ALMUSTAQBAL UNIVERSITY
COLLEGE OF ENGINEERING AND TECHNOLOGY
COMPUTER ENGINEERING TECHNIQUE DEPARTMENT

Subject: Control Engineering Fundamentals

Ninth lecture: Transient and Steady-State Response Analyses

By: prof. Dr. Abdulrahim Thiab Humod

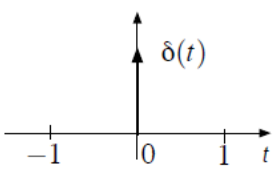
Transient and Steady-State Response Analyses

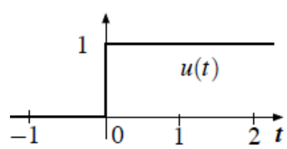
In analyzing and designing control systems, we must have a basis of comparison of performance of various control systems. This basis may be set up by specifying particular test input signals and by comparing the responses of various systems to these input signals.

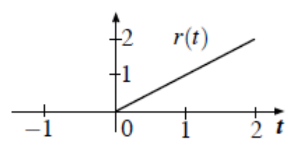
In this lecture we use test signals such as step, ramp, acceleration and impulse signals. With these test signals, mathematical and experimental analyses of control systems can be carried out easily, since the signals are very simple functions of time.

If the inputs to a control system are gradually changing functions of time, then a ramp function of time may be a good test signal.
If a system is subjected to sudden disturbances, a step function of time may be a good test signal.
If the system subjected to shock inputs, an impulse function may be best.

Once a control system is designed on the basis of test signals, the performance of the system in response to actual inputs is generally satisfactory.





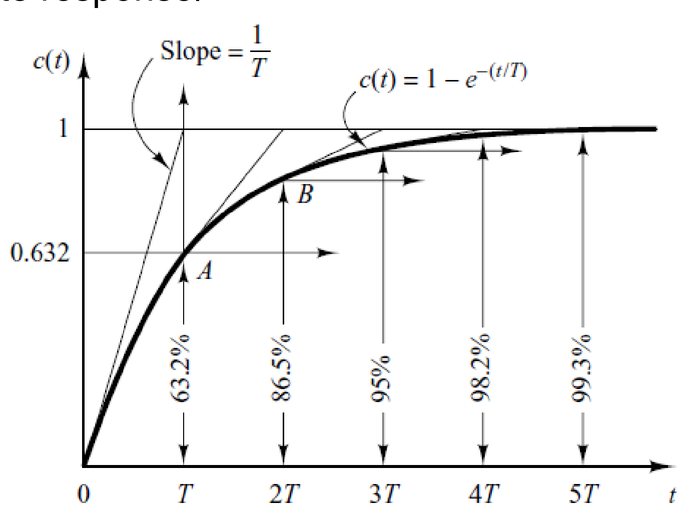


Transient Response and Steady-State Response. The time response of a control system consists of two parts: the transient response and the steady-state response.

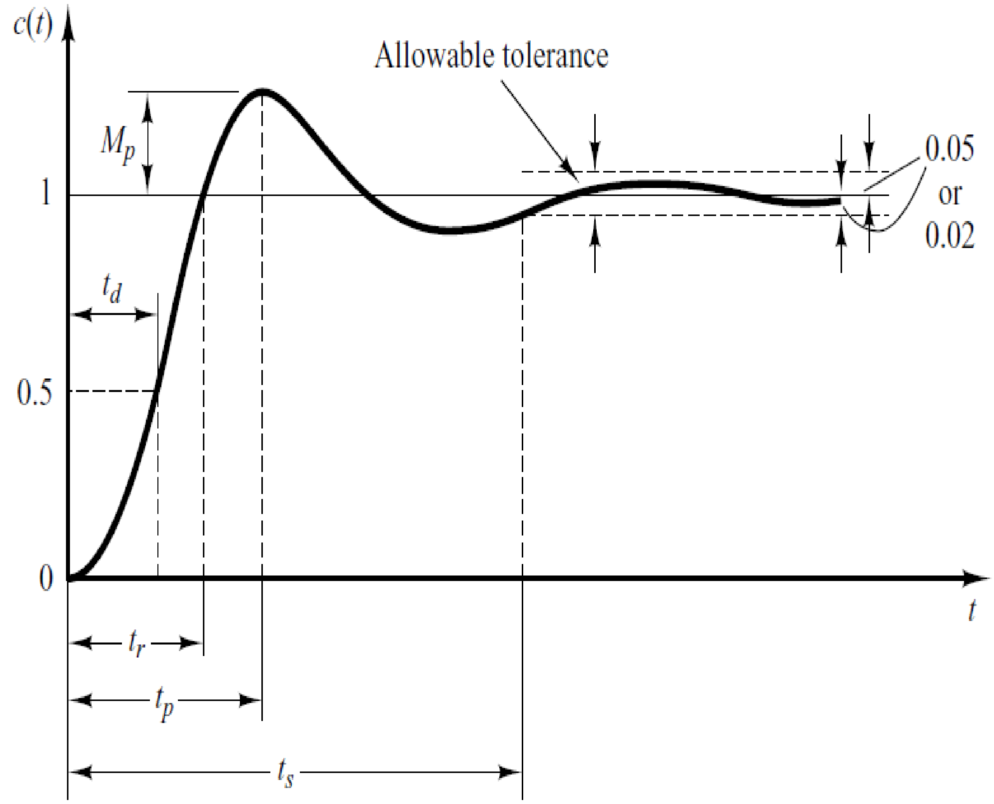
transient response: the response goes from the initial state to the final state.
steady-state response: we mean the manner in which the system output behaves as t approaches infinity.

