Discrete Mathematics

Lecture 4

Predicates and Quantifiers
Part 2

By

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Translate into English – Example1:

Express the statement “Every student in this class has studied calculus.

Solution $P(x)$: x has studied calculus.

$S(x)$: x is in this class.

The statement can be expressed as $\boxed{\forall x(S(x) \rightarrow p(x))}$

Translate into English – Example2:

Translate the statement $\forall x(C(x) \lor \exists y(C(y) \land F(x,y)))$ into English, where $C(x)$ is "x has a computer", $F(x,y)$ is "x and y are friends," and both x and y is the set of all students in your school.

Solution

Every student in your school has a computer or has a friend who has a computer.

Translate into English – Example3:

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \to F(x))$

Answer

a) Every comedian is funny.

Translate into English – Example3:

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

b) $\forall x(C(x) \wedge F(x))$

Answer

b) Every person is a funny comedian.

Translate into English – Example3:

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

c) $\exists x(C(x) \rightarrow F(x))$

Answer

c) There exists a person such that if she or he is a comedian, then she or he is funny.

Translate into English – Example3:

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

d) $\exists x(C(x) \wedge F(x))$

Answer

d) Some comedians are funny.

Translate into Logical Expression – Example1:

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

Let $P(x)$ be "x is perfect" let $F(x)$ be "x is your friend"

the domain be all people

a) No one is perfect.

Answer

a) $\forall x \neg P(x)$

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Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

Let $P(x)$ be "x is perfect" let $F(x)$ be "x is your friend"

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b) Not everyone is perfect.

Answer

b) $\neg\forall x\, P(x)$

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c) All your friends are perfect.

Answer

c) $\forall x(F(x) \rightarrow P(x))$

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Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

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d) At least one of your friends is perfect.

Answer

d) $\exists x(F(x) \land P(x))$

Precedence of Quantifiers:

The quantifiers $\forall$ and $\exists$ have higher precedence then all logical operators from propositional calculus.

For example, $\forall x P(x) \vee Q(x)$ is the disjunction of $\forall x P(x)$ and $Q(x)$.

In other words,

it means $(\forall x P(x)) \vee Q(x)$ rather than $\forall x(P(x) \vee Q(x))$.

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

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$\exists x P(x)$ **:**

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"Every student in this class has not taken calculus"

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$$x\ (x^2 \leq x)$$

Example2:

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What are the negations of the statements

$$\exists x(x^2 = 2)$$

$$\neg\exists x(x^2 = 2) \equiv \forall x\neg(x^2 = 2)$$

$$x\ (x^2 \neq 2)$$

Think you

Any questions ??

Quiz

Q1/ Determine the truth value of each of these statements if the domain consists of all real numbers.

a) $\exists x(x^3 = -1)$ **b)** $\exists x(x^4 < x^2)$

c) $\forall x((-x)^2 = x^2)$ **d)** $\forall x(2x > x)$

Q2/ Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology.