Class: 2nd Class
Subject: Mechanics of Materials
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Mechanics of Materials

Lec. 3/ Simple Strain

Simple strain

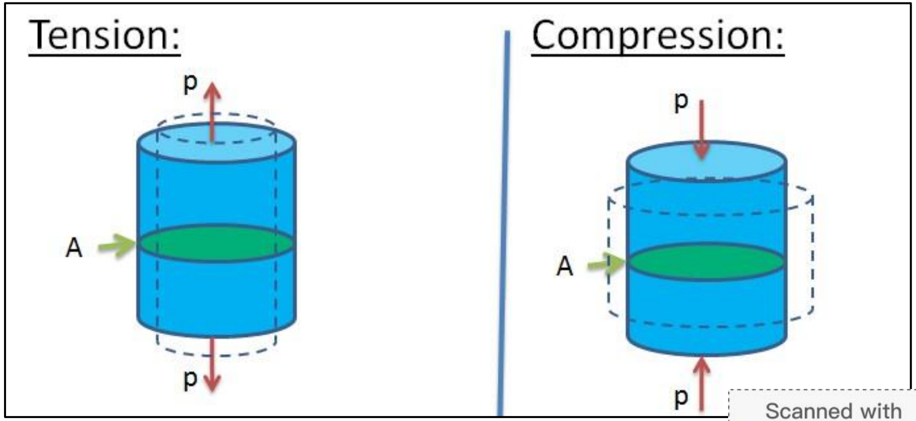
Deformation :
which means dimensional change.

