



3 – First order : linear

It is found in two forms

$$1- \frac{dy}{dx} + P(x).y = Q(x)$$

The $P(x)$ and $Q(x)$ are function for (x)

The solution is

$$\rho = e^{\int P(x)dx}$$

$$\rho.y = \int \rho Q(x)dx + c$$

$$2- \frac{dx}{dy} + P(y).x = Q(y)$$

The $P(y)$ and $Q(y)$ are function for (y)

$$\rho = e^{\int P(y)dy}$$

$$\rho.x = \int \rho Q(y)dy + c$$

Ex. 1.: Solve the differential equation:-

$$x \frac{dy}{dx} + 3y = x^2$$

Solution

$$\frac{x}{x} \frac{dy}{dx} + \frac{3y}{x} = \frac{x^2}{x}$$

$$\frac{dy}{dx} + \frac{3}{x}y = x$$

$$\text{Let } P(x) = \frac{3}{x}, \quad Q(x) = x$$

$$\rho = e^{\int P(x)dx}$$

$$\rho = e^{\int \frac{3}{x}dx} = e^{3\ln x} = x^3$$



$$\rho \cdot y = \int \rho Q(x) dx + c$$

$$x^3 \cdot y = \int x^3 \cdot x dx + c$$

$$x^3 \cdot y = \int x^4 dx + c$$

$$x^3 \cdot y = \frac{x^5}{5} + c$$

$$y = \frac{x^5}{5x^3} + \frac{c}{x^3}$$

$$y = \frac{x^2}{5} + cx^{-3}$$

Ex. 2.: Solve the differential equation:-

$$x \frac{dy}{dx} - 3y = x^2$$

Solution

$$\frac{x dy}{x dx} - \frac{3y}{x} = \frac{x^2}{x}$$

$$\frac{dy}{dx} - \frac{3y}{x} = x$$

$$\text{Let } P(x) = -\frac{3}{x}, \quad Q(x) = x$$

$$\rho = e^{\int P(x) dx}$$

$$\rho = e^{\int -\frac{3}{x} dx} = e^{-3 \ln x} = x^{-3}$$

$$\rho \cdot y = \int \rho Q(x) dx + c$$

$$x^{-3} \cdot y = \int x^{-3} \cdot x dx + c$$

$$x^{-3} \cdot y = \int x^{-2} dx + c$$



$$x^{-3} \cdot y = \frac{x^{-1}}{-1} + c$$

$$\frac{x^{-3}}{x^{-3}} y = \frac{-x^{-1}}{x^{-3}} + \frac{c}{x^{-3}}$$

$$y = -x^2 + cx^3$$

H.W.: Find a general solution of each the following equations:-

1) $\frac{dy}{dx} + 2y = e^{-x}$

2) $2\frac{dy}{dx} - y = e^{x/2}$

3) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

4) $(x - 2y) dy + y dx = 0$

4 - First order: Exact

The general formula of the equation $M(x,y) dx + N(x,y) dy = 0$

If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then the equation is **Exact**

If the equation was Exact the solve:

$$df(x, y) = 0$$

$$\int df(x, y) = \int 0$$

$$f(x, y) = c$$

$$M(x, y) dx + N(x, y) dy = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = df$$

Ex. 1.: Show that differential equation:-

$$(x^2 + y^2)dx + (2xy + \cos y)dy = 0 \quad \text{is exact and solve it?}$$



Solution

$$\frac{\partial M}{\partial y} = 2y, \quad \frac{\partial N}{\partial x} = 2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \textbf{Exact D.E.}$$

To find $f(x,y)$ such that

$$\frac{\partial f}{\partial x} = x^2 + y^2, \quad \frac{\partial f}{\partial y} = 2xy + \cos y$$

$$f(x,y) = \int M(x,y)dx + K(y)$$

$$f(x,y) = \int (x^2 + y^2)dx + K(y)$$

$$f(x,y) = \frac{x^3}{3} + xy^2 + K(y)$$

To find $K(y)$

$$\frac{\partial f}{\partial y} = 2xy + K'y = 2xy + \cos y$$

$$K'(y) = \cos y \quad \text{تكامل}$$

$$K(y) = \sin y$$

$$f(x,y) = \frac{x^3}{3} + xy^2 + \sin y$$

$$f(x,y) = c$$

$$\frac{x^3}{3} + xy^2 + \sin y = c$$

Ex. 2.: State the type of D.E. $(y \sec^2 x + \sec x \tan x) dx + (\tan x + 2y) dy = 0$

Solution

$$M = y \sec^2 x + \sec x \tan x$$



$$\frac{\partial M}{\partial y} = \sec^2 x$$

$$N = \tan x + 2y$$

$$\frac{\partial N}{\partial x} = \sec^2 x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{Exact D.E.}$$

$$\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = df = (y \sec^2 x + \sec x \tan x) dx + (\tan x + 2y) dy$$

$$\frac{\partial f}{\partial x} = y \sec^2 x + \sec x \tan x$$

$$df = \int (y \sec^2 x + \sec x \tan x) dx$$

$$f(x,y) = y \tan x + \sec x + K(y)$$

$$\frac{\partial f}{\partial y} = \tan x + 0 + K'(y) = \tan x + 2y$$

$$K'(y) = 2y$$

$$K(y) = \int 2y dy$$

$$K(y) = \frac{2y^2}{2}$$

$$f(x,y) = y \tan x + \sec x + y^2$$

$$f(x,y) = c$$

$$y \tan x + \sec x + y^2 = c$$

$$\text{H.W.: Solve equation } \frac{dx}{x^2 y} + \frac{dy}{xy^2} = 0$$