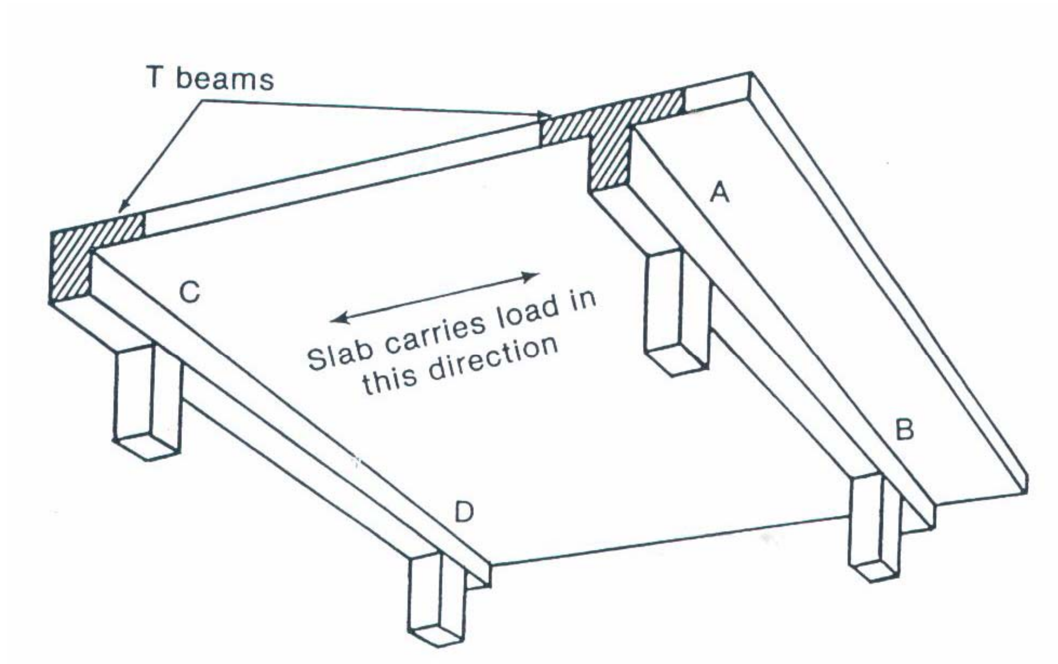
Lecture 11 - Flexure

Lecture Goals

- Basic Concepts
- T Beams and L Beams

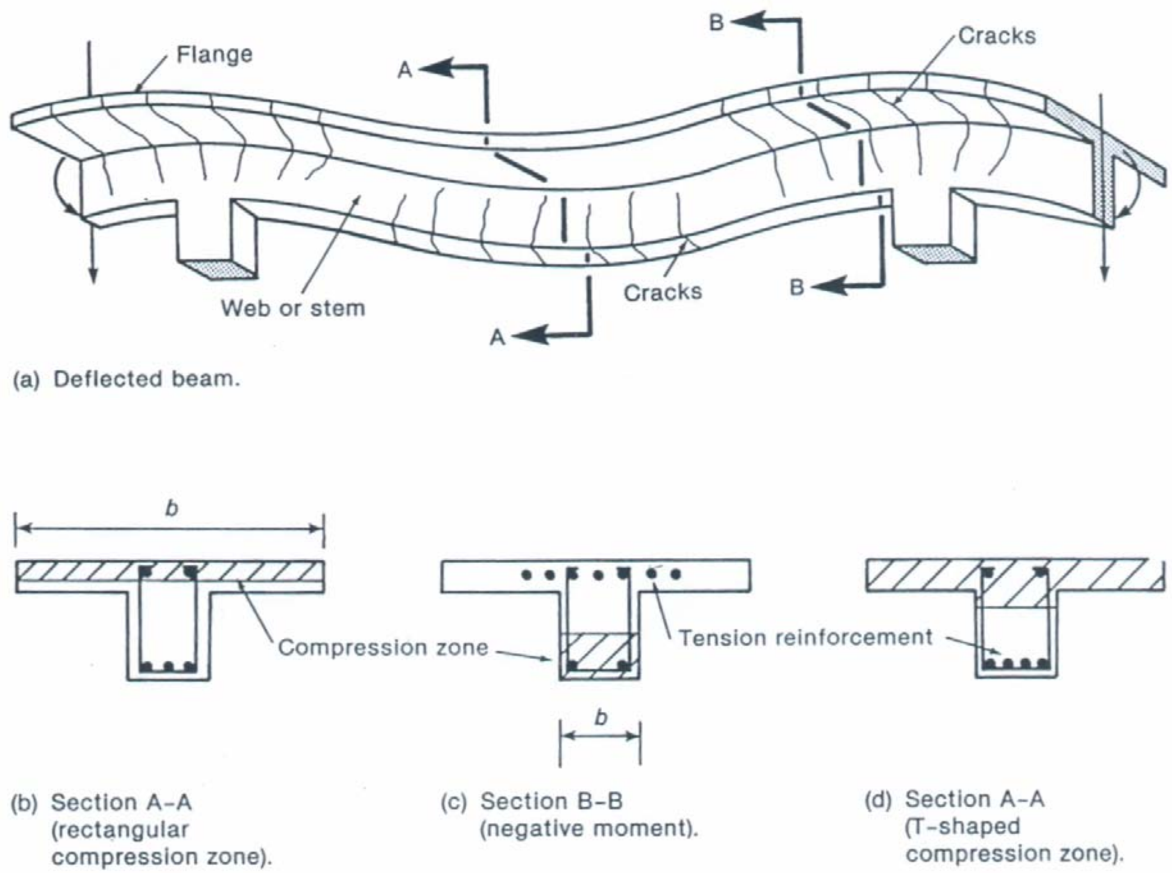
Analysis of Flanged Section

- Floor systems with slabs and beams are placed n monolithic pour.
- Slab acts as a top flange to the beam; ***T-beams***,and ***Inverted L(Spandrel) Beams***.



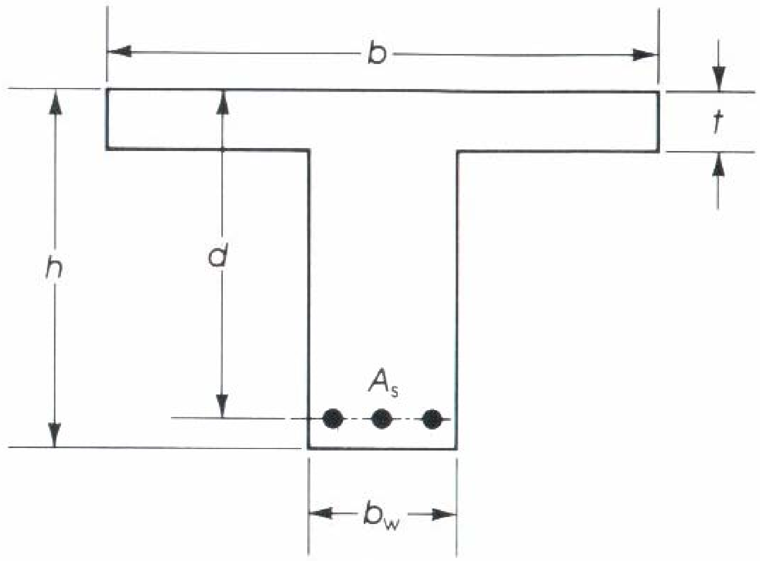
Analysis of Flanged Sections

Positive and Negative Moment Regions in a T-beam



Analysis of Flanged Sections

If the neutral axis falls within the slab depth analyze the beam as a rectangular beam, otherwise as a T-beam.

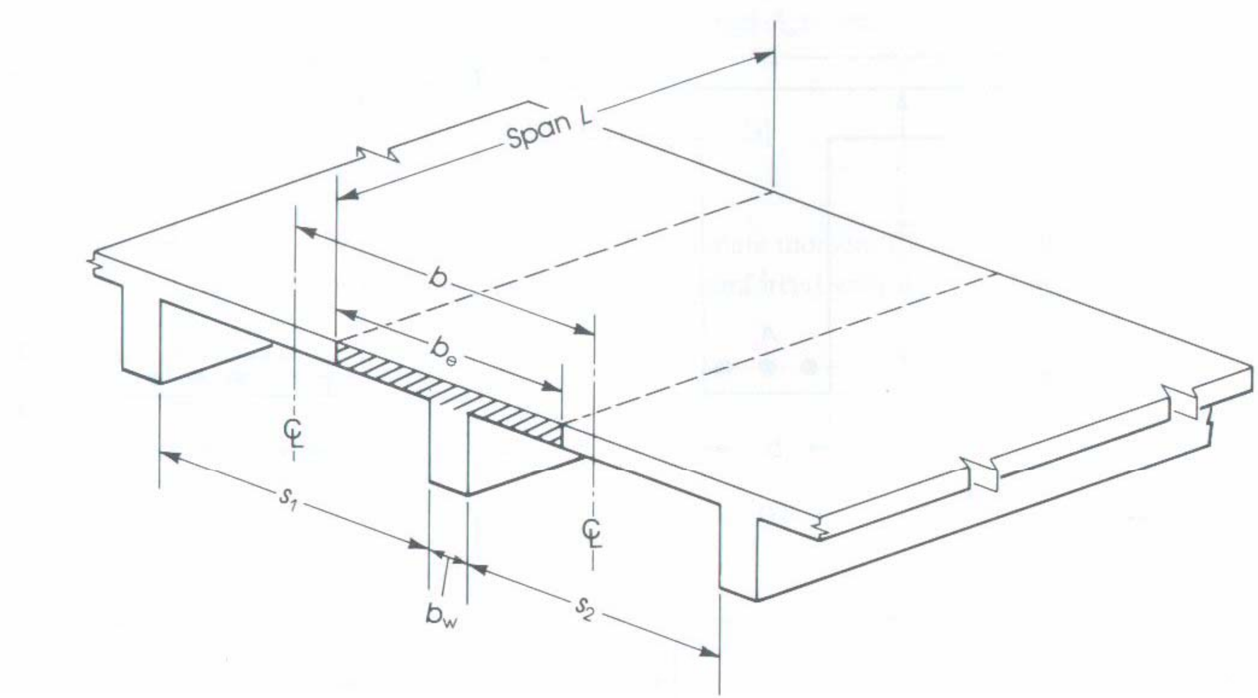


(a)

Analysis of Flanged Sections

Effective Flange Width

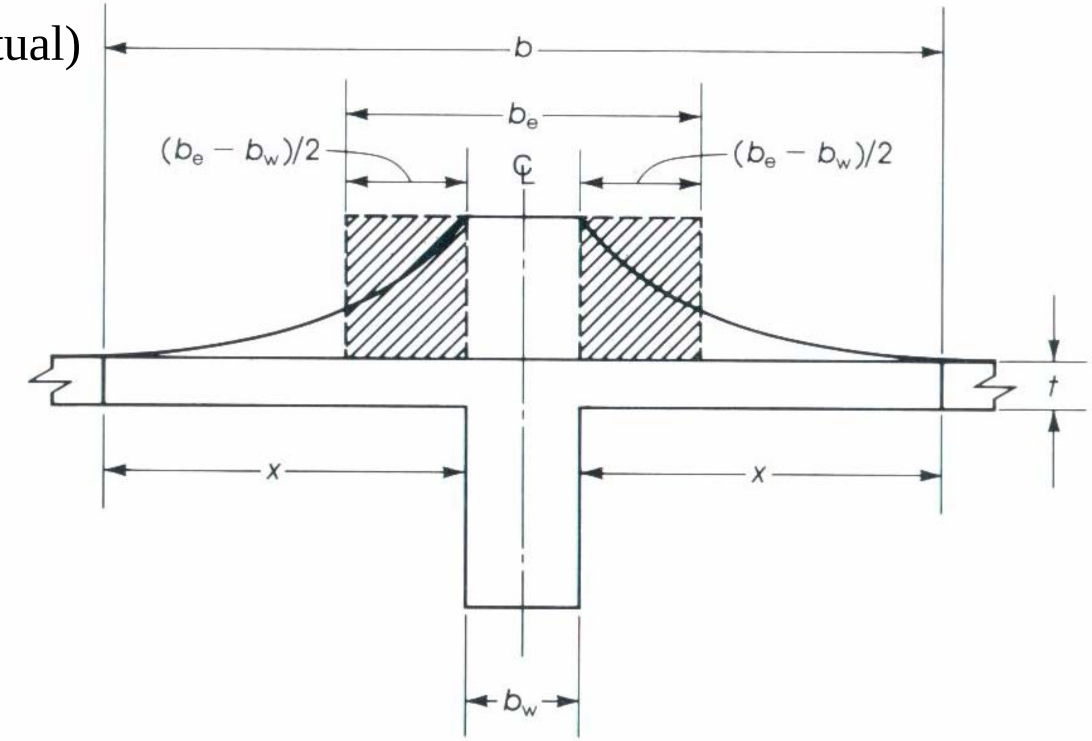
Portions near the webs are more highly stressed than areas away from the web.



Analysis of Flanged Sections

<u>Effective width</u> (b_{eff})

b_{eff} is width that is stressed uniformly to give the same compression force actually developed in compression zone of width $b_{(actual)}$



ACI Code Provisions for Estimating b_{eff}

From ACI 318, Section 8.10.2

T Beam Flange:

$$b_{\mathrm{eff}} \le \frac{L}{4}$$

$$\le 16h_{\mathrm{f}} + b_{\mathrm{w}}$$

$$\le b_{\mathrm{actual}}$$

ACI Code Provisions for Estimating b_{eff}

From ACI 318, Section 8.10.3

Inverted L Shape Flange

$$
\begin{aligned}
b_{\text{eff}} &\le \frac{L}{12} + b_{\text{w}} \\
&\le 6h_{\text{f}} + b_{\text{w}} \\
&\le b_{\text{actual}} = b_{\text{w}} + 0.5*(\text{clear distance to next web})
\end{aligned}
$$

ACI Code Provisions for Estimating b_{eff}

From ACI 318, Section 8.10

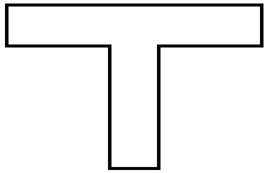
Isolated T-Beams

$$h_f \geq \frac{b_w}{2}$$

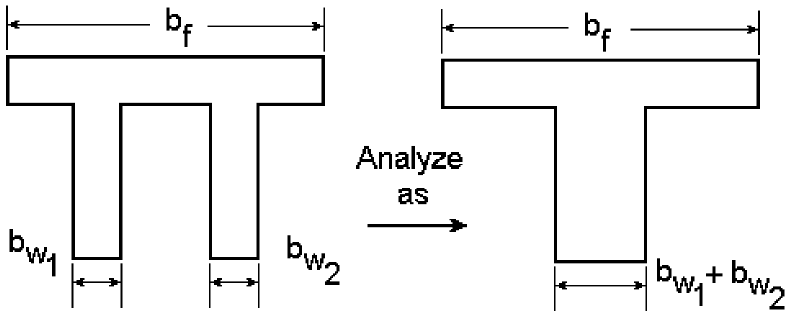
$$b_{eff} \leq 4b_w$$

Various Possible Geometries of T-Beams

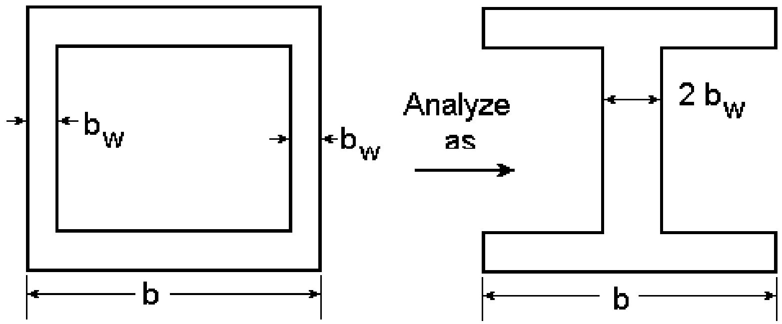
Single Tee



Twin Tee



Box



Analysis of T-Beam

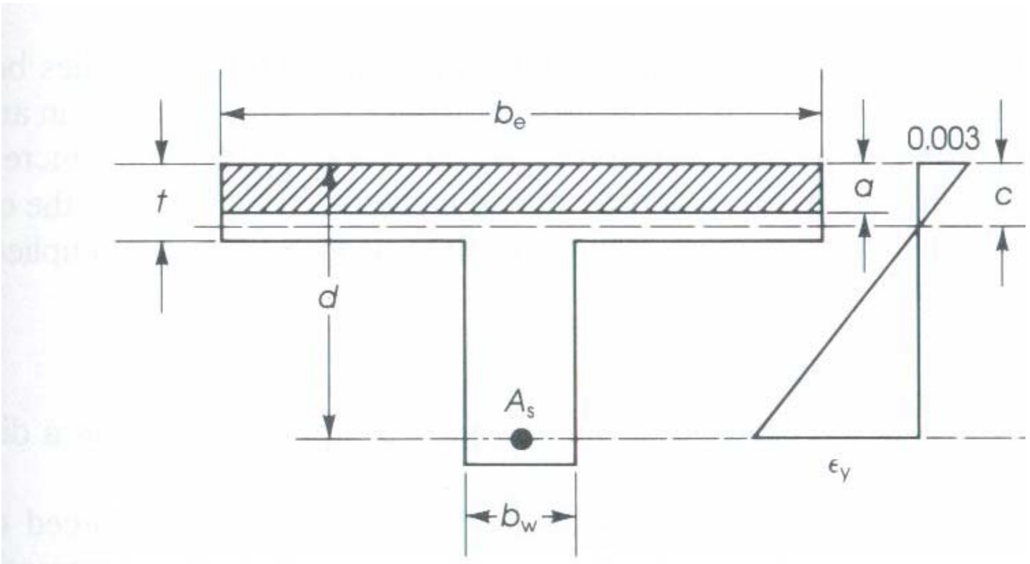
<u>Case 1</u>: $a \leq h_f$ Same as rectangular section

Assume $\varepsilon_s \geq \varepsilon_y \Rightarrow f_s = f_y$

Steel is yielding under reinforced

Check

$$a \leq h_f$$

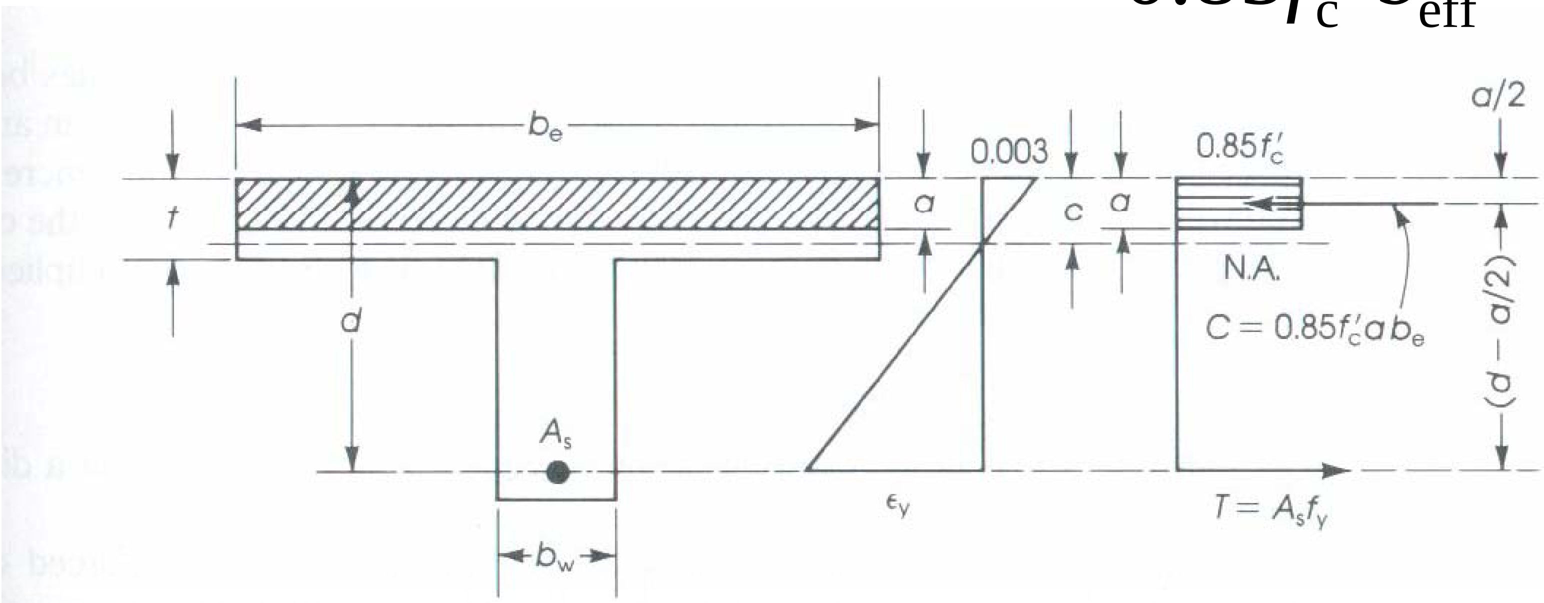


Analysis of T-Beam

Case 1: $a \le h_f$

Equilibrium

$$T = C \Rightarrow a = \frac{A_s f_y}{0.85 f_c' \, b_{eff}}$$



Analysis of T-Beam

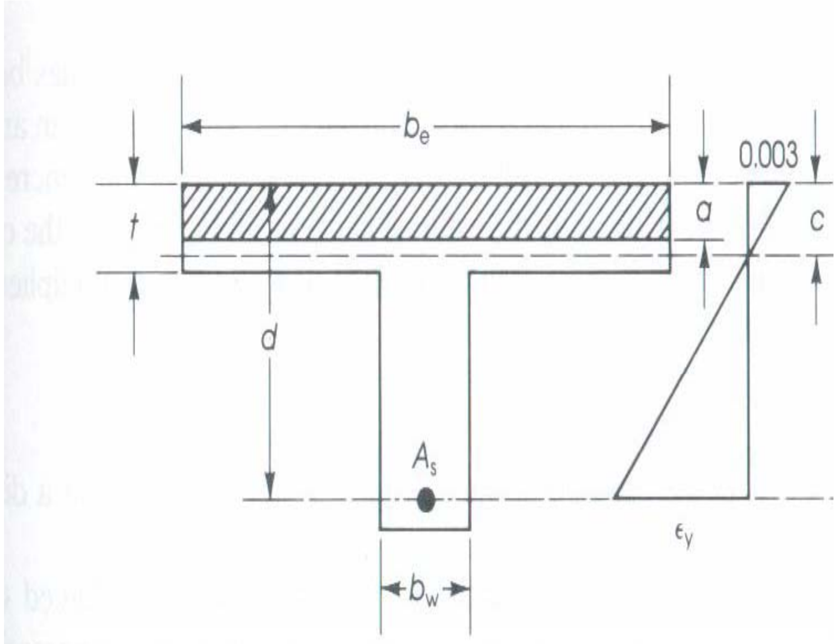
Case 1: $a \le h_f$

Confirm

$$\varepsilon_s \ge \varepsilon_y$$

$$c = \frac{a}{\beta_1}$$

$$\varepsilon_s = \left(\frac{d-c}{c}\right)\varepsilon_{cu} \ge \varepsilon_y$$

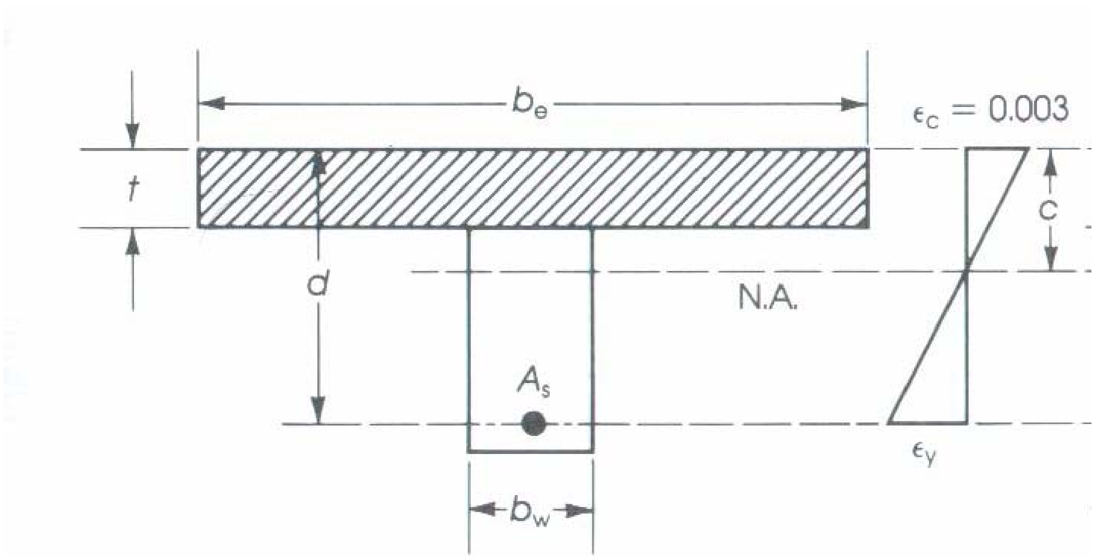


Analysis of T-Beam

Case 1: $a \le h_f$

Calculate M_n

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$



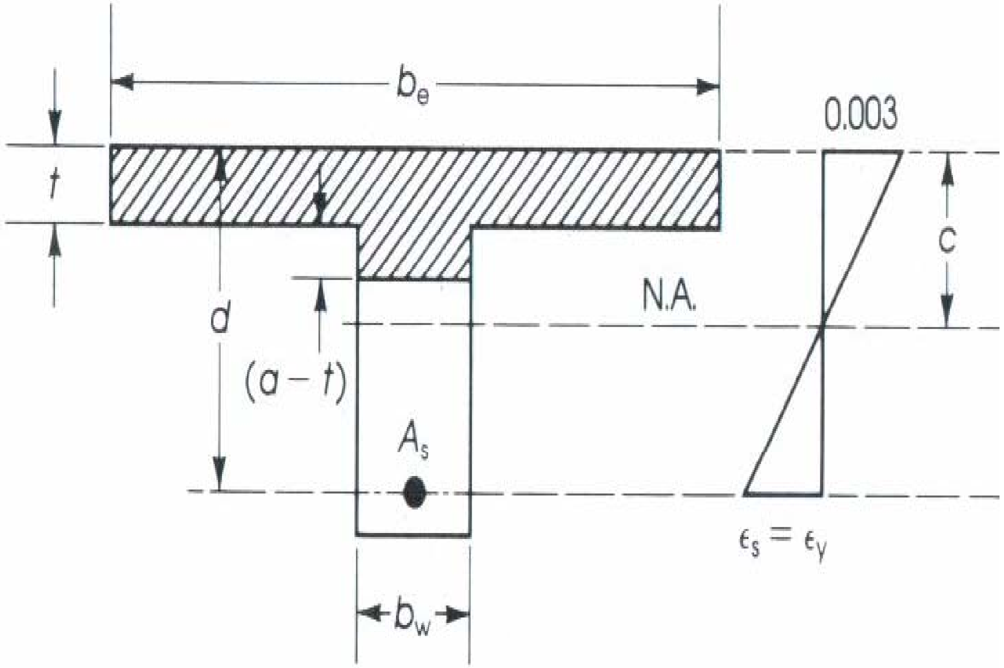
Analysis of T-Beam

Case 2: $a > h_f$ Assume steel yields

$$C_f = 0.85 f_c' (b - b_w) h_f$$

$$C_w = 0.85 f_c' b_w a$$

$$T = A_s f_y$$



Analysis of T-Beam

Case 2: $a > h_{f}$ Assume steel yields

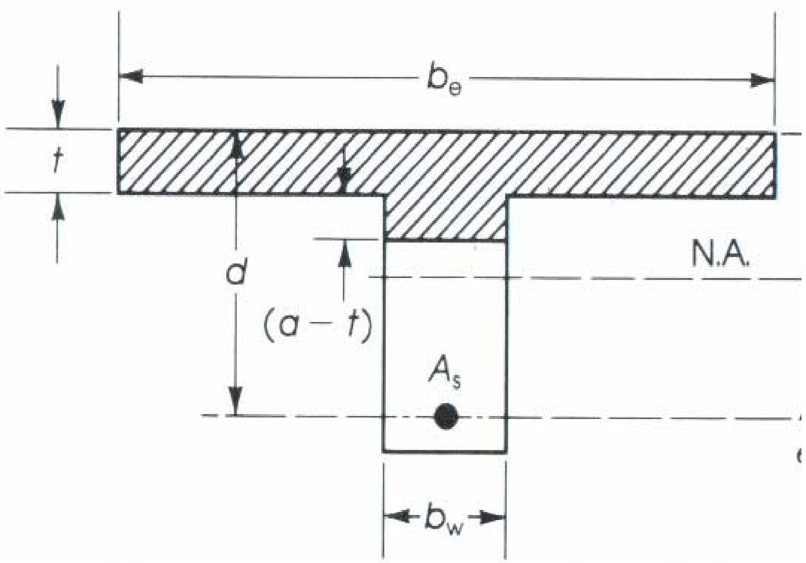
$$A_{sf} = \frac{0.85 f_{c}'\left(b - b_{w}\right) h_{f}}{f_{y}}$$

The flanges are considered to be equivalent compression steel.

Analysis of T-Beam

Case 2: $a > h_f$ Equilibrium

$$T = C_f + C_w \Rightarrow a = \frac{\left(A_s - A_{sf}\right) f_y}{0.85 f_c' b_w}$$



Analysis of T-Beam

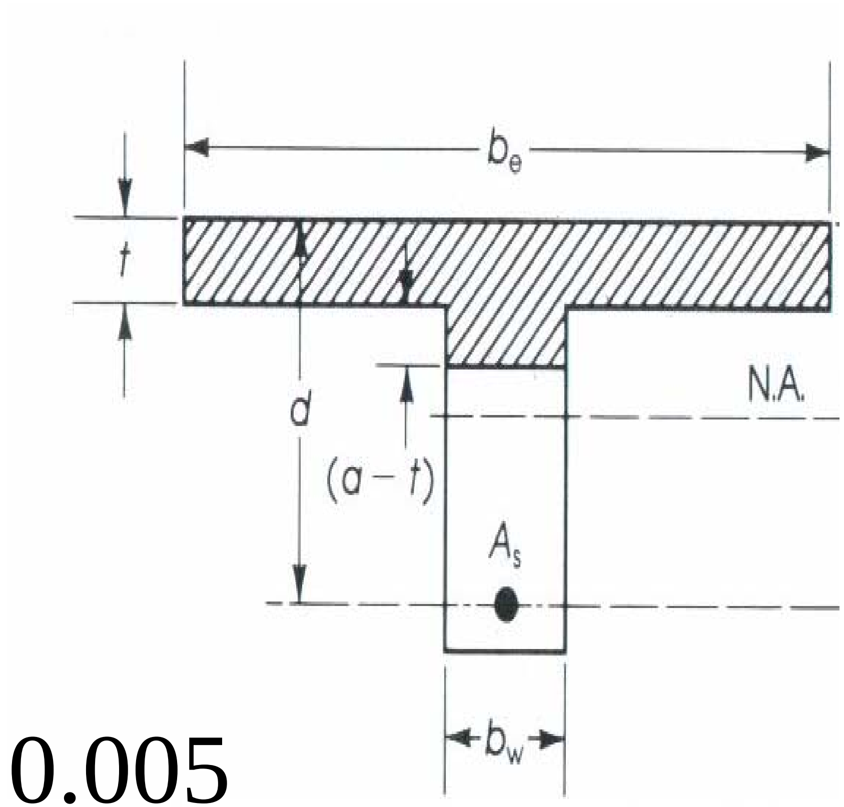
Case 2: $a > h_f$

Confirm

$$a > h_f$$

$$c = \frac{a}{\beta_1}$$

$$\varepsilon_s = \left(\frac{d - c}{c}\right)\varepsilon_{cu} \geq 0.005$$



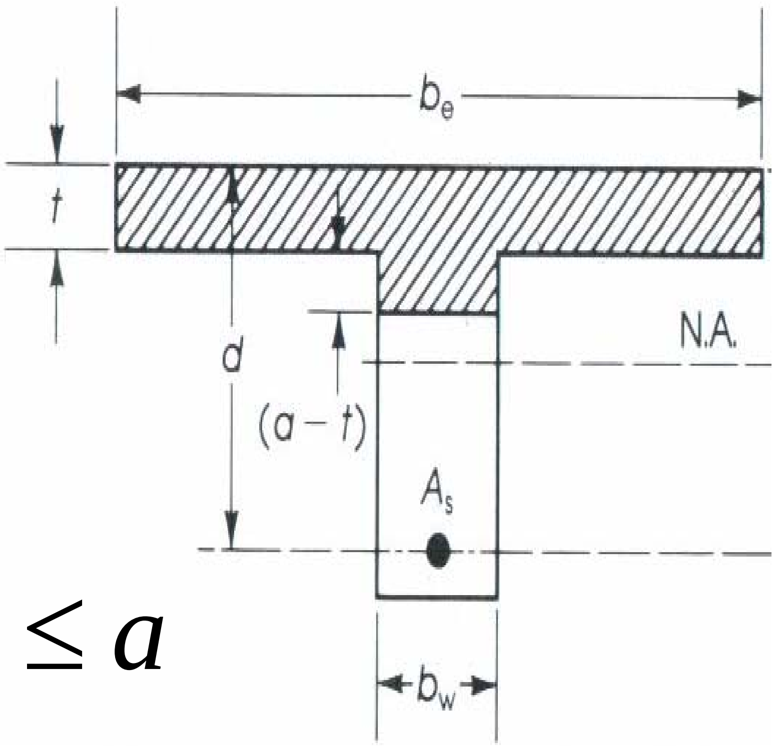
Analysis of T-Beam

Case 2: $a > h_f$

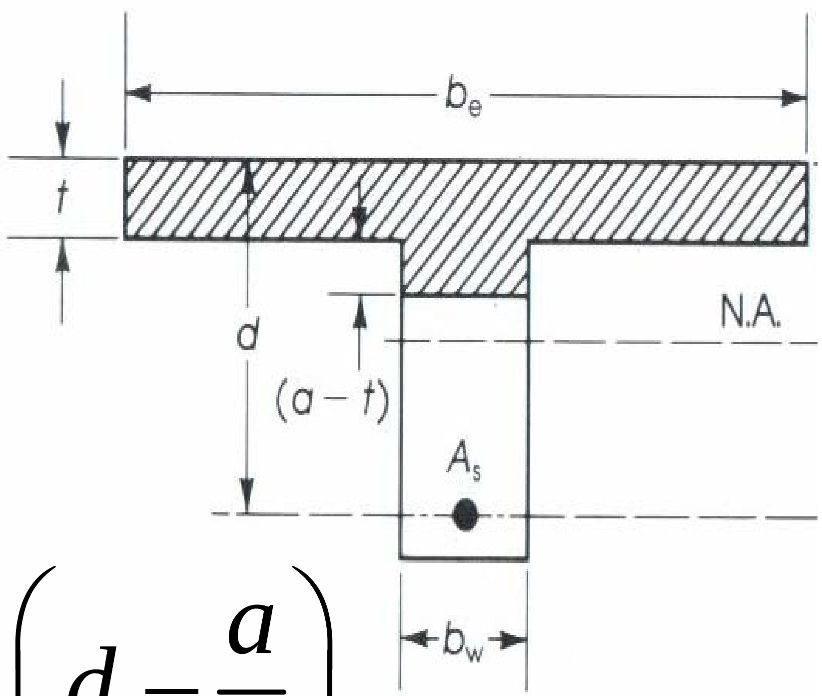
Confirm

$$\varpi = \rho \frac{f_y}{f_c'}$$

$$h_f \le \frac{1.18 \varpi d}{\beta_1} \text{ or } h_f \le a$$



Analysis of T-Beam



Case 2: $a > h_f$

Calculate nominal moments

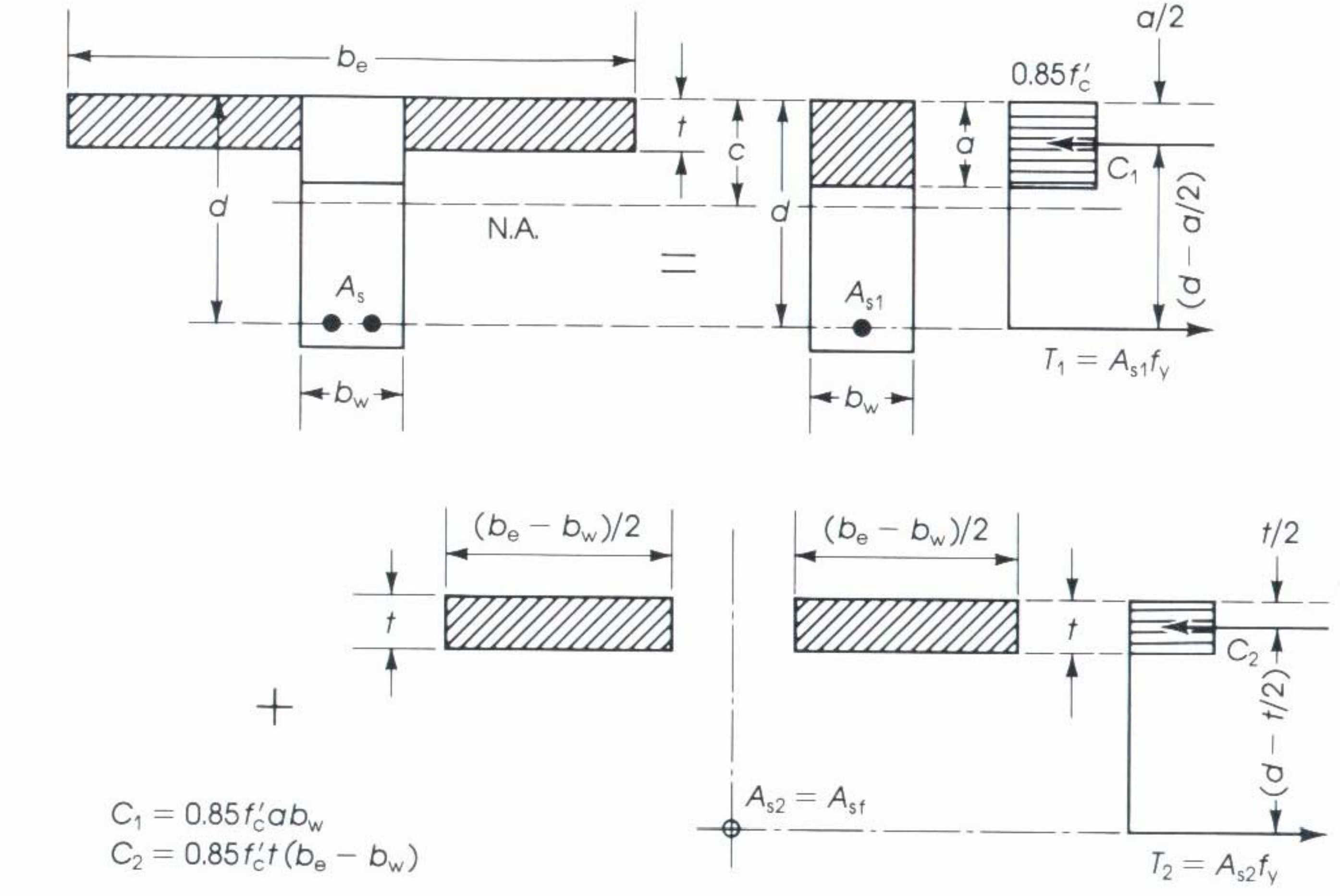
$$M_n = M_{n1} + M_{n2}$$

$$M_{n1} = \left(A_s - A_{sf}\right) f_y \left(d - \frac{a}{2}\right)$$

$$M_{n2} = A_{sf}\, f_y \left(d - \frac{h_f}{2}\right)$$

Analysis of T-Beams

The definition of M_{n1} and M_{n2} for the T-Beam are given as:



Analysis of T-Beams

The ultimate moment M_u for the T-Beam are given as:

$$M_u = \phi M_n$$

$$\phi = 0.9$$

For a T-Beam with the tension steel yielded.

Limitations on Reinforcement for Flange Beams

- Lower Limits
  - Flange in compression

$$\rho_{\min} = \frac{A_s}{b_w d} = \text{ larger of } \begin{cases} \dfrac{3\sqrt{f_c'}}{f_y} \\ \dfrac{200}{f_y} \end{cases}$$

Limitations on Reinforcement for Flange Beams

- Lower Limits
  - Flange in tension

$$A_{s(\min)} = \text{smaller of} \begin{cases} \dfrac{6\sqrt{f_c'}}{f_y} b_w d \\ \text{larger of} \begin{cases} \dfrac{3\sqrt{f_c'}}{f_y} b_{eff} d \\ \dfrac{200}{f_y} b_{eff} d \end{cases} \end{cases}$$

Limitations on Reinforcement for Flange Beams

- <u>Lower Limits</u>
  - If $A_{s(provided)} \geq 4/3\ A_{s(req'd)}$ based on analysis

    then $A_{s(min)}$ is not required (i.e.)

    $$\phi M_n \geq 4/3 M_u \text{ for } A_{s(provided)}$$

    See ACI 10.5.3

Example - T-Beam

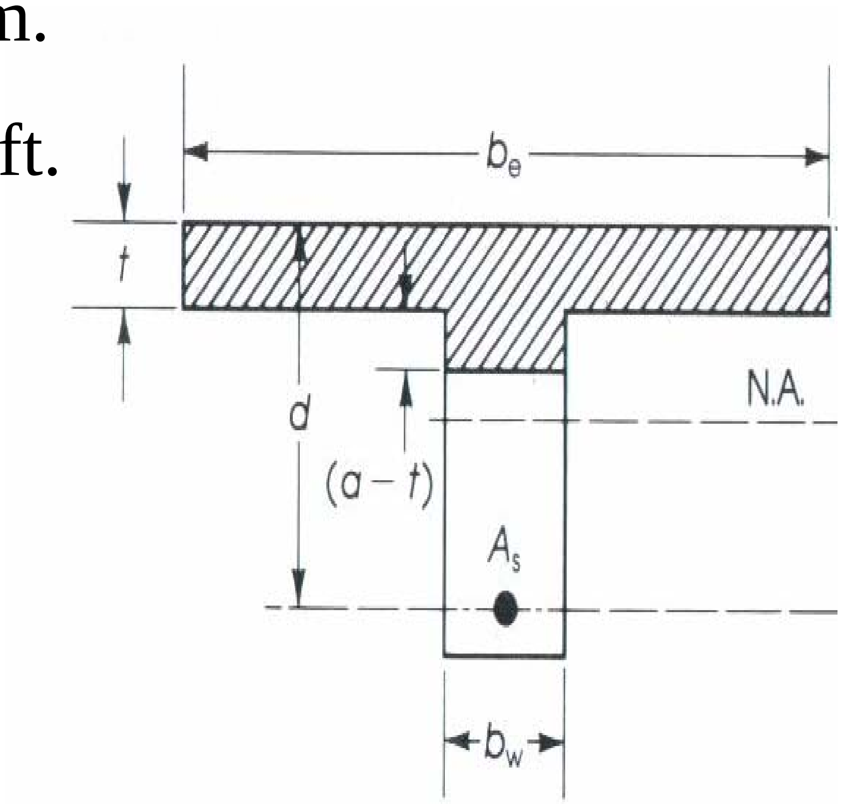
Find M_n and M_u for T-Beam.

b_{eff} = 54 in. h_f = 3 in. b = 7 ft.

d = 16.5 in. A_s = 8.5 in^2

f_y = 50 ksi f_c = 3 ksi

b_w= 12 in L = 18 ft



Example of L-Beam

Confirm b_{eff}

$$b_{eff} \leq \frac{L}{4} = \frac{18\text{ ft}\left(\dfrac{12\text{ in.}}{\text{ft.}}\right)}{4} = 54\text{ in.}$$

$$\leq 16h_f + b_w = 16\left(3\text{ in.}\right) + 12\text{ in.} = 60\text{ in.}$$

$$\leq b = 7\text{ ft.}\left(\frac{12\text{ in.}}{\text{ft.}}\right) = 84\text{ in.}$$

Example - T-Beam

Compute the equivalent c value and check the strain in the steel, ε_s

$$a = \frac{A_s f_y}{0.85 f_c' b_{eff}} = \frac{\left(8.5 \text{ in}^2\right)\left(50 \text{ ksi}\right)}{0.85\left(3 \text{ ksi}\right)\left(54 \text{ in.}\right)} = 3.09 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{3.09 \text{ in}}{0.85} = 3.63 \text{ in.}$$

$$\varepsilon_s = \left(\frac{d}{c} - 1\right)0.003 = \left(\frac{16.5 \text{ in.}}{3.63 \text{ in.}} - 1\right)0.003 = 0.0106 > 0.005$$

Steel will yield in the tension zone.

