

Strength of Materials
Chapter 02
Forces and Moments

Introduction

- The transverse loading of a beam may consist of *concentrated loads* P_1, P_2, \dots , expressed in newton, pounds, or their multiples, kilo newton and kips (Figure 2.1a), of a *distributed load* w , expressed in N/m, kN/m, lb/ft, or kips/ft (Figure 2.1b), or of a combination of both. When the load w per unit length has a constant value over part of the beam (as between A and B in Figure 2.1b), the load is said to be *uniformly distributed* over that part of the beam.

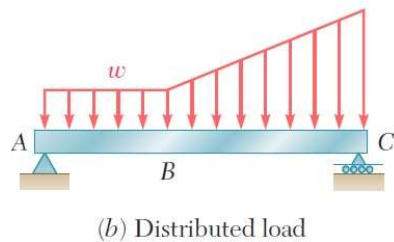
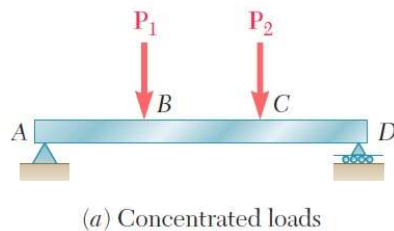
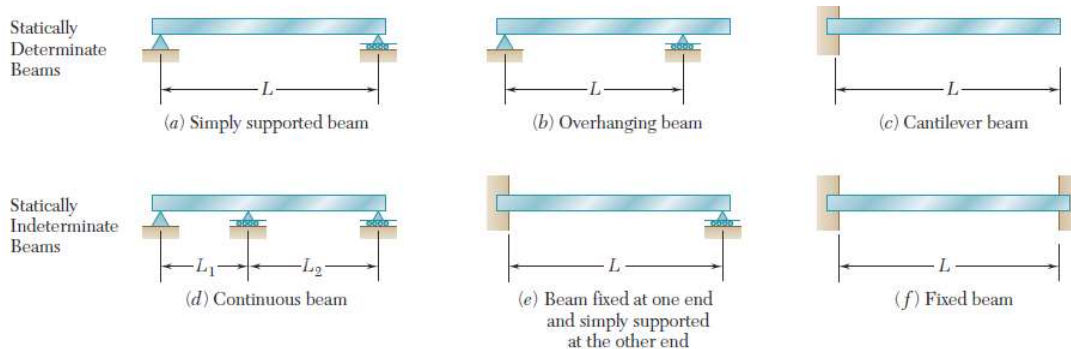


Figure 2.1: Transversely Loaded Beams

Classification of Beams

- Beams are classified according to the way in which they are supported. Several types of beams frequently used are shown in Figure 2.2. The distance L shown in the various parts of the figure is called the span.



- Sometimes two or more beams are connected by hinges to form a single continuous structure. Two examples of beams hinged at a point H are shown in Figure 2.3.

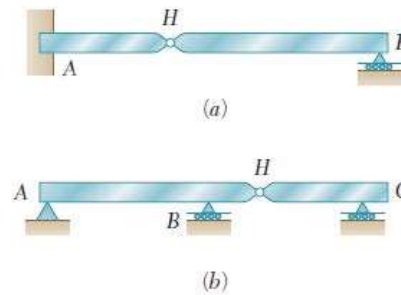


Figure 2.3: Beams Connected by Hinges

- When a beam is subjected to transverse loads, the internal forces in any section of the beam will generally consist of a shear force V and a bending couple M .
- Consider, for example, a simply supported beam AB carrying two concentrated loads and a uniformly distributed load (Figure 2.4a).
- To determine the internal forces in a section through point C we first draw the free-body diagram of the entire beam to obtain the reactions at the supports (Figure 2.4b).
- Passing a section through C , we then draw the free-body diagram of AC (Figure 2.4c), from which we determine the shear force V and the bending couple M .

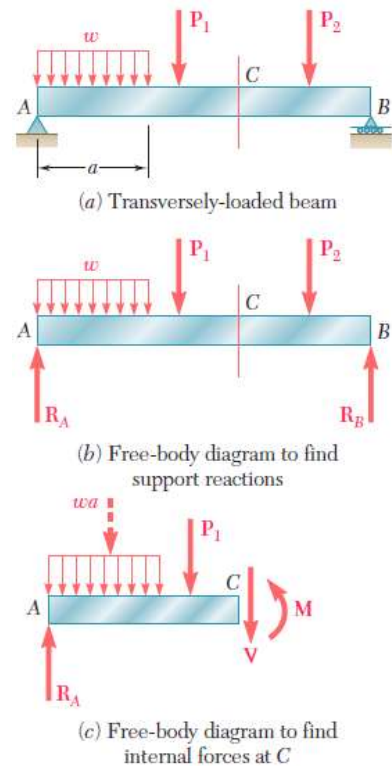


Figure 2.4

Sign Conventions

- The shear V and the bending moment M at a given point of a beam are said to be positive when the internal forces and couples acting on each portion of the beam are directed as shown in Figure 2.5.