the system response $c(t)$ may be written as $c(t) = c_{tr}(t) + c_{ss}(t)$
where the first term on the right-hand side of the equation is the transient response and the second term is the steady-state response.

Exponential response curve.



Transient and Steady-State Response Analyses



A control system is in **equilibrium** if, in the absence of any disturbance or input, the output stays in the same state.

A linear time-invariant control system is **stable** if the output eventually comes back to its equilibrium state when the system is subjected to an initial condition.

A linear time-invariant control system is **critically stable** if oscillations of the output continue forever.

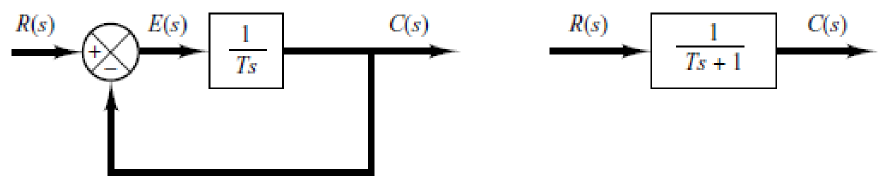
The system is **unstable** if the output diverges without bound from its equilibrium state when the system is subjected to an initial condition.

If the output of a system at steady state does not exactly agree with the input, the system is said to have **steady state error.**

FIRST-ORDER SYSTEMS

Consider the first-order system shown in Figure below; Physically, this system may represent an RC circuit, thermal system, or the like.

Figure (1)



The input-output relationship is given by

$$\frac{C(s)}{R(s)} = \frac{1}{Ts + 1} \qquad \ldots\ldots \quad (1)$$

Unit-Step Response of First-Order Systems. Since the Laplace transform of the unit-step function is **1/s**, substituting **R(s)=1/s** into Equation (1), we obtain

$$C(s) = \frac{1}{Ts + 1}\frac{1}{s}$$

Expanding C(s) into partial fractions gives

$$C(s) = \frac{1}{s} - \frac{T}{Ts + 1} = \frac{1}{s} - \frac{1}{s + (1/T)} \qquad \ldots\ldots \quad (2)$$

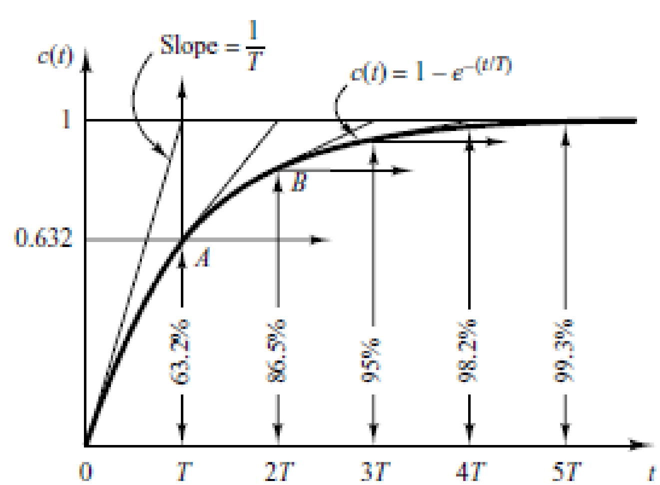
Taking the inverse Laplace transform of Equation (2), we obtain

$$c(t) = 1 - e^{-t/T}, \qquad \text{for } t \geq 0 \qquad \ldots\ldots \quad (3)$$

Equation (5–3) states that initially the output c(t) is zero and finally it becomes unity.

One important characteristic of such an exponential response curve c(t) is that at t=T the value of c(t) is 0.632, or the response c(t) has reached 63.2% of its total change. This may be easily seen by substituting t=T in c(t). That is,

$$c(T) = 1 - e^{-1} = 0.632$$



Note that the smaller the time constant T, the faster the system response.