External forces

Internal forces

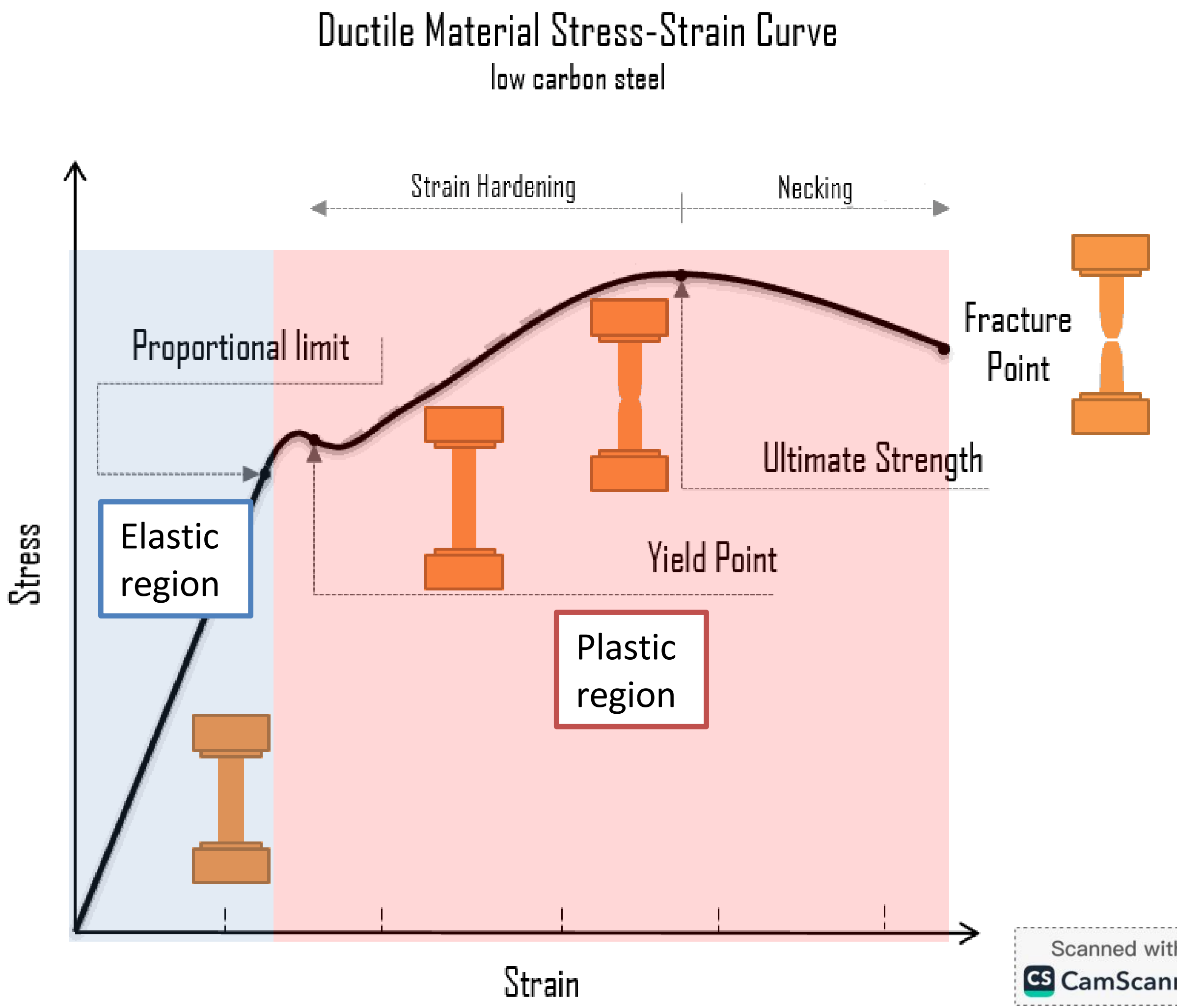
Stress

strain

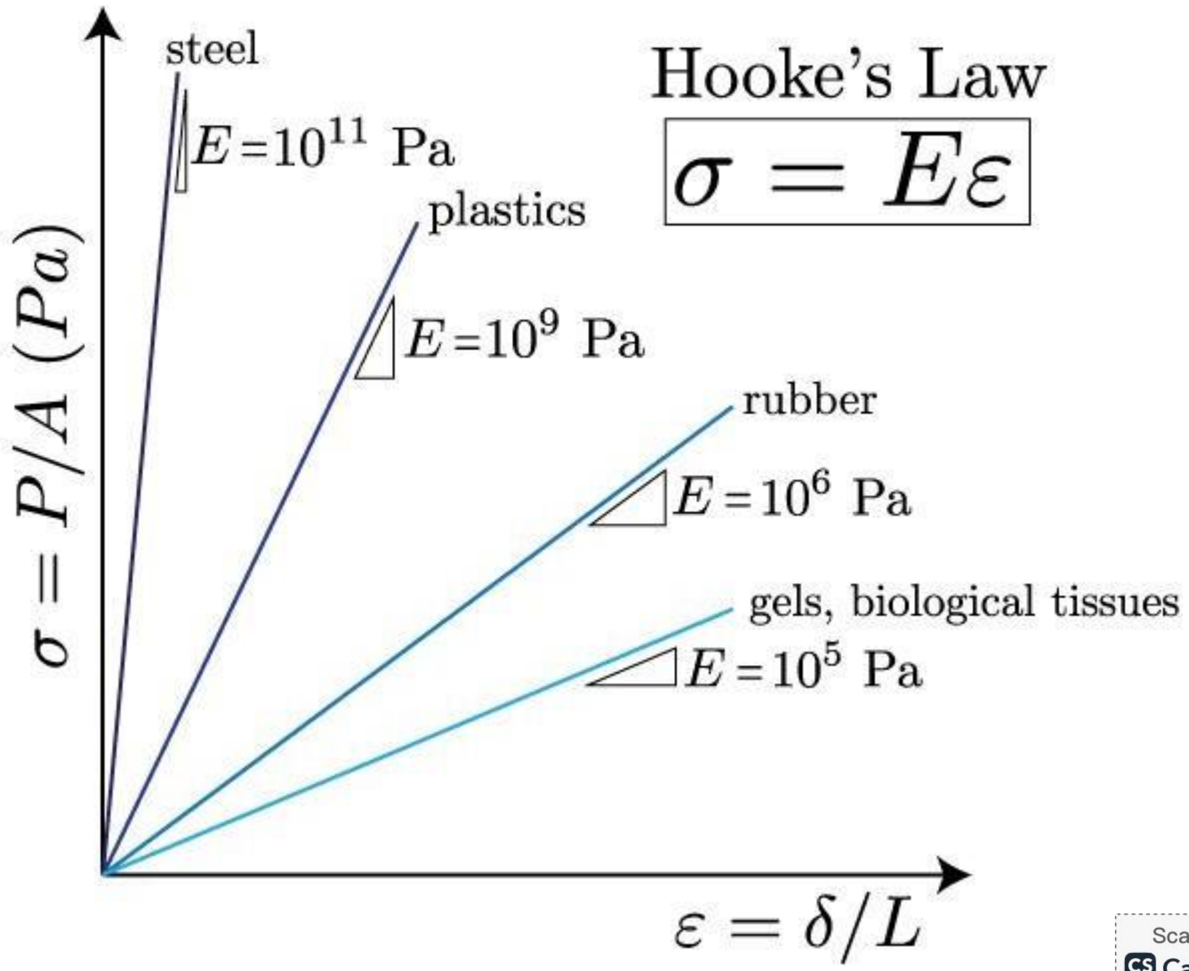


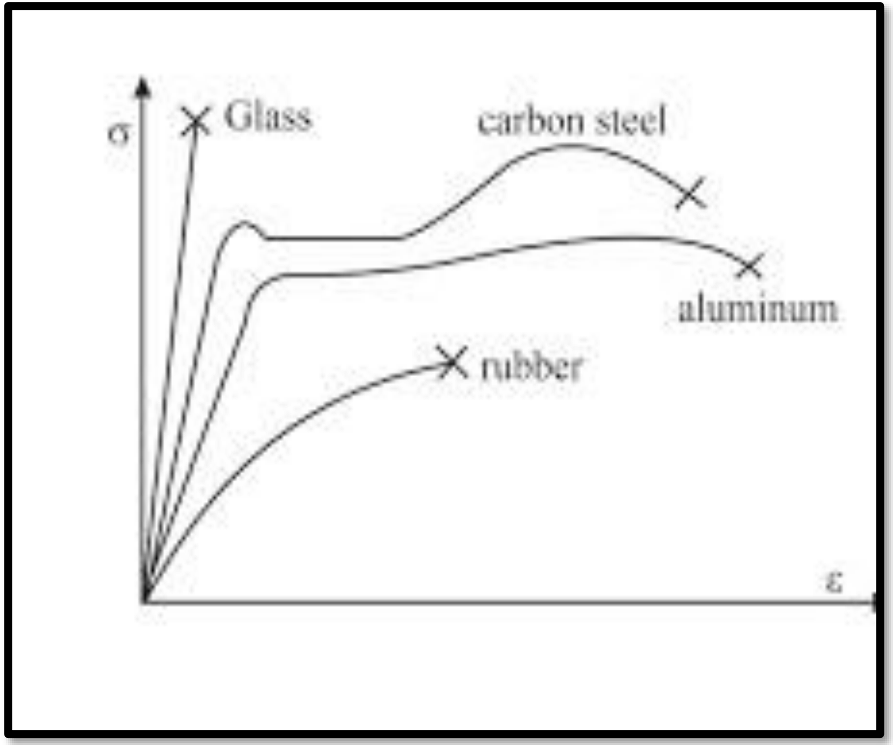
Simple strain

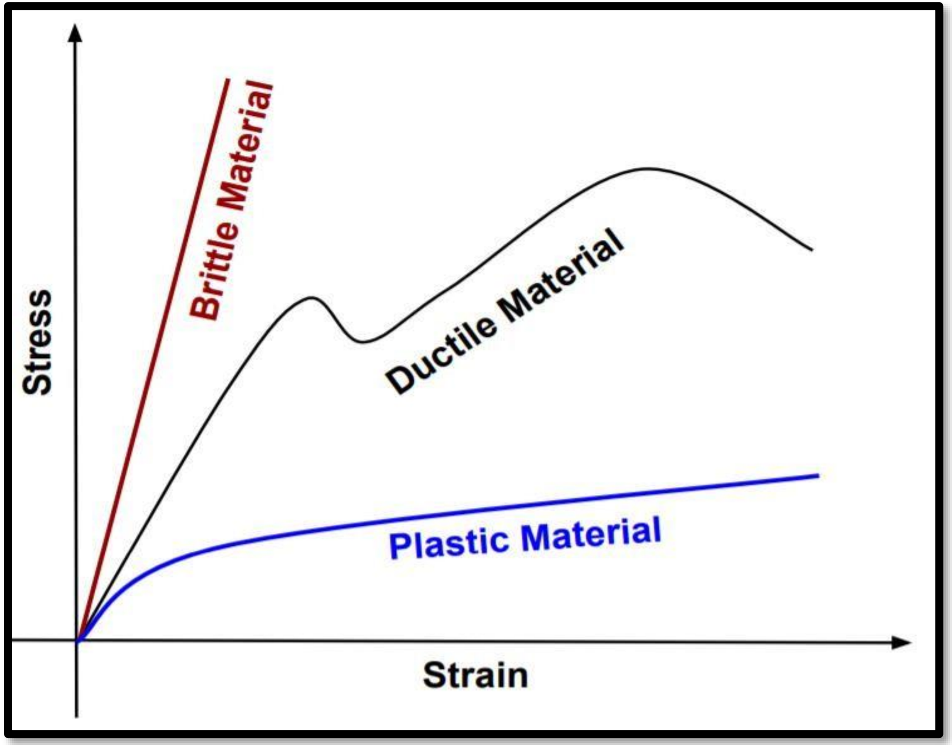
Tensile test



Simple strain

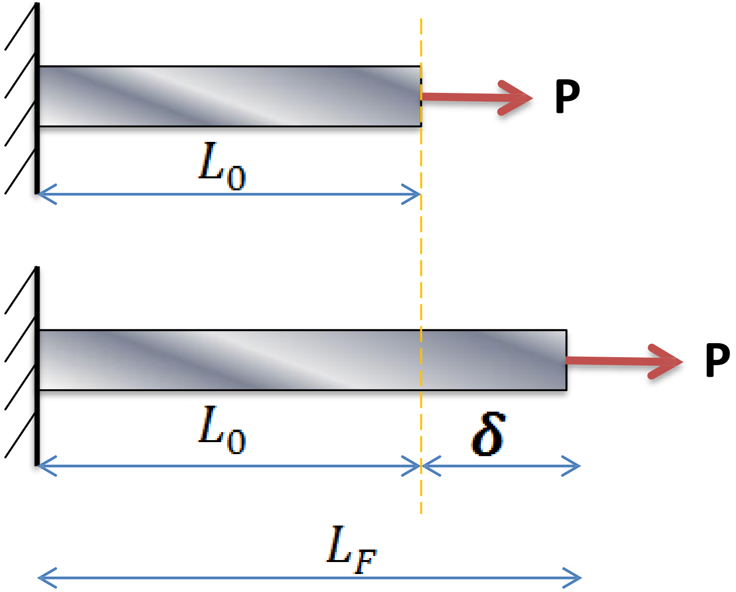



σ
Glass
carbon steel
aluminum
rubber
ε


Brittle Material
Ductile Material
Plastic Material
Stress
Strain

Simple strain

Also known as unit deformation, strain is the ratio of the change in length caused by the applied force, to the original length.



$$\epsilon = \frac{\delta}{L_0} = \frac{L_F - L_0}{L_0}$$

(Unit less)

Hooke's Law

$$\epsilon = \frac{\sigma}{E} = \frac{P/A}{E}$$

Deformation law

$$\frac{\delta}{L_0} = \frac{P/A}{E}$$

$$\delta = \frac{PL}{EA}$$

Unit ;

mm

Simple strain

Deformation (mm)

Load (N)

Original length (mm)

$$\delta = \frac{PL}{EA}$$

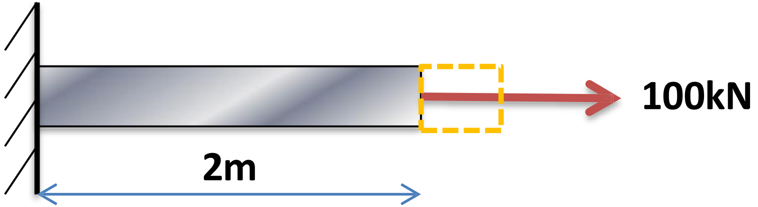
Modulus of elasticity (Gpa)

Cross-sectional area (mm2)

Simple strain

Q/ a steel bar 2m long, with diameter 50mm, its subjected to axial load 100kN and a steel bar having 200Gpa modulus of elasticity, So determine the deformation in this bar?