Example - T-Beam

Compute the reinforcement ρ and check to make sure it is greater than ρ_{min}

$$\rho = \frac{A_s}{b_w d} = \frac{\left(8.5 \text{ in}^2\right)}{\left(12 \text{ in.}\right)\left(16.5 \text{ in.}\right)} = 0.0429$$

$$\rho_{min} = \begin{cases} \dfrac{200}{f_y} = \dfrac{200}{50000} = 0.004 \\ \dfrac{3\sqrt{f_c}}{f_y} = \dfrac{3\sqrt{3000}}{50000} = 0.00329 \end{cases} \Rightarrow \rho_{min} = 0.004$$

$$\boxed{0.0429 > 0.004}$$

Section works for minimum reinforcement.

Example - T-Beam

Compute ω and check that the c value is greater than h_f

$$\varpi = \rho \frac{f_y}{f'_c} = 0.0429\left(\frac{50 \text{ ksi}}{3 \text{ ksi}}\right) = 0.7155$$

$$h_f \leq \frac{1.18\varpi d}{\beta_1} \Rightarrow 3 \text{ in.} \leq \frac{1.18(0.7155)(16.5 \text{ in.})}{0.85} = 16.388$$

$$h_f \leq a \Rightarrow 3 \text{ in.} \leq 3.09 \text{ in.}$$

Analysis the beam as a T-beam.

Example - T-Beam

Compute ω and check that the c value is greater than h_f

$$A_{sf} = \frac{0.85 f_c' \left(b_{eff} - b_w\right) h_f}{f_y} = \frac{0.85\left(3\text{ ksi}\right)\left(54\text{ in.} - 12\text{ in.}\right)\left(3\text{ in.}\right)}{\left(50\text{ ksi}\right)}$$

$$= 6.426\text{ in}^2$$

Compute a

$$a = \frac{\left(A_s - A_{sf}\right) f_y}{0.85 f_c' b_w} = \frac{\left(8.5\text{ in}^2 - 6.426\text{ in}^2\right)\left(50\text{ ksi}\right)}{0.85\left(3\text{ ksi}\right)\left(12\text{ in.}\right)}$$

$$= 3.889\text{ in.}$$

Example - T-Beam

Compute nominal moment components

$$M_{n1} = \left(A_s - A_{sf}\right) f_y \left(d - \frac{a}{2}\right)$$

$$= \left(8.5\ \text{in}^2 - 6.426\ \text{in}^2\right)\left(50\ \text{ksi}\right)\left(16.5\ \text{in.} - \frac{3.889\ \text{in.}}{2}\right)$$

$$= 1535.34\ \text{k-in.}$$

$$M_{n2} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = \left(6.426\ \text{in}^2\right)\left(50\ \text{ksi}\right)\left(16.5\ \text{in.} - \frac{3\ \text{in.}}{2}\right)$$

$$= 4819.5\ \text{k-in.}$$

Example - T-Beam

Compute nominal moment

$$M_{\mathrm{n}} = M_{\mathrm{n1}} + M_{\mathrm{n2}}$$

$$= 1535.34 \text{ k-in.} + 4819.5 \text{ k-in.}$$

$$= 6354.84 \text{ k-in.} \Rightarrow 529.57 \text{ k-ft.}$$

Compute ultimate moment

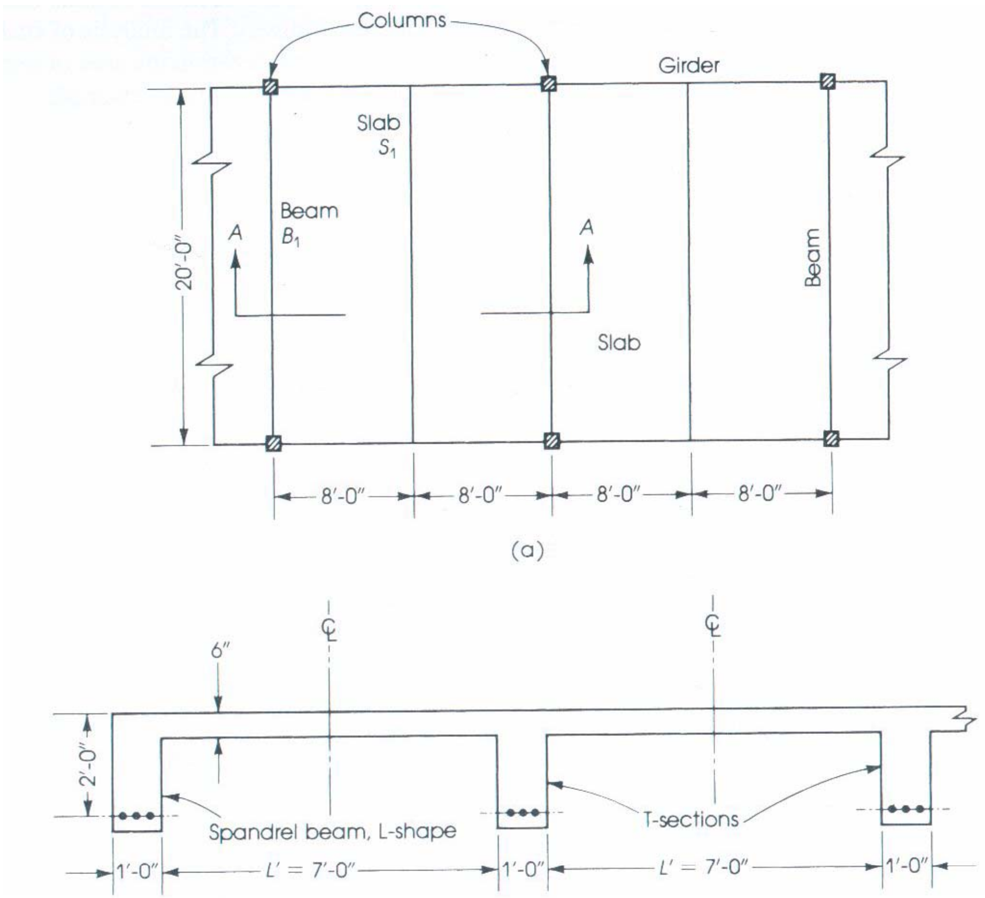
$$M_{\mathrm{u}} = \phi M_{\mathrm{n}} = 0.9\left(529.57 \text{ k-ft.}\right)$$

$$= 416.6 \text{ k-ft.}$$

Example of L-Beam

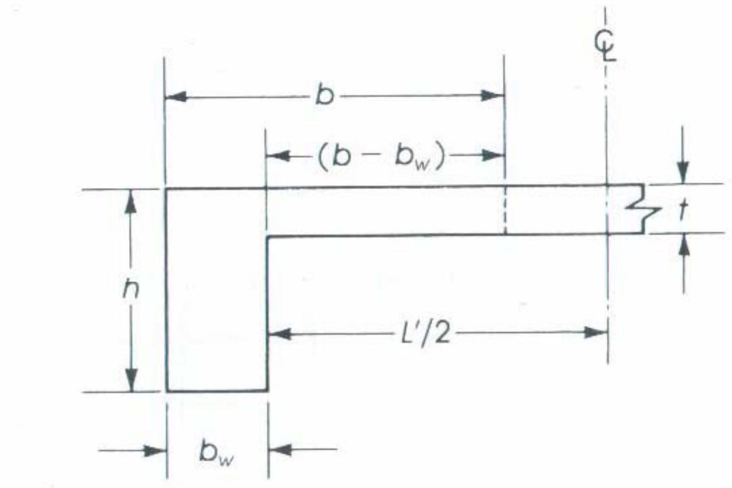
Determine the effective b for the spandrel beam and do the analysis.