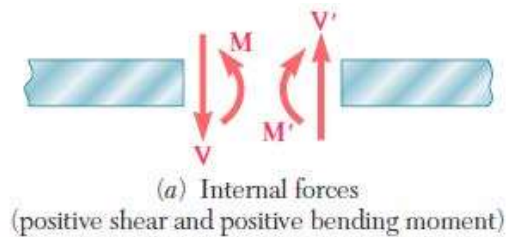


Figure 2.5

These conventions can be more easily remembered if we note that:

- The shear at any given point of a beam is positive when the external forces (loads and reactions) acting on the beam tend to shear off the beam at that point as indicated in Figure 2.6.

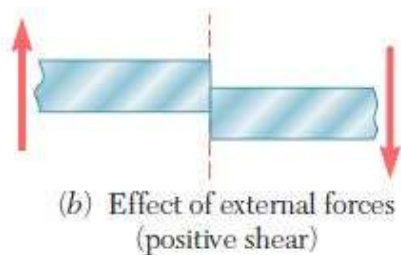


Figure 2.6

- The bending moment at any given point of a beam is positive when the external forces acting on the beam tend to bend the beam at that point as indicated in Figure 2.7.

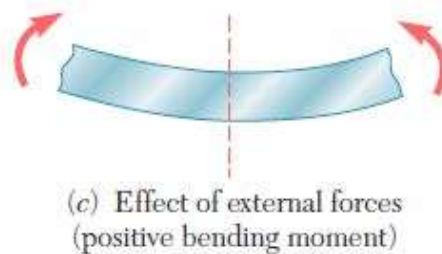


Figure 2.7

Basic Relationship between the Rate of Loading, Shear Force and Bending Moment

- A simple beam with a varying load indicated by $w(x)$ is sketched in Figure 2.8. The coordinate system with origin at the left end A is established and distances to various sections in the beam are denoted by the variable x .

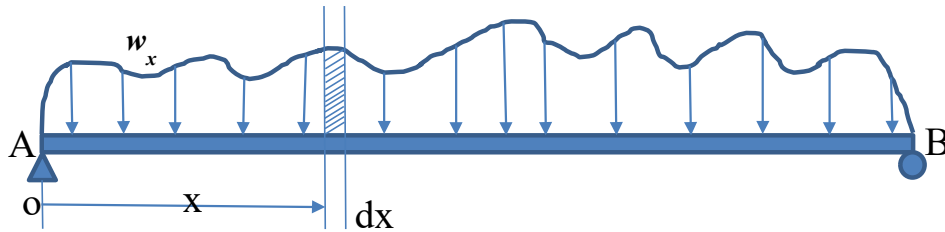


Figure 2.8

- Basic Relationship Between the Rate of Loading, Shear Force and Bending Moment
- For any value of x , the relationship between the load $w(x)$, the shearing force V and bending moment M is:

$$w = -\frac{dV}{dx} = -\frac{d^2M}{dx^2} \quad (1)$$

- And for the relationship between shearing force V and bending moment M is:

$$V = \frac{dM}{dx} \quad (2)$$

- Conclusions: From the above relations, the following important conclusions may be drawn:
- From Equ. (2), the area of the shear force diagram between any two points, from the basic calculus is the bending moment diagram.

$$M = \int V dx$$

- The slope of bending moment diagram is the shear force, thus:

$$V = \frac{dM}{dx}$$

► Thus, if $V=0$, the slope of the bending moment diagram is zero and the bending moment is therefore constant.

□ The maximum or minimum bending moment occurs where

$$\frac{dM}{dx} = 0$$

□ The slope of the shear force diagram is equal to the magnitude of the intensity of the distributed loading at any position along the beam. The (-ve) sign is as a consequence of our particular choice of sign conventions.

Examples

There are two methods to solve problems of shear and bending diagrams:

- 1) Method of Equations, and
- 2) Method of areas

Example (1): Draw the shear and bending moment diagrams for the beam shown in Figure (2.9).

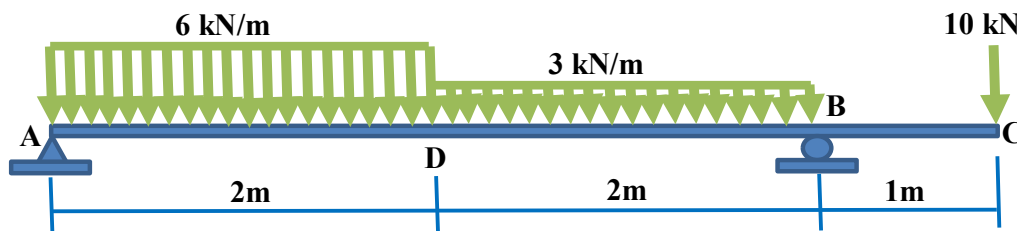


Figure (2.9)

Solution: Method of Equations

► First, find all reactions of the beam

$$\sum M_A = 0 \quad \rightarrow 6(2)(1) + 3(2)(3) + 10(5) - 4(R_B) = 0$$

$$\rightarrow R_B = 20 \text{ kN } \uparrow$$

$$\sum F_y = 0 \quad \rightarrow 6(2) + 3(2) + 10 - R_A - R_B = 0$$

$$\rightarrow R_A = 8 \text{ kN } \uparrow$$

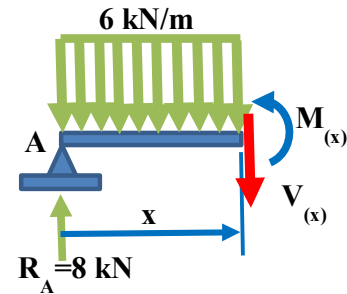
$$\sum F_x = 0 \quad \rightarrow A_x = 0$$

- Now divide the beam into three regions according to the change in loading. These regions are AD, DB, and BC.

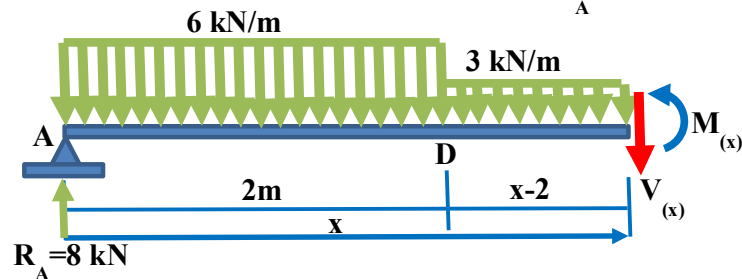
- Part AD: ($0 \leq x \leq 2$)

$$+\uparrow \sum F_y = 0 \rightarrow 8 - 6x = V_{(x)}$$

$$\sum M_{sec.} = 0 \rightarrow 8x - 6 \frac{x^2}{2} = M_{(x)}$$



- Part DB: ($2 \leq x \leq 4$)



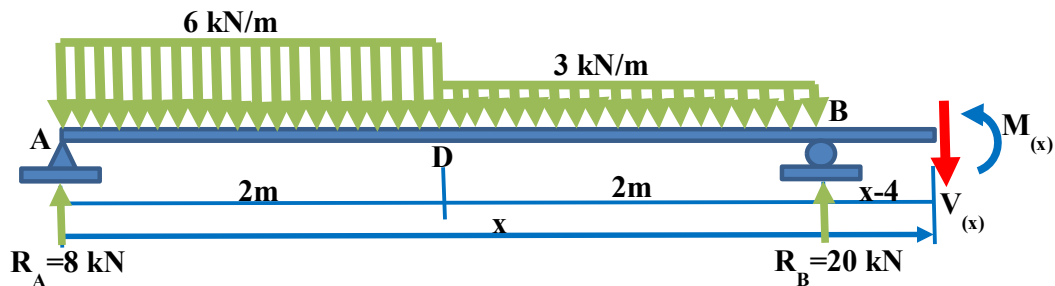
$$+\uparrow \sum F_y = 0 \rightarrow 8 - 6(2) - 3(x - 2) = V_{(x)}$$

$$\rightarrow V_{(x)} = 2 - 3x$$

$$\sum M_{sec.} = 0 \rightarrow 8x - 6(2)(x - 1) - 3 \frac{(x - 2)^2}{2} = M_{(x)}$$

$$\rightarrow M_{(x)} = 12 - 4x - 1.5(x - 2)^2$$

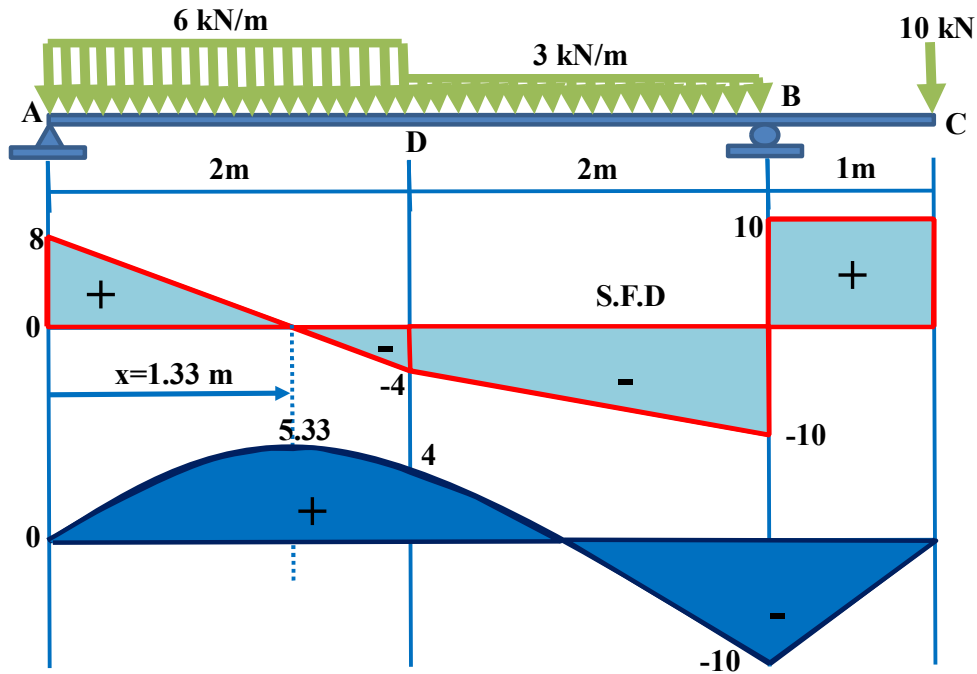
- Part BC: ($4 \leq x \leq 5$)