Sol:



$$\delta = \frac{PL}{AE}$$

$$\delta = \frac{100x2x10^3}{\frac{\pi}{4}(50)^2x200}\left(\frac{kN.mm}{mm^2\left(\frac{kN}{mm^2}\right)}\right)$$

$$\delta = 0.509\ mm$$

P= 100kN
L= 2m
d= 50mm
E= 200Gpa

$\delta = ?$

Simple strain

Q / A steel wire **10m** long, hanging vertically supports a tensile load of **2000 N**. Neglecting the weight of the wire, Determine the required diameter if the stress is not to exceed **140Mpa** and the total elongation is not o exceed **5mm**. Assume **E=200 Gpa**?

Sol:

$$\delta = \frac{PL}{EA} \Rightarrow A = \frac{PL}{E\delta}$$

$$A = \frac{(2000)(10)}{(200x10^{9})(5x10^{-3})} \left(\frac{N.m}{\frac{N}{m^2}.m}\right)$$

$$A = 2x10^{-5} m^2$$

$$A = 20\ mm^2$$

$$A = \frac{\pi}{4} d^2 \Rightarrow d = \sqrt{\frac{4A}{\pi}} = 5.05mm$$

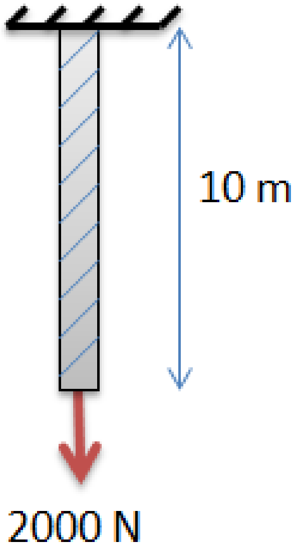
$\delta = 5\ mm$

P= 2000N

L= 10m

E= 200Gpa

d=?



Simple strain

Q/ A steel wire 10m long, hanging vertically supports a tensile load of 2000 N. Neglecting the weight of the wire, Determine the required diameter if the stress is not to exceed 140Mpa and the total elongation is not o exceed 5mm. Assume E=200 Gpa?

