to check about which axis is symmetric, we do the follows:

① put $r = -r \Longrightarrow -r = 4\cos\theta$ (not symm. about the origin)

② put $\theta = \pi - \theta \Longrightarrow r = 4\cos(\pi - \theta)$

$= 4(\cos\pi\cos\theta + \sin\theta\sin\pi)$

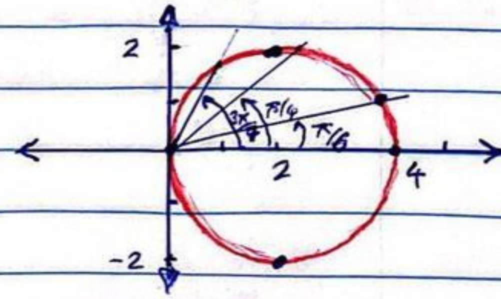
$r = -4\cos\theta$ (not symm. about y-axis)

③ Put $\theta = -\theta \Rightarrow r = 4\cos(-\theta) = \underline{4\cos\theta}$

(symmetry about the x-axis)

★ من خلال المعادلات العامة لرسم المعادلات القطبية ، الشكل هو دائرة تقع على x-axis بقطر 4 وحدات.

r	θ
4	0
3.46	$\pi/6$
2.83	$\pi/4$
0	$\pi/2$
-2.83	$3\pi/4$
-4	π

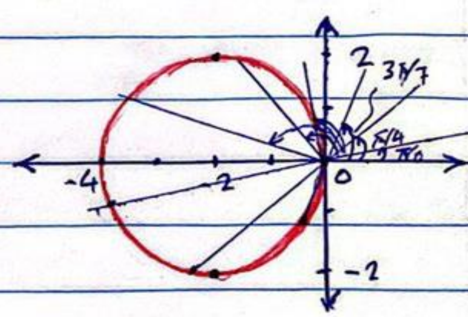


② Sketch $r = -4\cos\theta$

sol.:

هنا نطبق نفس الخطوات في المثال ① لمعرفة مع اي اجابة يكون الشكل متناظر. ومن خلال المعادلات العامة لرسم المعادلات القطبية ، الشكل هو دائرة بقطر 4 وحدات تقع على x-axis السالب.

r	θ
-4	0
-3.46	$\pi/6$
-2.83	$\pi/4$
0	$\pi/2$
2.83	$3\pi/4$
4	π



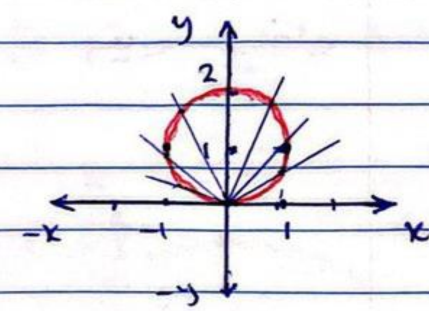
③ Graph $r = 2\sin\theta$

الدالة هي دالة جيب ، وتعطي رسم لشكل متناظر حول y-axis بوجود ثابت ذلك نعوض مكان θ بـ $\pi-\theta$

$\therefore$ put $\theta = \pi-\theta \longrightarrow \boxed{r = 2\sin(\pi-\theta) = 2\sin\theta}$

من خلال المعادلات العامة فالمعادلة $r = 2\sin\theta$ يعطي رسم دائرة متناظرة حول y-axis الموجب ويقطر 2 وحدة

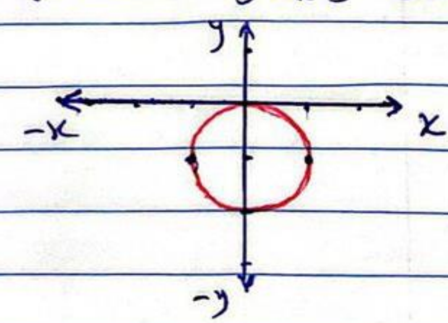
r	θ
0	0
1	$\pi/6$
1.414	$\pi/4$
2	$\pi/2$
1.414	$3\pi/4$
0	π



