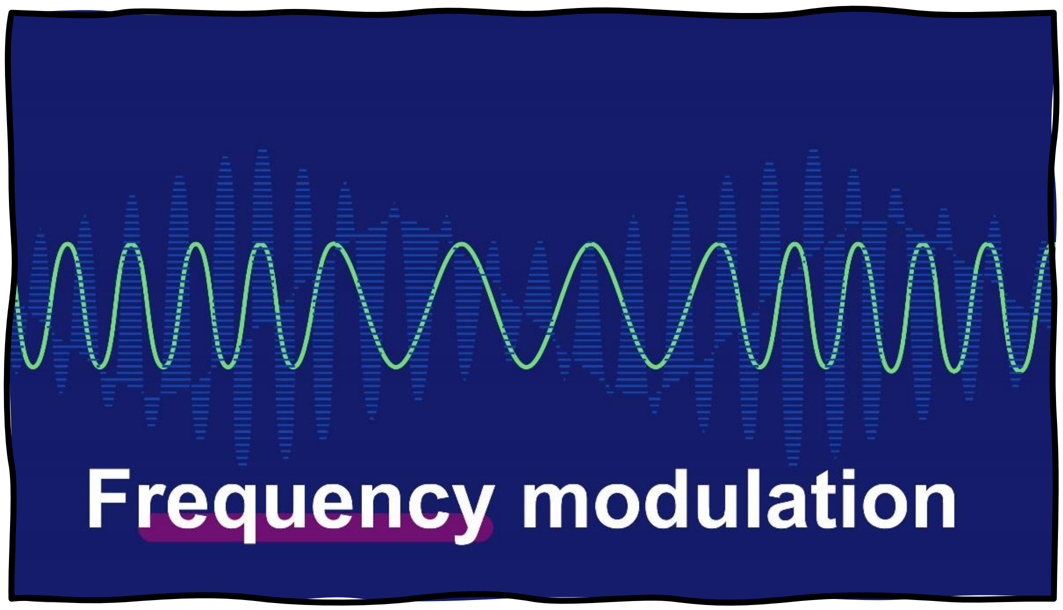
Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Prof. Dr. Bayan Mahdi Sabbar & M.Sc. Huda Wasfi Hassoon
Lecture: 7

Lecture 7

FM Modulation



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Introduction:

In Frequency Modulation (FM), the frequency of the carrier signal is varied according to the instantaneous value of the modulating signal, while the amplitude and phase remain constant. To generate a frequency-modulated signal, the carrier's frequency is adjusted based on the amplitude of the incoming audio signal. As the voltage of the modulating signal increases, the carrier frequency rises; as the voltage decreases, the carrier frequency lowers.

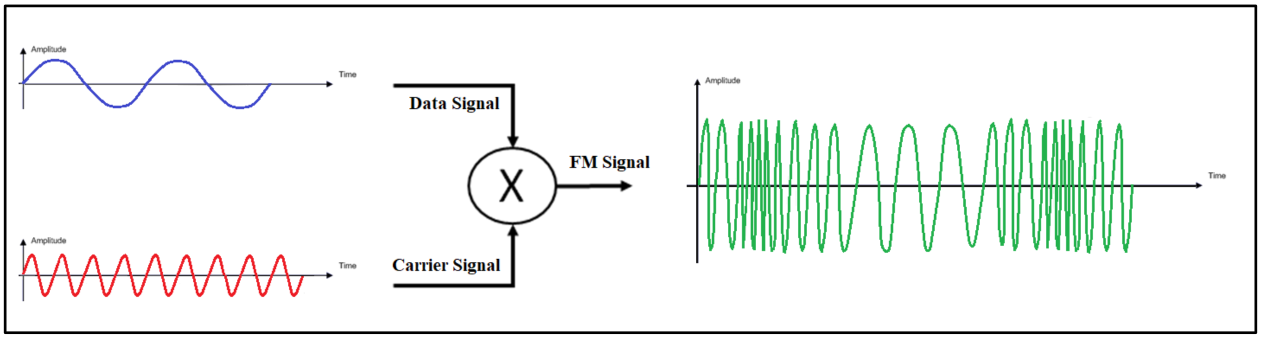

Fig. 1: FM Modulation.