Sol:

$$\sigma \leq 140Mpa$$

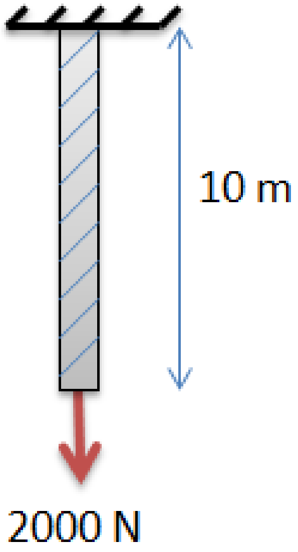
$\delta = 5\ mm$

P= 2000N

L= 10m

E= 200Gpa

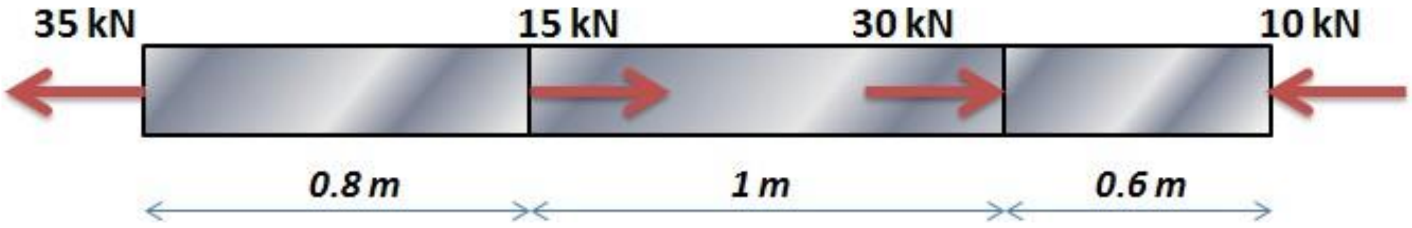
d=?



$$\sigma = \frac{P}{A} = \frac{2000}{\frac{\pi}{4}(5.05)^2} = 99.90Mpa$$

$$99.90Mpa \leq 140Mpa$$

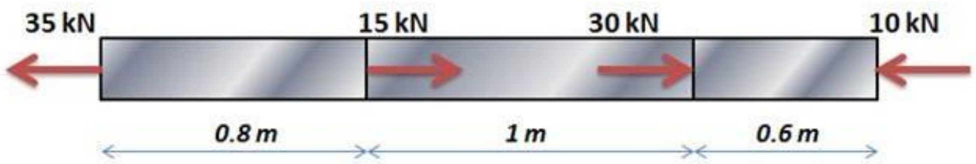
Q / An-aluminim bar having a cross-sectional area of 160mm^2 carries the axial loads at the positions shown in Figure below. If E=70 Gpa, compute the total Deformation of the bar. Assume that the bar is suitably braced to prevent buckling?



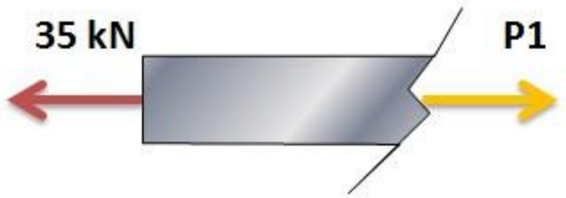
A= 160mm^2
E= 70Gpa
$\delta = ?$

$$\sum F_X = 0$$

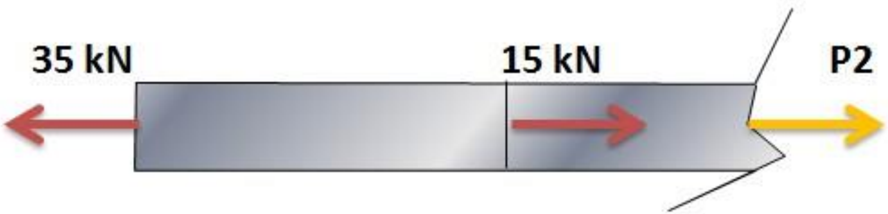
A= 160mm^2
E= 70Gpa
$\delta = ?$



$$P1 - 35 = 0$$
$$P1 = 35\ kN \quad (+Tension)$$

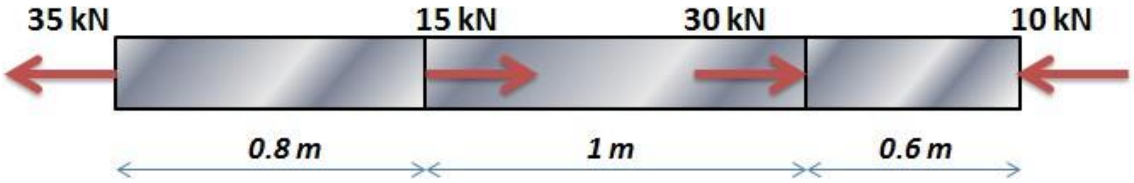


$$P2 - 35 + 15 = 0$$
$$P2 = 20\ kN \quad (+Tension)$$



$$P3 - 10 = 0$$
$$P3 = 10\ kN \quad (-Comprssion)$$

$$P1 = 35\ kN \quad (+Tension)$$
$$P2 = 20\ kN \quad (+Tension)$$
$$P3 = 10\ kN \quad (-Comprssion)$$