④ Sketch $r = -2\sin\theta$

الحل تماماً نفس الموجود في المثال السابق لكن يقع على y-axis السالب كدائرة بقطر 2 وحدة .

r	θ
0	0
-1	$\pi/6$
-1.414	$\pi/4$
-2	$\pi/2$
-1.414	$3\pi/4$
0	π



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(5) Graph $r = 3 + 5\cos\theta$

Sol/ Compare to $r = a + b\cos\theta$

Its a type of a limason's graphs! and as

$a = 3$, $b = 5 \Longrightarrow a < b \Longrightarrow$ "Inner loop"

and as $+\cos\theta$ has +ve sign, then it is located on +ve x-axis an symmetric on it.

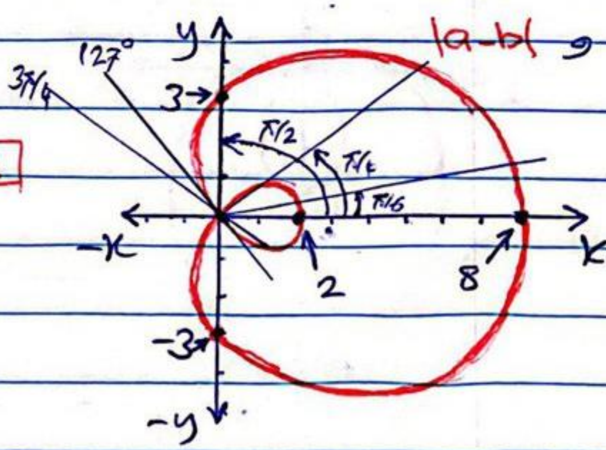
من خلال المعادلة المعطاة فيمكن القول بأن الشكل هو ليماسونا بحلقة داخلية ويقع على الموجب x-axis ومتناظر حوله.

هنا يجب تحديد اربع تقاطعات وهي +a و -a و a+b و |a-b|

$a = \boxed{3}$

$a + b = 3 + 5 = \boxed{8}$

$|a - b| = |3 - 5| = |-2| = \boxed{2}$



r	θ
8	0
7.33	$\pi/6$
6.53	$\pi/4$
3	$\pi/2$
0	127
-0.535	$3\pi/4$
-2	π

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⑥ Sketch $r = 2 - 5\sin\theta$

Sol/

Compare it to $r = a + b\sin\theta$

$a = 2$, $b = 5$ ⟶ $a < b$ ⟹ "inner loop"

as it $-\sin\theta$, the graph located on -ve y-axis and symmetric on it.

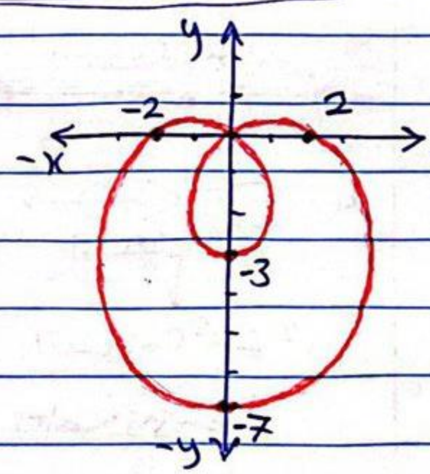
بما أن الدالة جيب (sin) وسالبة فالشكل يكون واقع على y-axis السالب ومتناظر حوله.