$$+\uparrow \sum F_y = 0 \rightarrow 8 - 6(2) - 3(2) = V_{(x)}$$

$$\rightarrow V_{(x)} = 10$$

$$\begin{aligned}\sum M_{sec.} = 0 &\rightarrow 8x - 6(2)(x - 1) - 3(2)(x - 3) + 20(x - 4) = M_{(x)} \\ &\rightarrow M_{(x)} = 10x - 50\end{aligned}$$



Notes:

- ☐ If load is to the n degree, then:
 - ☐ The shear is to the degree (n+1), and
 - ☐ The moment is to the degree (n+2)
- For this example,
- The load is to the zero degree (n=0),
 - The shear is to the 1st degree (n=1), and
 - The moment is to the 2nd degree (n=2)

Increasing and decreasing in slopes

Figure (2.10) shows how you can draw the increasing curve and the decreasing curve.

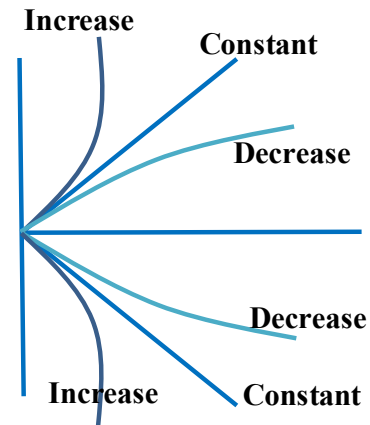


Figure (2.10)

Example (2): Draw the shear force and bending moment diagrams for the beam shown in Figure (2.11).

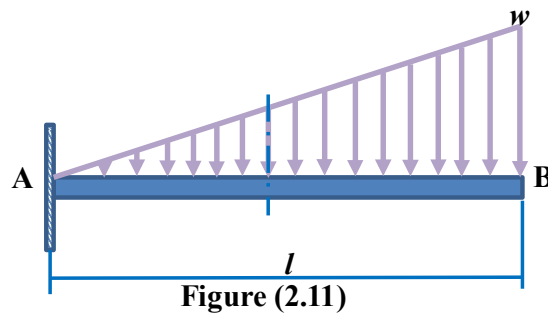


Figure (2.11)

Solution:

$$\sum M_A = 0 \rightarrow \frac{wl}{2} \left(\frac{2}{3}l \right) = M_A$$

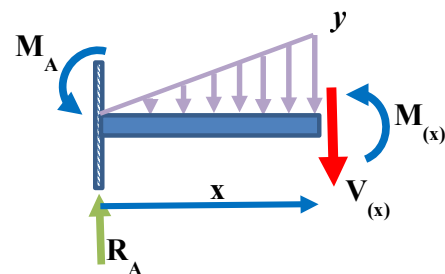
$$\rightarrow M_A = \frac{wl^2}{3}$$

$$+\uparrow \sum F_y = 0 \rightarrow R_A = \frac{wl}{2}$$

Now, for section:

$$\frac{y}{x} = \frac{w}{l} \rightarrow y = \frac{x}{l}w$$

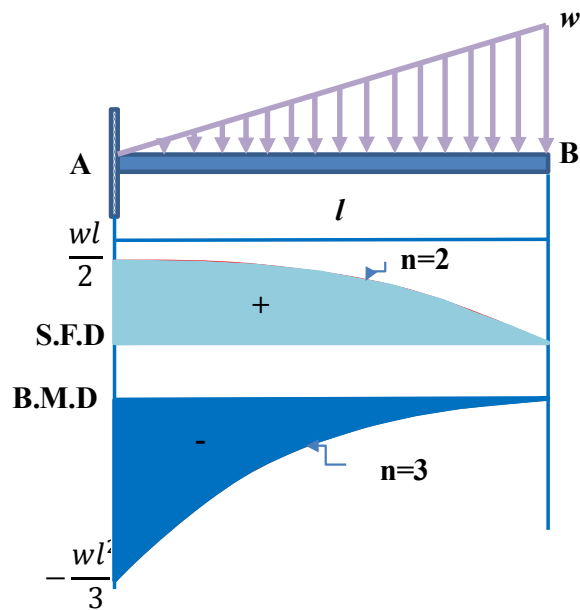
$$+\uparrow \sum F_y = 0 \rightarrow V_{(x)} = \frac{wl}{2} - \frac{1}{2}xy$$