When the audio signal is modulated onto the carrier wave, the new carrier frequency oscillates up and down. This change in frequency is called **frequency deviation**, typically measured in kilohertz (kHz). For instance, if the deviation is ±3 kHz, the carrier frequency shifts up by 3 kHz or down by 3 kHz from its original frequency. We can write an FM wave in the form:

$$E_{FM}(t) = E_c \cos\left(w_c t + \beta \sin(w_m t)\right)$$

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Modulation-index:

In modulation systems, the modulation index represents the extent to which the modulated parameter deviates from its unmodulated state. Specifically, in Frequency Modulation (FM), the modulation index measures the variation in the carrier frequency relative to its unmodulated frequency. It is mathematically defined as:

$$\boldsymbol{\beta = \frac{Peak\ frequency\ deviation}{modulating\ frequency}}$$

$$\boldsymbol{\beta = \frac{\Delta f}{f_m}}$$

Common Application:

Frequency Modulation (FM) is widely used in radio and television broadcasting. The FM spectrum is allocated for various applications. Analog television channels (0 to 72) occupy frequencies between 54 MHz and 825 MHz, where FM is used to transmit audio signals.

Additionally, FM radio operates within the 88 MHz to 108 MHz band. Each FM radio station utilizes a 200 kHz bandwidth, with approximately 38 kHz dedicated to audio transmission, while the remaining spectrum is used for stereo and auxiliary signal.

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The spectrum of FM:

Any modulated signal generates sidebands. While determining sidebands for an amplitude-modulated (AM) signal is straightforward, analyzing sidebands in frequency modulation (FM) is more complex. The sidebands in FM depend not only on the frequency deviation but also on the modulation index (β), which represents the ratio of the deviation to the modulating frequency.

The total spectrum of an FM signal comprises an infinite series of discrete spectral components, described mathematically using Bessel functions of the first kind. Based on Bessel functions, the signal
$s(t)$ can be expressed as a series expansion:

$$s(t) = E_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[(w_c + n w_m t)\right]$$

Where $J_n(\beta)$ is the n-th order Bessel function of the first kind. These functions can be computed by the series:

$$J_n(\beta) = \sum_{m=0}^{\infty} (-1)^m \frac{(\frac{1}{2}\beta)^{n+2m}}{m!\,(n+m)!}$$

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A summary of the Bessel function of the first kind, for order n and discrete values of the argument β, is provided in Table 5.1. Additionally, a graphical representation of the function is shown in Figure 2. It is important to note that for very small values of β, $J_0(\beta)$ approaches unity, while through $J_1(\beta)$ to $J_n(\beta)$ approach to zero.