Use 4 #9 bars and find the ultimate moment capacity. f_y=50 ksi, f_c = 3 ksi



Example of L-Beam

Compute b_{eff}

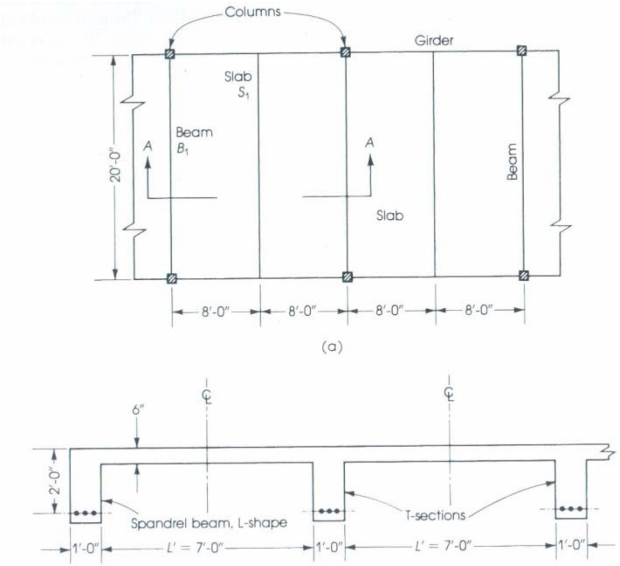


$$b_{eff} \leq \frac{L}{12} + b_w$$

$$\leq 6h_f + b_w$$

$$\leq b_{actual} = b_w + 0.5*(\text{clear distance to next web})$$

Example of L-Beam



Compute b_{eff}

$$b_{eff} \leq \frac{L}{12} + b_w = \frac{20\text{ ft}\left(\frac{12\text{ in.}}{\text{ft}}\right)}{12} + 12\text{ in.} = 32\text{ in.}$$

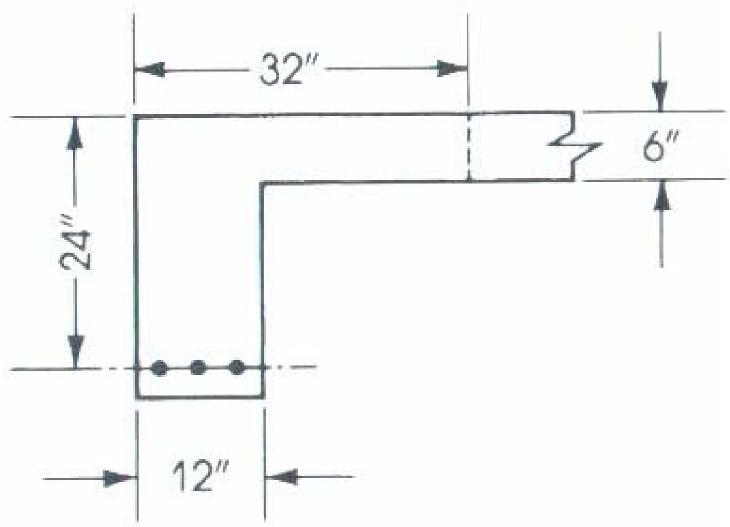
$$\leq 6h_f + b_w = 6\left(6\text{ in.}\right) + 12\text{ in.} = 48\text{ in.}$$

$$\leq b_{actual} = b_w + 0.5 * \left(\text{clear distance to next web}\right)$$

$$= 12\text{ in.} + 0.5 * \left(7\text{ ft}\left(\frac{12\text{ in.}}{\text{ft}}\right)\right) = 54\text{ in.}$$

Example of L-Beam

The value b_{eff} and A_s



$$b_{eff} = 32 \text{ in.}$$

$$A_s = 4\left(1.0 \text{ in}^2\right) = 4.0 \text{ in}^2$$

Example - L-Beam

Compute the equivalent c value and check the strain in the steel, ε_s

$$a = \frac{A_s f_y}{0.85 f_c' \, b_{\text{eff}}} = \frac{\left(4.0 \text{ in}^2\right)\left(50 \text{ ksi}\right)}{0.85\left(3 \text{ ksi}\right)\left(32 \text{ in.}\right)} = 2.45 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{2.45 \text{ in}}{0.85} = 2.88 \text{ in.}$$

$$\varepsilon_s = \left(\frac{d}{c} - 1\right)0.003 = \left(\frac{24 \text{ in.}}{2.88 \text{ in.}} - 1\right)0.003 = 0.0220 > 0.005$$

Steel will yield in the tension zone.

Example - L-Beam

Compute the reinforcement ρ and check to make sure it is greater than ρ_{min}

$$\rho = \frac{A_s}{b_w d} = \frac{\left(4.0 \text{ in}^2\right)}{\left(12 \text{ in.}\right)\left(24 \text{ in.}\right)} = 0.0139$$

$$\rho_{min} = \begin{cases} \dfrac{200}{f_y} = \dfrac{200}{50000} = 0.004 \\ \dfrac{3\sqrt{f_c}}{f_y} = \dfrac{3\sqrt{3000}}{50000} = 0.00329 \end{cases} \Rightarrow \rho_{min} = 0.004$$

$$0.0139 > 0.004$$

Section works for minimum reinforcement.

Example - T-Beam

Compute ω and check that the c value is greater than h_f

$$\varpi = \rho \frac{f_y}{f_c'} = 0.0139\left(\frac{50 \text{ ksi}}{3 \text{ ksi}}\right) = 0.2315$$

$$h_f \le \frac{1.18\varpi d}{\beta_1} \Rightarrow 6 \text{ in.} \le \frac{1.18(0.2315)(24 \text{ in.})}{0.85} = 7.71 \text{ in.}$$

$$h_f \le a \Rightarrow 6 \text{ in.} \le 2.45 \text{ in.}$$

False!

Analysis the beam as a Singly reinforced beam.

Example - L-Beam

Compute a

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{\left(4.0 \text{ in}^2\right)\left(50 \text{ ksi}\right)}{0.85\left(3 \text{ ksi}\right)\left(32 \text{ in.}\right)}$$

$$= 2.451 \text{ in.}$$

Example - L-Beam

Compute nominal moment

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

$$= \left(4.0 \text{ in}^2\right)\left(50 \text{ ksi}\right)\left(24.0 \text{ in.} - \frac{2.451 \text{ in.}}{2} \right)$$

$$= 4554.9 \text{ k-in.} \Rightarrow 379.58 \text{ k-ft.}$$

Example - L-Beam

Compute ultimate moment

$$M_{\mathrm{u}} = \phi M_{\mathrm{n}} = 0.9\left(379.58 \text{ k-ft.}\right)$$
$$= 341.62 \text{ k-ft.}$$

Homework

Work Problems

1. 5.10 parts a and b
2. 5.11 parts a and b
3. 5.12 parts a and b.