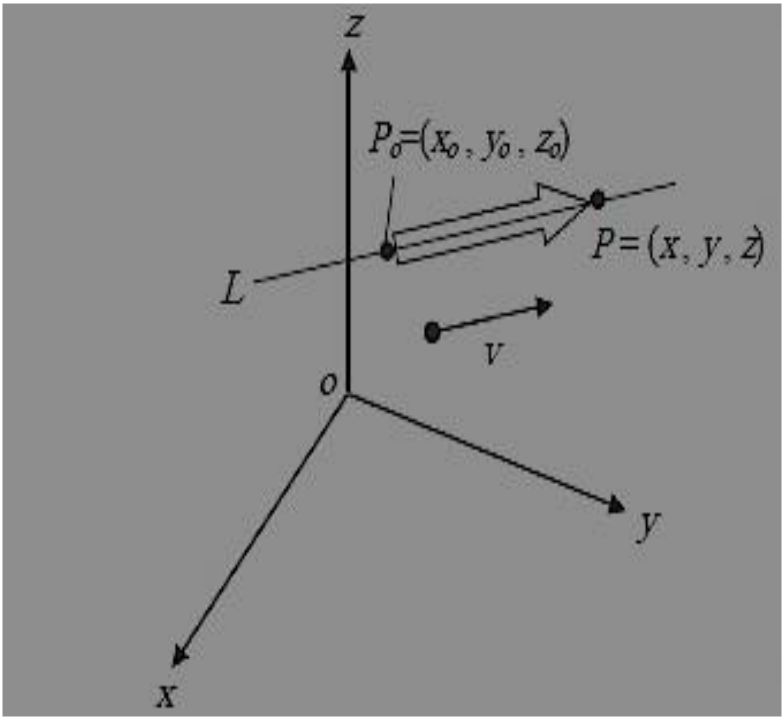
4.Vectors analysis including parametric equations for lines in space

Equation lines in space.

Suppose that L is a line in space through a point P_o (x_o , y_o , z_o) Parallel to a vector v = ai + bj + ck ,then L is the set of all points p (x , y , z)for which P_oP is parallel to v. thus P_oP = tv for some scalar parameter t. the value of t depend on the location of the point P along the line. the expanded from of the equation:



$\overrightarrow{P_oP} = tv$

$(x - x_o)i + (y - y_o)j + (z - z_o)k = t(ai + bj + ck)$

$x - x_o = ta$

$y - y_o = tb$

$z - z_o = tc$

From equation above:

The parametric equation for the line through $P_o(x_o, y_o, z_o)$ parallel to $v = ai + bj + ck$:

$$x = x_o + ta$$
$$y = y_o + tb$$
$$z = z_o + tc$$

Example: find parametric equation for the line through the points $P(-3,2,-3)$ and $Q(1,-1,4)$

Solution: the vector

$$\overrightarrow{PQ} = (1-(-3))i + (-1-2)j + (4-(-3))k$$

$$\overrightarrow{PQ} = 4i - 3j + 7k$$

$$\therefore \quad a = 4 \; , \; b = -3 \; , c = 7$$

$\therefore \quad x_o = -3 \quad , \quad y_o = 2 \quad , z_o = -3$

$\therefore$

$x = x_o + at \Longrightarrow x = -3 + 4t$

$y = y_o + bt \Longrightarrow y = 2 - 3t$

$z = z_o + ct \Longrightarrow z = -3 + 7t$

Example: Find parametric equations for the line through the point $(-2,0,4)$ parallel to the vector $v = 2i + 4j - 2k$

Solution:

With $P_o(x_o, y_o, z_o) = (-2,0,4)$

$x_o = -2 \quad , y_o = 0 \quad , z_o = 4$

$and \; v = ai + bj + ck = 2i + 4j - 2k$

$a = 2 \quad , b = 4 \quad , c = -2$

$\therefore$

$x = x_o + at \Longrightarrow x = -2 + 2t$

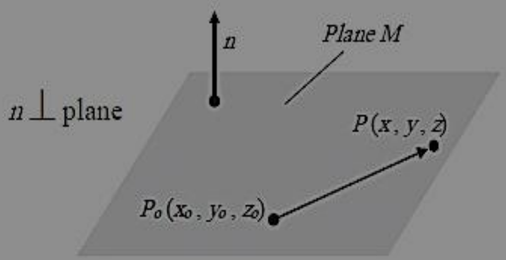
$y = y_o + bt \Longrightarrow y = 4t$

$z = z_o + ct \Longrightarrow z = 4 - 2t$

4.2 Vectors analysis including parametric equations for planes in space

Equation for plane in space:

Suppose that plane M passes through a point $P_o(x_o, y_o, z_o)$ and is normal to the nonzero vector $n = Ai + Bj + Ck$, then M is the set of all points $P(x,y,z)$ for which $\overrightarrow{P_oP}$ is orthogonal to n. Thus the dot product



$n.\overrightarrow{P_oP} = 0$

This equation is equivalent to:

$$(Ai + Bj + Ck).[(x - x_o)i + (y - y_o)j + (z - z_o)k] = 0$$

$$\boxed{A(x - x_o) + B(y - y_o) + C(z - z_o) = 0} \Longrightarrow \textit{Component equation}$$

This becomes:

$$Ax + By + Cz = Ax_o + By_o + Cz_o$$

$$\boxed{Ax + By + Cz = D} \Longrightarrow \textit{Component equation simplified}$$

Where $D = Ax_o + By_o + Cz_o$

Example: Find an equation for the plane through *Po* (4,2,1) normal to $\overline{N = 5\imath + 2\jmath - 3k}$.

Solution//

$D = AX_0 + BY_0 + CZ_O = 5*4 + 2*2 - 3*1 = 21$

Ax+By+Cz=D

5x+2y-3z=21

Example: Find an equation for the plane through *A*(1,1,1) ,*B*(3,2,4) and *C*(3,0,3).

Solution//

=(3-1)i+(2-1)j+(4-1)*k*=*2i*+*j*+*3k*$\overline{AB}$

=(3-1)i+(0-1)j+(3-1)*k*=*2i*-*j*+*2k*$\overline{AC}$

$$\overline{AB}\ X\ \overline{AC} = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 2 & -1 & 2 \end{vmatrix}$$
$$= [2-(-3)i - (4-6)j + (-2-2)k$$
$$= 5i + 2j - 4k]$$

$D = Ax_o + By_o + Cz_o = 5*1 + 2*1 - 4*1 = 3$

5x+2y-4z=3