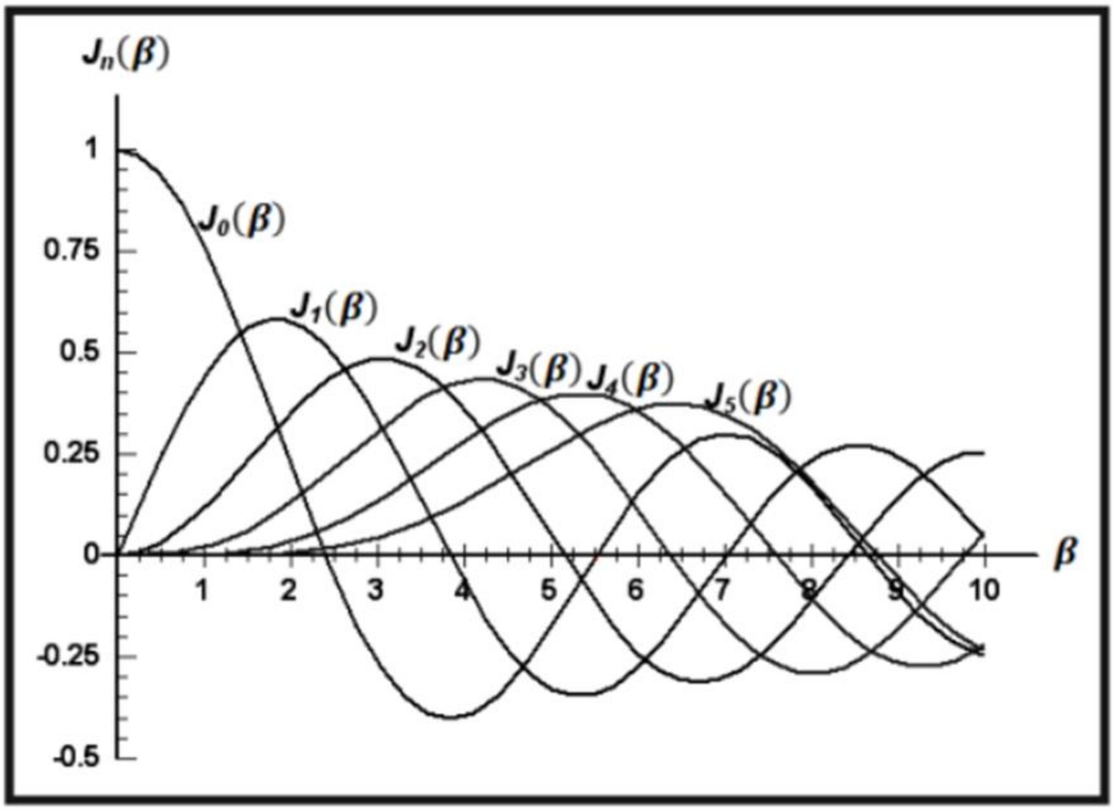

Fig. 2: Bessel function of kind 1 and of order 1 to 10.

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Table 1: Amplitude Variation of the Carrier and Sidebands at Different Modulation Indices.

Modulation Index	Sideband										
	carrier	1	2	3	4	5	6	7	8	9	10
0.00	1.00										
0.25	0.98	0.12									
0.5	0.94	0.24	0.03								
1.0	0.77	0.44	0.11	0.02							
1.5	0.51	0.56	0.23	0.06	0.01						
2.0	0.22	0.58	0.35	0.13	0.03						
2.41	0	0.52	0.43	0.20	0.06	0.02					
2.5	- 0.05	0.50	0.45	0.22	0.07	0.02	0.01				
3.0	- 0.26	0.34	0.49	0.31	0.13	0.04	0.01				
4.0	- 0.40	- 0.07	0.36	0.43	0.28	0.13	0.05	0.02			
5.0	- 0.18	- 0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02		
5.53	0	- 0.34	- 0.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01	
6.0	0.15	- 0.28	- 0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	
7.0	0.30	0.00	- 0.30	- 0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02
8.0	0.17	0.23	- 0.11	- 0.29	- 0.10	0.19	0.34	0.32	0.22	0.13	0.06