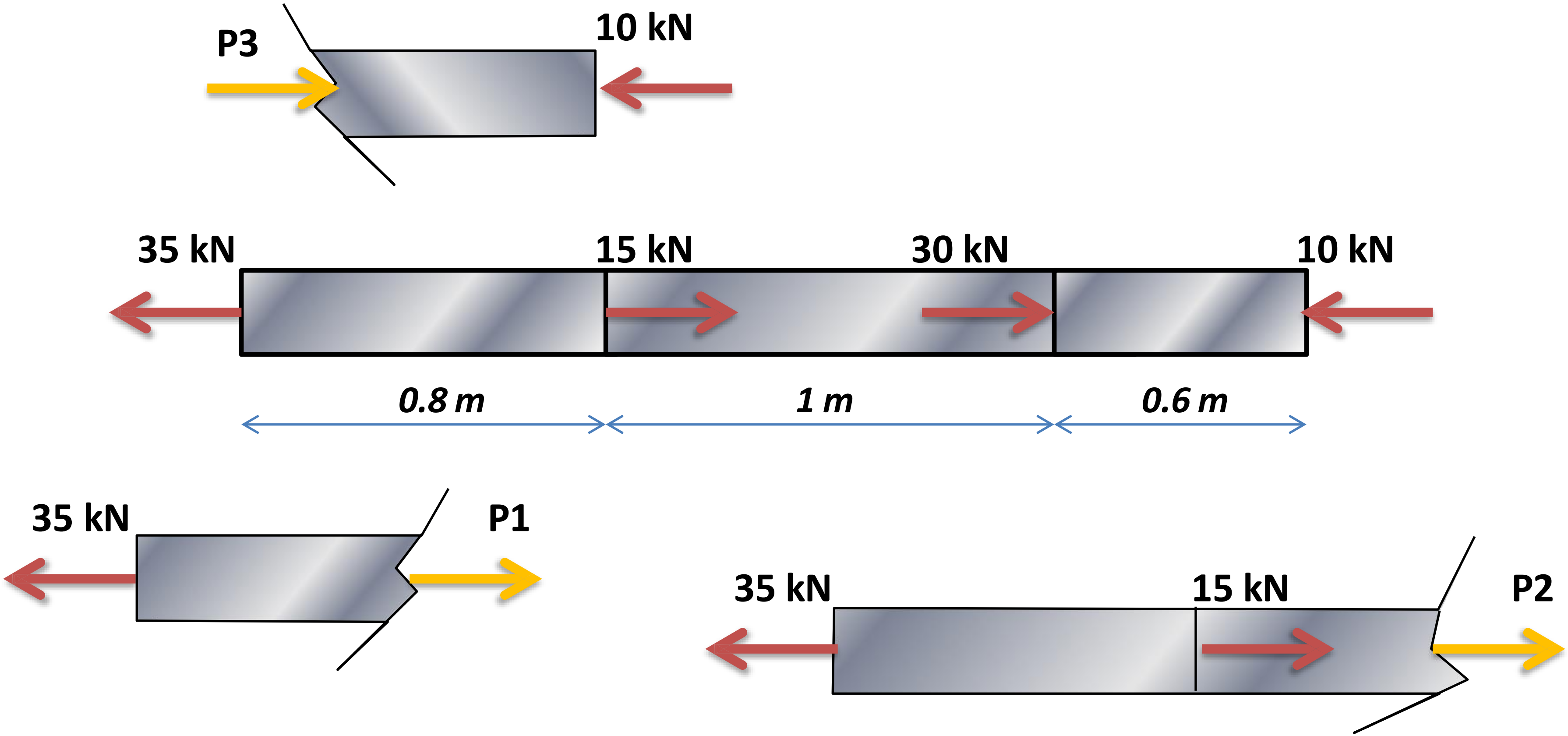
A= 160mm^2
E= 70Gpa
$\delta = ?$

$$\delta = \sum \frac{PL}{AE}$$

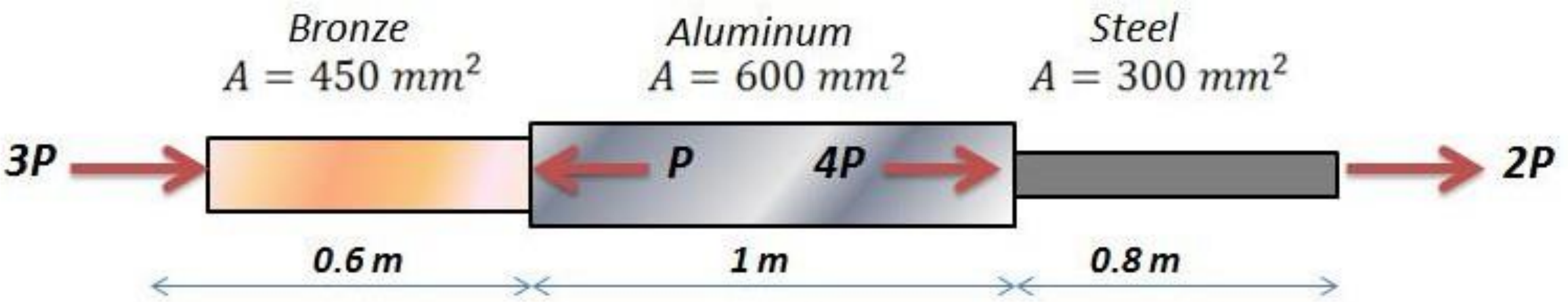
$$\delta = \frac{1}{AE} \sum PL$$

$$\delta = \frac{10^3}{(160x10^{-6})(70x10^9)}[35(0.8) + 20(1) - 10(0.6)]$$

$$\delta = 0.00375\ m$$
$$\delta = 3.75\ mm$$


P3
10 kN
35 kN
15 kN
30 kN
10 kN
0.8 m
1 m
0.6 m
35 kN
P1
35 kN
15 kN
P2

Q / An aluminum tube is fastened between a steel rod and a bronze as shown in Figure below. Axial loads are applied at the positions indicated. Find the value of *P* that will not exceed a maximum overall deformation of 2mm or a stress in the steel 140Mpa, the aluminum of 80Mpa, or in the bronze of 120Mpa?



$\sigma_s \leq 140\ Mpa$	$E_s = 200\ Gpa$
$\sigma_a \leq 80\ Mpa$	$E_a = 70\ Gpa$
$\sigma_b \leq 120\ Mpa$	$E_b = 83\ G$