$$V_{(x)} = \frac{wl}{2} - \frac{1}{2}x\left(\frac{x}{l}w\right) = \frac{wl}{2} - \frac{1}{2}\frac{x^2}{l}w$$

$$\sum M_{sec} = 0 \rightarrow M_{(x)} = \frac{wl}{2}(x) - \frac{wl^2}{3} - \frac{1}{2}(xy)\left(\frac{x}{3}\right)$$

$$\rightarrow M_{(x)} = \frac{wl}{2}x - \frac{wl^2}{3} - \frac{wx^2}{6l}$$



2- Method of Areas

$$V_B = V_A + \text{area of load between A and B}$$

$$M_B = M_A + \text{area of shear between A and B}$$

Example (3): Resolve example (1) by method of areas.

Solution:

$$V_B = 8 + (-6 \times 2) = -4 \text{ kN}$$

$$V_C = -4 + (-3 \times 2) = -10 \text{ kN}$$

$$V_D = -10 + 20 + 0 = 10 \text{ kN}$$

$$\frac{8+4}{2} = \frac{8}{x} \rightarrow x = 1.33 \text{ m}$$

$$M_E = M_A + \frac{1}{2}(8)(1.33) = 5.33 \text{ kN.m}$$

$$M_B = 5.33 - \frac{1}{2}(4)(2 - 1.33) = 4 \text{ kN.m}$$

$$M_C = 4 - \left(\frac{4+10}{2}\right)(2) = -10 \text{ kN.m}$$

$$M_D = -10 + 10 \times 1 = 0$$

Example (4): Draw the shear force and bending moment diagrams for the beam shown in Figure (2.12).

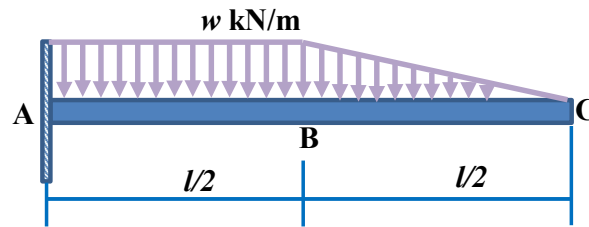


Figure (2.12)

Solution:

$$+\uparrow \sum F_y = 0 \rightarrow R_A = \frac{3wl}{4}$$

$$\sum M_A = 0 \rightarrow M_A = w \frac{l}{2} \left(\frac{l}{4}\right) + \frac{1}{2}(w) \left(\frac{l}{2}\right) \left(\frac{l}{2} + \frac{1}{3} \frac{l}{2}\right) = \frac{7}{24} wl^2$$

$$V_B = \frac{3wl}{4} - \frac{wl}{2} = \frac{wl}{4}$$

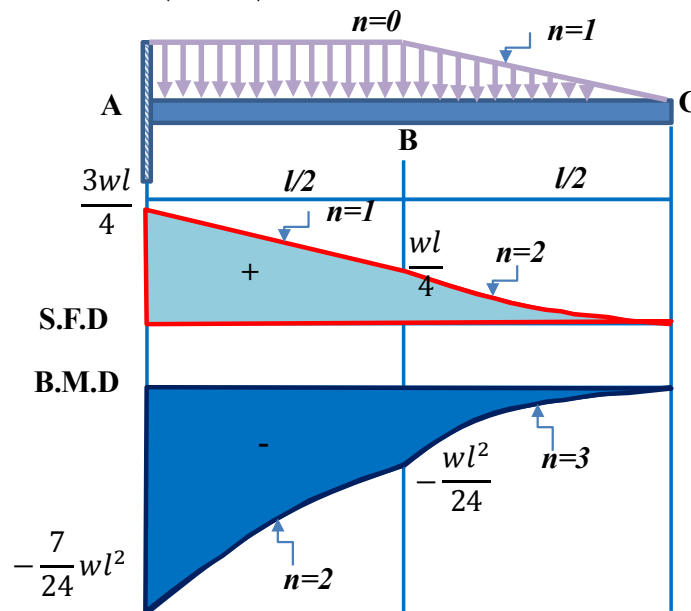
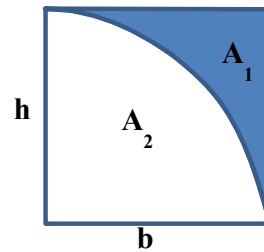
$$V_C = \frac{wl}{4} - \frac{1}{2}(w) \left(\frac{l}{2}\right) = 0$$

$$M_B = -\frac{7}{24} wl^2 + \left(\frac{\frac{3wl}{4} + \frac{wl}{4}}{2}\right) \frac{l}{2} = -\frac{7}{24} wl^2 + \frac{wl^2}{4} = -\frac{wl^2}{24}$$

$$A_1 = \frac{bh}{n+1}$$

$$A_2 = \frac{nbh}{n+1}$$

$$M_c = -\frac{wl^2}{24} + \left(\frac{\frac{l}{2} \times \frac{wl}{4}}{2+1} \right) = 0$$



Example (5): Draw the shear force and bending moment diagrams for the beam shown in Figure (2.13).