لتحديد نقاط التقاطع الاربعة نعمل التالي

$a = \boxed{2}$

$a + b = 2 + 5 = \boxed{7}$

$|a - b| = |2 - 5| = \boxed{3}$



r	θ
2	0
-1.535	$\pi/4$
-3	$\pi/2$
-1.535	$3\pi/4$
2	π
5.535	225°
7	$3\pi/2$

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(7) Graph $r = 3 - 7\cos\theta$

Sol)

compare to $r = a + b\cos\theta \rightarrow$ "limasons"

$a = 3,\ b = 7 \rightarrow a < b \rightarrow$ "Inner loop"

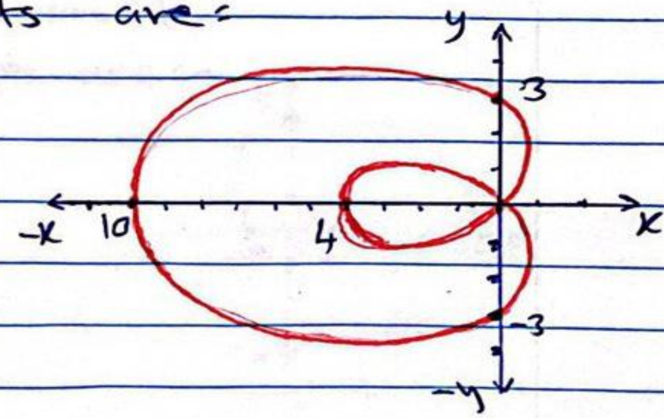
$-\cos\theta \rightarrow$ located on -ve x-axis and symmetric on it

The Four intercepts are:

$a = \boxed{\mp 3}$

$a + b = 3 + 7 = \boxed{10}$

$|a - b| = |3 - 7| = \boxed{4}$



(8) Sketch $r = 3 + 3\cos\theta$

Sol)

compare to $r = a + b\cos\theta \rightarrow$ "limasons"

$a = 3,\ b = 3 \rightarrow a = b \rightarrow$ "cardioid"

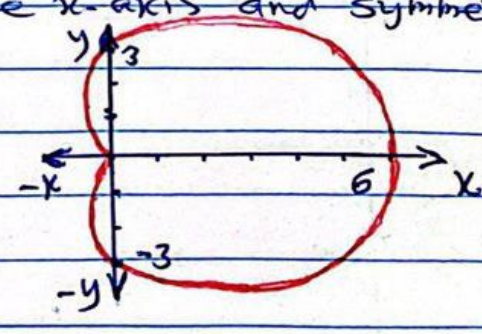
$+\cos\theta \rightarrow$ located on +ve x-axis and symmetric on it.

The intercepts are:

$a = \mp 3$

$a + b = 6$

$a - b = \text{zero}$



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(a) Graph $r = 3 + 2\cos\theta$

Sol

$a = 3$, $b = 2 \Longrightarrow a > b \longrightarrow$ case III

$\frac{3}{2} = 1.5 < 2 \longrightarrow$ "Dimple cardioid"

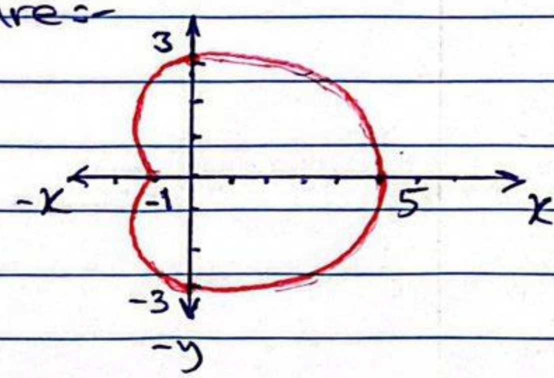
$+\cos\theta \Longrightarrow$ located on the x-axis & symmetric on it.