$$\sum F_X = 0$$

$$P_b - 3P = 0$$

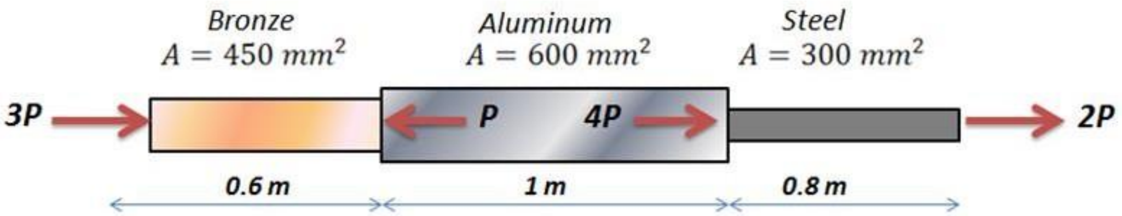
$$P_b = 3P \ \ (-compression)$$

$$P_a + P - 3P = 0$$

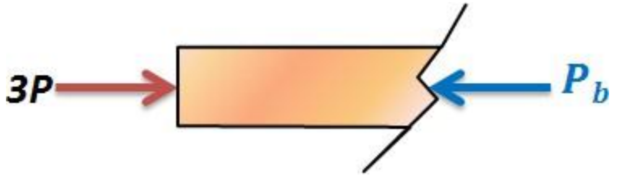
$$P_a = 2P \ \ (-compression)$$

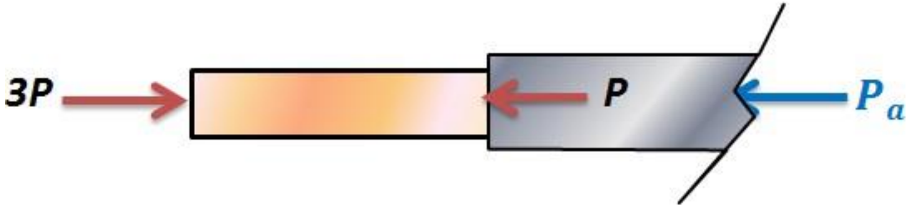
$$P_s - 2P = 0$$

$$P_s = 2P \ \ (+Tension)$$

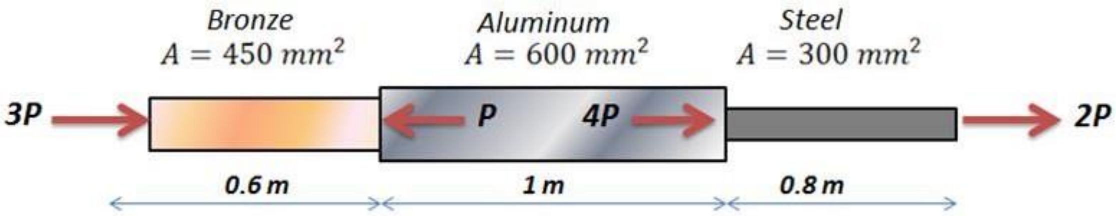


$\sigma_s \le 140\ Mpa$	$E_s = 200\ Gpa$
$\sigma_a \le 80\ Mpa$	$E_a = 70\ Gpa$
$\sigma_b \le 120\ Mpa$	$E_b = 83\ Gpa$





$$P_b = 3P \ (-compression)$$
$$P_a = 2P \ (-compression)$$
$$P_s = 2P \ (+Tension)$$



$\sigma_s \le 140\ Mpa$	$E_s = 200\ Gpa$
$\sigma_a \le 80\ Mpa$	$E_a = 70\ Gpa$
$\sigma_b \le 120\ Mpa$	$E_b = 83\ Gpa$

Based on allowable deformation:

$$\delta = \sum \frac{PL}{AE}$$

$$0.002 = \frac{1}{(10^{-6})(10^{9})}\left[\frac{-3P(0.6)}{450(83)} - \frac{2P(1)}{600(70)} + \frac{2P(0.8)}{300(200)}\right]$$

$$0.002 = \frac{P}{(10^{-6})(10^{9})}\left[\frac{-3(0.6)}{450(83)} - \frac{2(1)}{600(70)} + \frac{2(0.8)}{300(200)}\right]$$

$$P = -28.9\ kN$$
$$\boxed{P = 28.9\ kN\ (\text{Contraction})}$$

Based on allowable stresses:

$P = \sigma A$

$P_b = 3P \ (-compression)$

$P_a = 2P \ (-compression)$

$P_s = 2P \ (+Tension)$

$\sigma_s \leq 140\ Mpa$	$A = 300\ mm^2$
$\sigma_a \leq 80\ Mpa$	$A = 600\ mm^2$
$\sigma_b \leq 120\ Mpa$	$A = 450\ mm^2$

Bronze:

$$3P = (120x10^6)(450x10^{-6}) = 18\ kN$$

Aluminum:

$$2P = (80x10^6)(600x10^{-6}) = 24\ kN$$

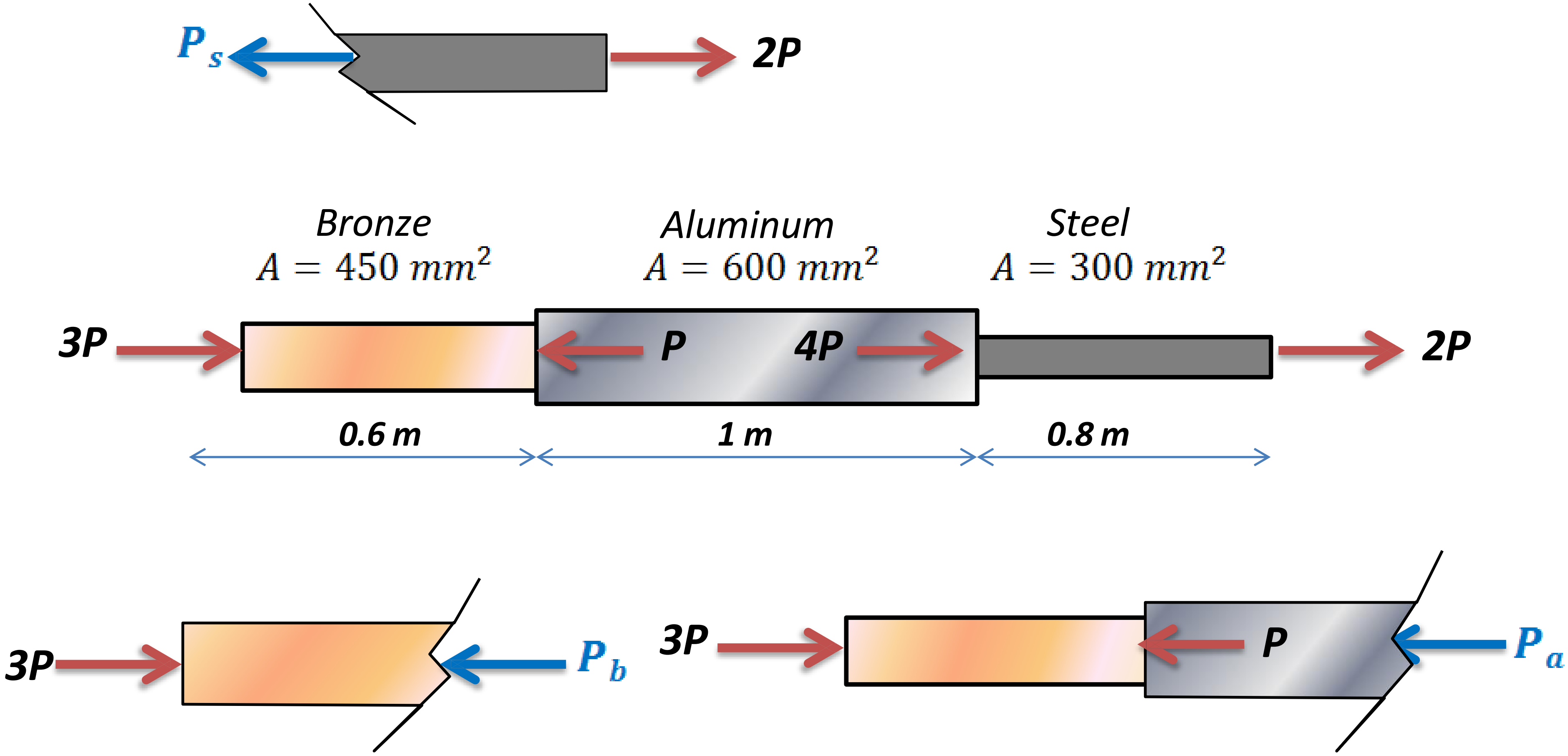
steel:

$$2P = (140x10^6)(300x10^{-6}) = 21\ kN$$

$P = 28.9\ kN$ (Contraction)

use the smallest value of P,

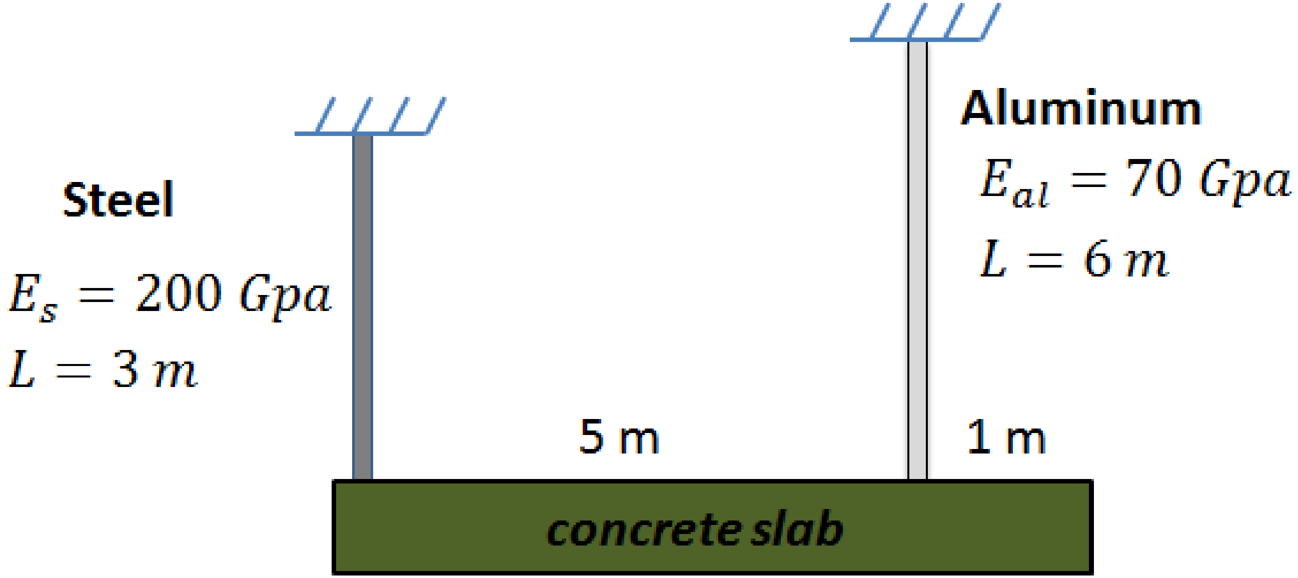
$$P = 18\ kN$$



P_s
2P
Bronze
$A = 450\ mm^2$
Aluminum
$A = 600\ mm^2$
Steel
$A = 300\ mm^2$
3P
P
4P
2P
0.6 m
1 m
0.8 m
3P
P_b
3P
P
P_a

Q/ A uniform concrete slab of mass M is to be attached, as shown in Fig. to two rods whose lower ends are initially at the same level. Determine the ratio of the areas of the rods so that the slab will remain level after it is attached to the rods?

$$\frac{A_{al}}{A_{st}} = ?$$



Static:

$$\sum M_{st} = 0$$

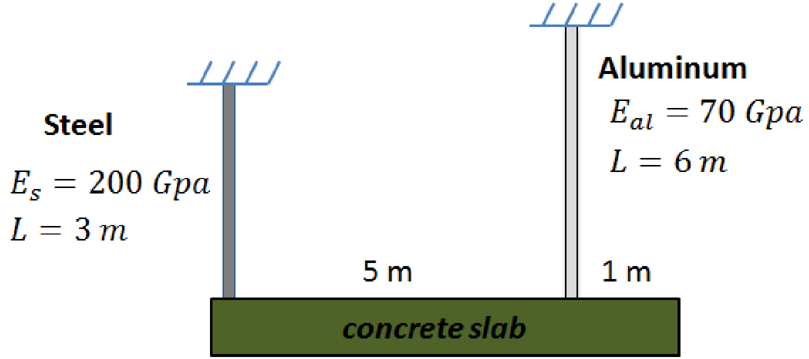
$$W(3) - T_{al}(5) = 0$$

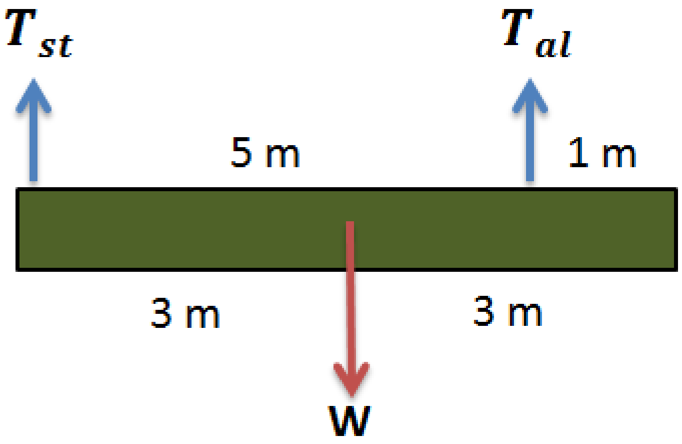
$$T_{al} = 0.6W$$

$$\sum F_y = 0$$

$$0.6W + T_{st} - W = 0$$

$$T_{st} = 0.4W$$





$$T_{al} = 0.6W$$

$$T_{st} = 0.4W$$

Deformation:

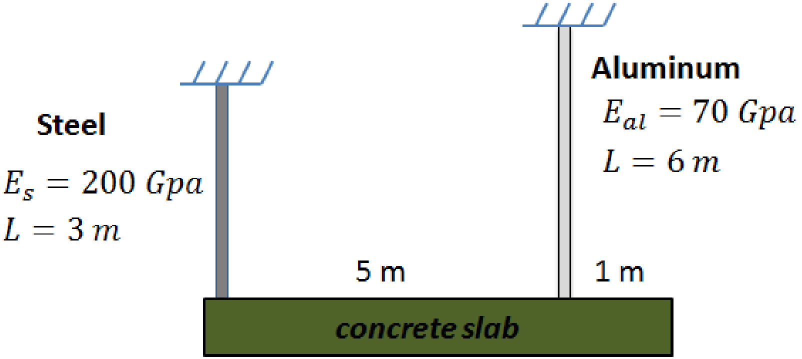
$$\delta_{st} = \delta_{al}$$

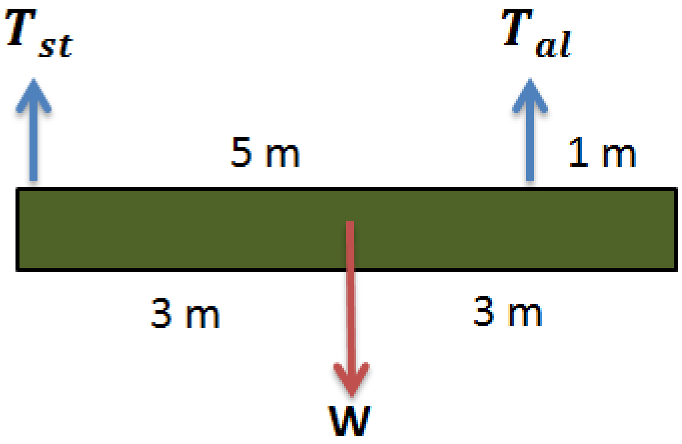
$$\frac{T_{st}\, L}{E\, A_{st}} = \frac{T_{al}\, L}{E\, A_{al}}$$

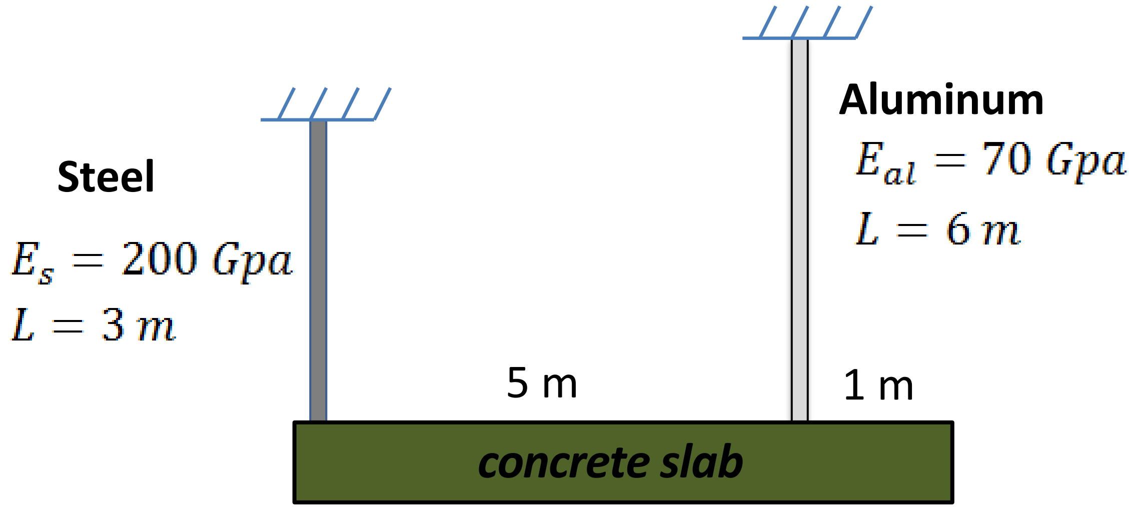
$$\frac{0.4W(3)}{200\,(A_{st})} = \frac{0.6W(6)}{70\,(A_{al})}$$

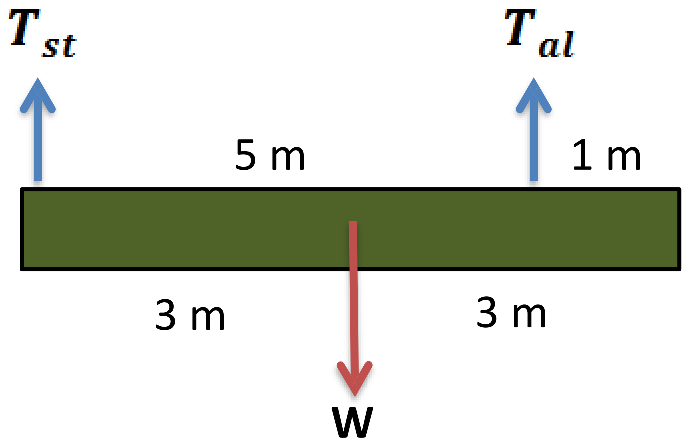
$$84(A_{al}) = 720\,(A_{st})$$

$$\boxed{\frac{A_{al}}{A_{st}} = 8.57}$$






Steel
$E_s = 200\ Gpa$
$L = 3\ m$
Aluminum
$E_{al} = 70\ Gpa$
$L = 6\ m$
5 m
1 m
concrete slab


T_{st}
T_{al}
5 m
1 m
3 m
3 m
W

Q /

Rigid beam AB rests on the two short posts shown in Figure.. AC is made of steel and has a diameter of 20 mm, and BD is made of aluminum and has a diameter of 40 mm. Determine the displacement of point F on AB if a vertical load of 90 kN is applied over this point. Take E_{st} = 200 GPa, E_{al} = 70 GPa.