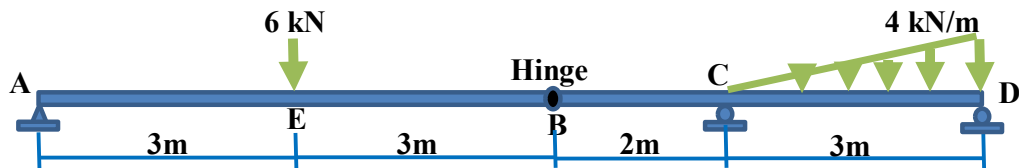
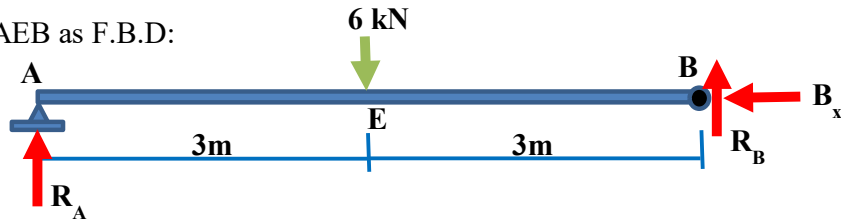


Figure (2.13)

Solution:

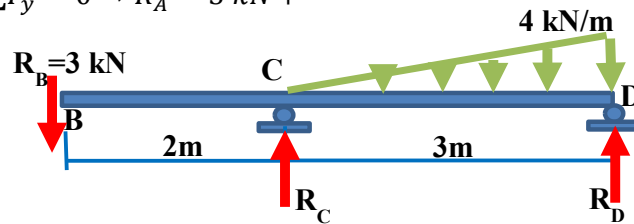
► Part AEB as F.B.D:



$$\sum M_A = 0 \rightarrow R_B = 3 \text{ kN} \uparrow$$

$$\sum F_y = 0 \rightarrow R_A = 3 \text{ kN} \uparrow$$

► Part BCD as F.B.D:



$$\sum M_A = 0 \rightarrow R_C = 7 \text{ kN}$$

$$\sum F_y = 0 \rightarrow R_D = 2 \text{ kN} \uparrow$$

$$V_E = 3 + 0 = 3 \text{ kN}$$

$$V_B = -3 + 0 = -3 \text{ kN}$$

$$V_C = -3 + 0 = -3 \text{ kN}$$

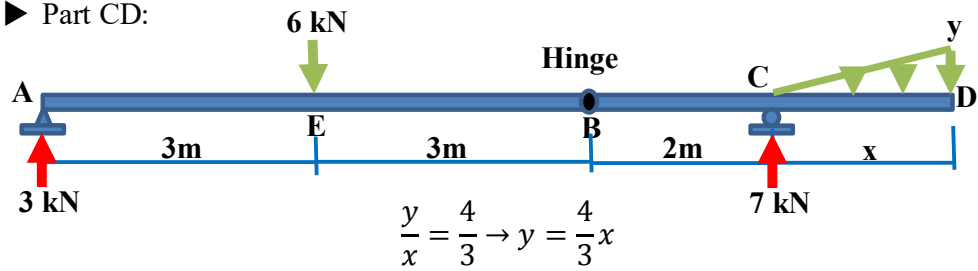
$$V_D = 4 - \frac{1}{2}(4)(3) = -2 \text{ kN}$$

$$M_E = 0 + 3(3) = 9 \text{ kN.m}$$

$$M_B = 9 - 3(3) = 0$$

$$M_C = 0 - 3(2) = -6 \text{ kN.m}$$

► Part CD:

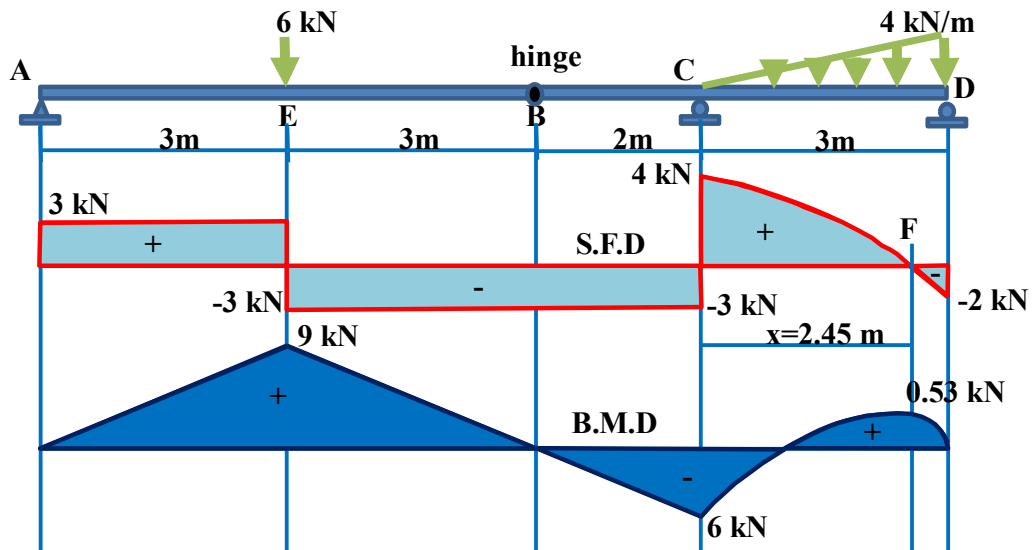


$$\frac{y}{x} = \frac{4}{3} \rightarrow y = \frac{4}{3}x$$

$$3 - 6 + 7 - \frac{1}{2}(xy)$$

$$V_{(x)} = 4 - \frac{2}{3}x^2 = 0 \rightarrow x = 2.45 \text{ m}$$

$$M_F = -6 + \frac{4(2.45)(2)}{2 + 1} = 0.53 \text{ kN.m}$$



Example (6): Find loading of the beam if B.M.D is as shown in Figure (2.14) simply support.

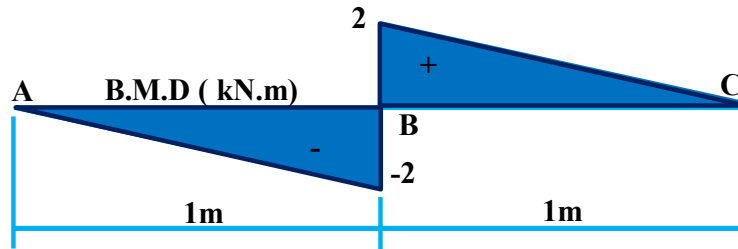


Figure (2.14)

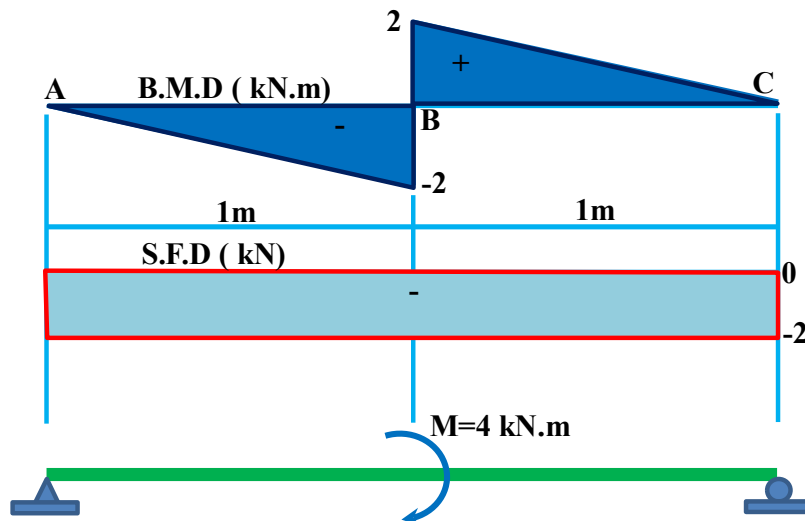
Solution:

$$M_B = M_A + \text{area of she } AB$$

$$-2 = 0 + V(1) \rightarrow V = -2 \text{ kN}$$

$$M_{B,\text{new}} = M_{B,\text{ol}} + M$$

$$2 = -2 + M \rightarrow M = 4$$



Example (7): A beam is supported at left end only and have S.F.D as shown in Figure (2.15). Draw loading and B.M.D for this beam.

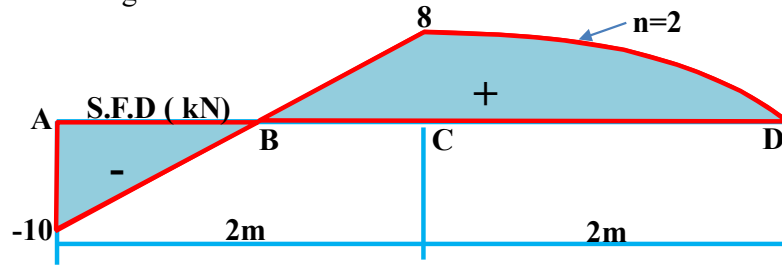


Figure (2.15)