The four intercepts are:-

$a = \boxed{\mp 3}$

$a+b = 3+2 = \boxed{5}$

$|a-b| = |3-2| = \boxed{1}$



H.W

(1) Graph $r = -2 + 3\sin\theta$

(2) Graph $r = 5 - 2\sin\theta$

(3) sketch $r = 7 + 3\cos\theta$

(5) sketch $r = 4\cos 3\theta$, $r = 4\sin 2\theta$,
$r^2 = 2^2\cos 2\theta$, $r^2 = -3^2\cos 2\theta$

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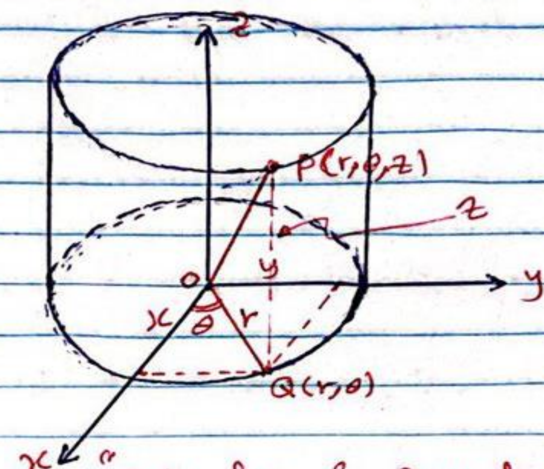
Cylindrical Coordinates الإحداثيات الاسطوانية

Cylindrical coordinates are a simple extension of two-dimensional polar coordinates to three dimensions.

الإحداثيات الاسطوانية هي امتداد بسيط للإحداثيات القطبية ثنائية الابعاد الى ثلاث ابعاد.

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$



"cylindrical coordinates"

In terms of the Cartesian coordinates (x, y, z) similar to the polar coordinates:

	Ranges
$r = \sqrt{x^2+y^2}$,	$r \to [0, \infty)$
$\theta = \tan^{-1}\frac{y}{x}$,	$\theta \to [0, 2\pi)$
$z = z$,	$z \to (-\infty, \infty)$

The inverse tangent "$\tan^{-1}$" must be suitably defined to take the correct quadrant of (x, y) into account.

ملاحظة: عند حساب الزاوية باستخدام معكوس دالة الظل، يجب الأخذ بالاعتبار تصحيح الربع في الاحداثيات x ، y.

Ex) ① convert the point $(1, 1, 3)$ to cylindrical coordinates (r, θ, z)

Sol.)

In this example, we have given a point in (x, y, z) & asked to put it in (r, θ, z), & to do this we need to use the equations we showed in previous page.

$r^2 = x^2 + y^2 \quad , \quad \theta = \tan^{-1}\frac{y}{x} \quad , \quad z = z$

$(1, 1, 3) \longrightarrow x = 1 \;,\; y = 1 \;,\; \boxed{z = 3}$

$\therefore r^2 = 1^2 + 1^2 = 2 \longrightarrow \boxed{r = \sqrt{2}}$

$\theta = \tan^{-1}\frac{1}{1} = \tan^{-1} 1 = \boxed{45^\circ} = \boxed{\frac{\pi}{4}}$

$\therefore (1, 1, 3)$ in cartesian equals to $(\sqrt{2}, \frac{\pi}{4}, 3)$ in cylindrical coordinates

---

Ex) ② convert the point $(4, \pi/4, -2)$ to cartesian coordinates / rectangular coordinates.

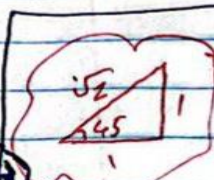
Sol.)

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

$(r, \theta, z) \longrightarrow (x, y, z)$

$(4), (\pi/4), (-2)$

$r = 4$
$\theta = \pi/4$
$z = -2$



$\cos \pi/4 = \frac{1}{\sqrt{2}}$

$\sin \pi/4 = \frac{1}{\sqrt{2}}$

$\therefore x = 4\cos \pi/4 = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$y = 4\sin \pi/4 = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$z = -2$