Another important characteristic of the exponential response curve is that the slope of the tangent line at t=0 is 1/T

$$\frac{dc}{dt} = \frac{1}{T}e^{-t/T}\bigg|_{t=0} = \frac{1}{T} \qquad \ldots\ldots \ (4)$$

From Equation (4) we see that the slope of the response curve c(t) decreases monotonically from 1/T at t=0 to zero at t=∞.
The exponential response curve c(t) given by Equation (3) is shown in Figure 2.
In one time constant, the response curve has gone from 0 to 63.2% of the final value.
In two time constants, the response reaches 86.5%of the final value.
At t=3T, 4*T*, and 5T, the response reaches 95%, 98.2%, and 99.3%, respectively, of the final value

Unit-Ramp Response of First-Order Systems. Since the Laplace transform of the unit-ramp function is $1/s^2$, we obtain the output of the system of Figure 1 as

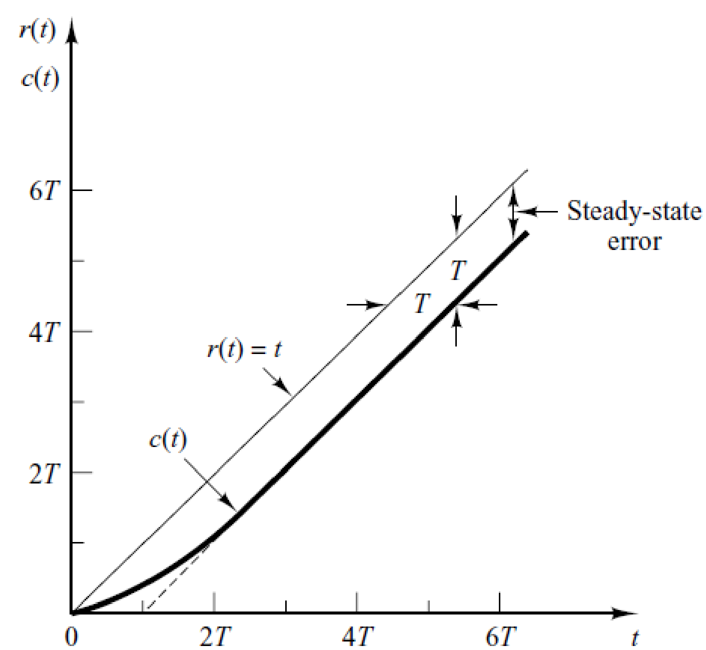
$$C(s) = \frac{1}{Ts + 1}\frac{1}{s^2}$$

Expanding C(s) into partial fractions gives

$$C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{Ts + 1} \qquad \ldots\ldots \quad (5)$$

Taking the inverse Laplace transform of Equation (5), we obtain

$$c(t) = t - T + Te^{-t/T}, \qquad \text{for } t \geq 0 \qquad \ldots. \quad (6)$$



The error signal e(t) is then

$$\begin{aligned} e(t) &= r(t) - c(t) \\ &= T(1 - e^{-t/T}) \end{aligned}$$

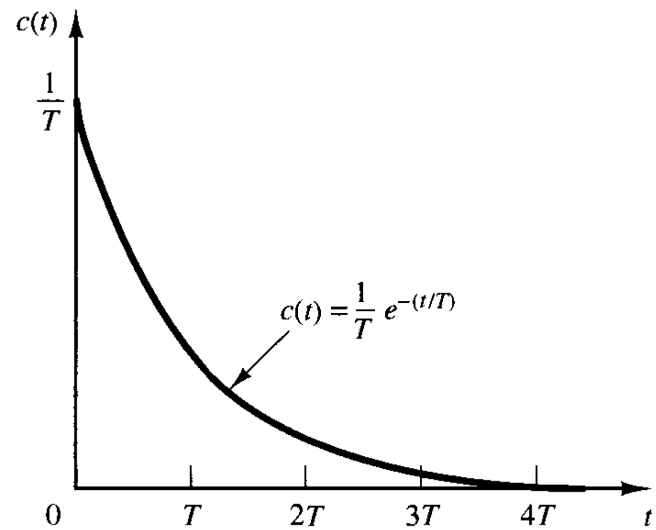
As t approaches infinity, $e^{-t/T}$ approaches zero, and thus the error signal e(t) approaches T or

$e(\infty) = T$

Unit-Impulse Response of First-Order Systems. For the unit-impulse input, **R(s)=1** and the output of the system of Figure (1) can be obtained as

$$C(s) = \frac{1}{Ts + 1}$$

$$c(t) = \frac{1}{T} e^{-t/T}, \qquad \text{for } t \geq 0$$



An Important Property of Linear Time-Invariant Systems. In the analysis above, it has been shown that for the unit-ramp input the output c(t) is

$$c(t) = t - T + Te^{-t/T}, \qquad \text{for } t \geq 0$$

For the unit-step input, which is the derivative of unit-ramp input, the output c(t) is

$$c(t) = 1 - e^{-t/T}, \qquad \text{for } t \geq 0$$

For the unit-impulse input, which is the derivative of unit-step input, the output c(t) is

$$c(t) = \frac{1}{T} e^{-t/T}, \qquad \text{for } t \geq 0$$

Comparing the system responses to these three inputs clearly indicates that the response to the derivative of an input signal can be obtained by differentiating the response of the system to the original signal.