Solution:

$$V_C = V_A + \text{area of shear AC}$$

$$8 = -10 + w(2) \rightarrow w = 9 \text{ kN/m}$$

$$V_D = V_C + \text{area of shear CD}$$

$$0 = 8 + \frac{1}{2}(w_1)(2)$$

$$\rightarrow w_1 = -8 \text{ kN/m}$$

$$\sum M_A = 0 \rightarrow M_A = 8.67 \text{ kN.m}$$

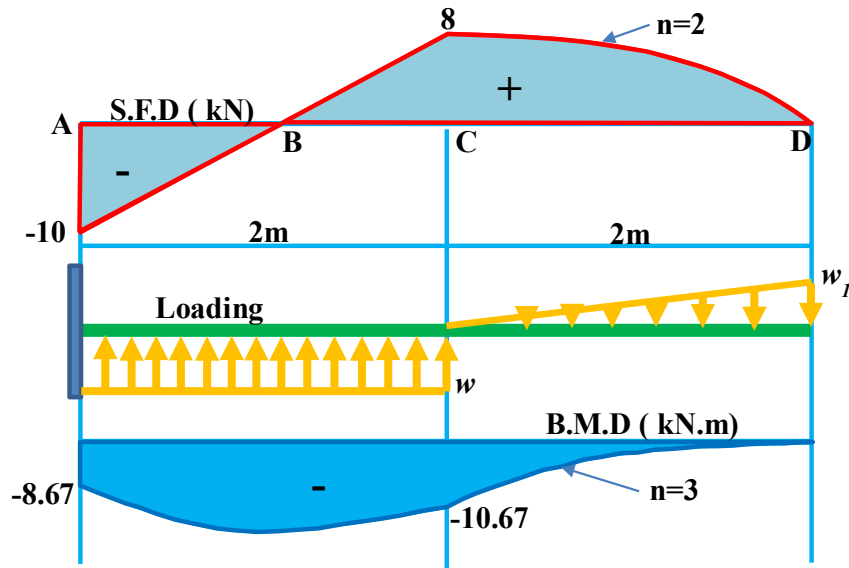
$$M_B = -8.67 + \frac{1}{2}(-10)(1.11)$$

$$\rightarrow M_B = -14.23 \text{ kN.m}$$

$$M_C = -14.23 + \frac{1}{2}(-8)(2 - 1.11)$$

$$\rightarrow M_C = -10.67 \text{ kN.m}$$

$$M_D = -10.67 + \frac{2(8)(2)}{2 + 1} = 0$$



Example (8): find the force (P) so that the B.M. becomes zero at point (C) for the beam shown in Figure (2.16).

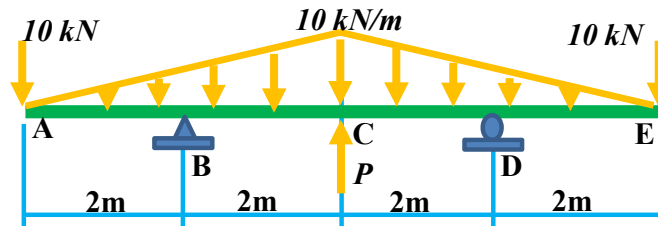


Figure (2.16)

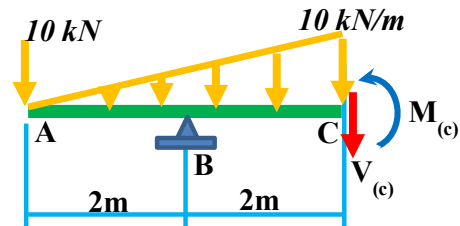
Solution:

$$\sum M_D = 0$$

$$\rightarrow 10(6) - R_B(4) - P(2) + \frac{1}{2}(10)(4)\left(\frac{4}{3} + 2\right) + \frac{1}{2}(10)(4)\left(2 - \frac{4}{3}\right) - 10(2) = 0$$

$$\rightarrow R_B = 30 - \frac{P}{2}$$

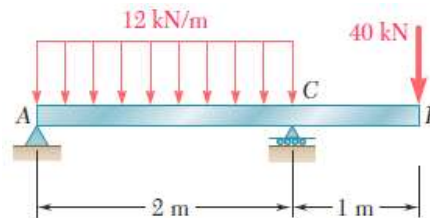
► Part ABC as F.B.D:



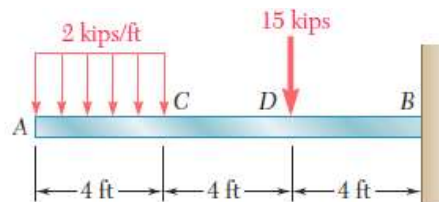
$$M_c = 0 = -10(4) + \left(30 - \frac{P}{2}\right)(2) - \frac{1}{2}(10)(4)\left(\frac{4}{3}\right)$$

$$\rightarrow P = -6.67 \text{ kN}$$

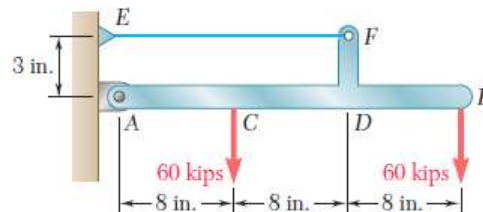
Problem 01: Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Problem 02: Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Problem 03: Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Problem 04: Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