$\Longrightarrow (2\sqrt{2}, 2\sqrt{2}, -2)$

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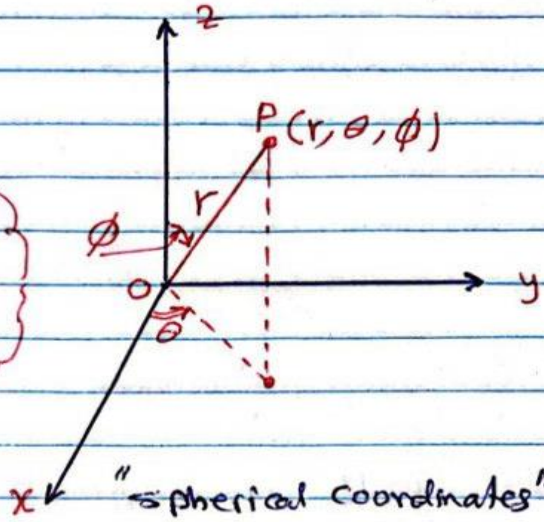
Spherical Coordinates: الاحداثيات الكروية

Spherical coordinates determine the position of a point in 3-dimensional space based on the distance "r" from the origion and two angles θ & ϕ.

$x = r\sin\phi\cos\theta$
$y = r\sin\phi\sin\theta$
$z = r\cos\phi$



"spherical coordinates"

θ is essentially the same as the angle θ from polar coordinates.

	Range
$r = \sqrt{x^2+y^2+z^2}$	$r \rightarrow [0, \infty)$
$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$	$\phi \rightarrow [0, 2\pi)$
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$	$\theta \rightarrow [0, 2\pi)$

Ex) ③ convert the point $(1, \sqrt{3}, 2)$ to spherical coordinates (r, θ, ϕ)

Sol.)

$r = \sqrt{x^2+y^2+z^2}$, $\theta = \tan^{-1}\frac{y}{x}$, $\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$(1, \sqrt{3}, 2) \longrightarrow x=1,\ y=\sqrt{3},\ z=2$

$\therefore r^2 = 1^2+(\sqrt{3})^2+(2)^2 = 8 \Longrightarrow r = \sqrt{8} = \boxed{2\sqrt{2}}$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{\sqrt{3}}{1} = \boxed{\frac{\pi}{3}}$

$\phi = \tan^{-1}\left(\frac{\sqrt{1^2+(\sqrt{3})^2}}{2}\right) = \tan^{-1}\left(\frac{\sqrt{4}}{2}\right) = \tan^{-1}\frac{\cancel{2}}{\cancel{2}} = \boxed{\frac{\pi}{4}}$

$\therefore (1, \sqrt{3}, 2)$ in Cartesian equals to $(2\sqrt{2}, \pi/3, \pi/4)$ in spherical coordinates

---

Ex) ④ Convert the point $(3, \pi/4, \frac{3\pi}{4})$ to Cartesian coordinates.

Sol.)

$x = r \sin\phi \cos\theta$, $(r, \theta, \phi) \to (x, y, z)$

$y = r \sin\phi \sin\theta$, $(3, \pi/4, 3\pi/4)$

$z = r \cos\phi$

$r = 3$

$\theta = \pi/4$

$\phi = \frac{3\pi}{4}$

$\therefore x = 3 \cdot \sin\frac{3\pi}{4} \cdot \cos\pi/4 = 3 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \boxed{\frac{3}{2}}$

$y = 3 \cdot \sin\frac{3\pi}{4} \cdot \sin\pi/4 = 3 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \boxed{\frac{3}{2}}$

$z = 3 \cos\frac{3\pi}{4} = 3 \cdot \frac{-1}{\sqrt{2}} = \boxed{-\frac{3}{\sqrt{2}}}$

$\rightarrow \left(\frac{3}{2}, \frac{3}{2}, \frac{-3}{\sqrt{2}}\right)$

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-- نهاية محاضرة " Polar Coordinates, Polar Functions, Graph of Polar Functions, Polar Coordinates and the Relations with Cartesian, Cylinderical, and Spherical Coordinated, الأحداثيات القطبية، الدوال القطبية، رسم الدوال القطبية، الاحداثيات القطبية وعلاقتها مع الأحداثيات الكارتيزية والاسطوانية والكروية" --

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