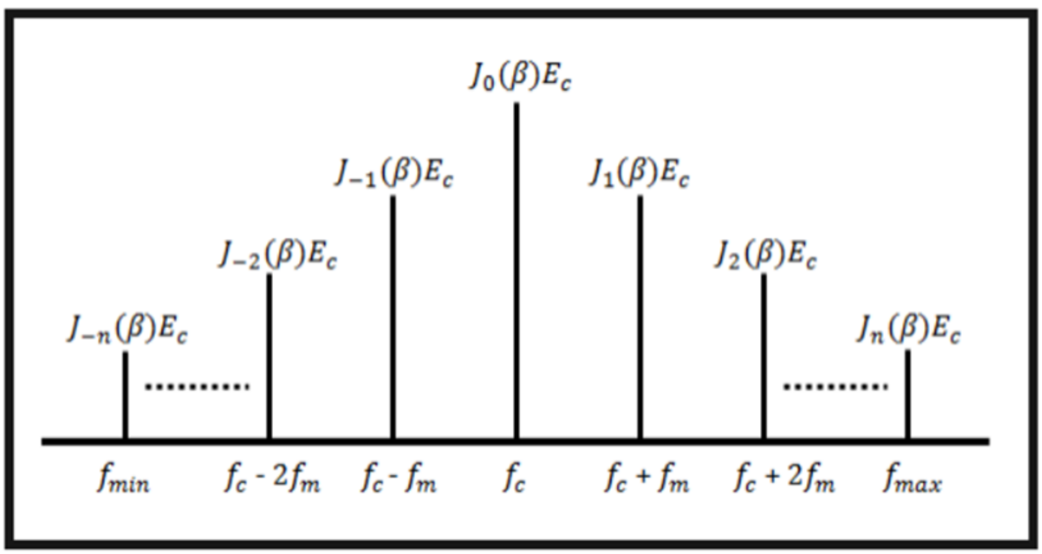

Fig. 3: Frequency Spectrum of FM Signal Showing Carrier and Sidebands at Different Harmonics.

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Important notes:

Infinite Sidebands and Bandwidth:

- An FM wave theoretically contains an infinite number of sidebands, which implies an infinite bandwidth requirement for transmission or reception.
- The sidebands are located at frequencies $(f_c \pm f_m)$, $(f_c \pm 2f_m)$, $(f_c \pm 3f_m)$, and so on.

Determination of Sidebands:

- The number of significant sideband components depends on the value of the modulation index (β).

Sideband Amplitudes:

- The amplitude of each sideband is determined by the coefficient $J_n(\beta)$, as provided in the Bessel table (e.g., Table 1).

Symmetry of Sidebands:

- The sidebands are symmetric around the carrier frequency (f_c).
- Two sidebands at equal distances from f_c will have identical amplitudes.
- A negative value of $J_n(\beta)$ indicates a phase difference of 180°.

Narrowband FM (NBFM) and Wideband FM (WBFM):

An FM signal with a low modulation index $(\beta \ll 1 \; or \; \Delta f \ll f_m)$ is referred to as a narrowband FM (NBFM) signal. For most practical purposes, the contributions of higher-order Bessel functions can be ignored, allowing its spectrum to be approximated as follows:

$$\boldsymbol{s(t) = E_c\, j_0(\beta)\cos(w_c t) + E_c\, j_1(\beta)\cos\big((w_c + w_m)t\big) - E_c\, j_{-1}(\beta)\, cos\big((w_c - w_m)t\big)}$$

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Narrowband FM shares similarities with AM in that it has sideband components at frequencies $(f_c \pm f_m)$, requiring a transmission bandwidth of $2f_m$. However, the key difference lies in the sideband components of narrowband FM, which are 180 degrees out of phase with each other, unlike in AM.

In contrast, an FM signal with a high modulation index ($\beta \gg 1 \ or \ \Delta f \gg f_m$)) is known as wideband FM (WBFM). The bandwidth of a wideband FM signal is approximately $2\Delta f$.

By combining the principles of both WBFM and NBFM, we arrive at Carson's Rule, which states that the minimum practical bandwidth required to transmit an FM signal is:

$$\boldsymbol{BW = 2(f_m + \Delta f)}$$

Bandwidth:

represents the range of frequencies required for signal transmission. The bandwidth of an FM signal depends on the modulation index (β), with higher modulation indices requiring a wider system bandwidth. The relationship between FM signal bandwidth and modulation index can be determined using the equations below:

$$\boldsymbol{Carson\ Law: BW = 2(\beta + 1)f_m}$$

$$\boldsymbol{Bessel\ Law: BW = 2nf_m}$$

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Power of FM signal:

The total power in the infinite spectrum is:

$$P_t = \frac{E_c{}^2}{2R} \sum_{n=-\infty}^{\infty} (J_n(\beta))^2 = \frac{E_c{}^2}{2R}$$

Hence, the carrier power is:

$$P_c = \frac{E_c{}^2}{2R} (J_0(\beta))^2$$

Generation of NBFM:

Narrowband FM (NBFM), similar to DSB-LC, is a type of linear modulation. An NBFM signal can be generated using a phase shifter and a balanced modulator, as illustrated in the figure below.

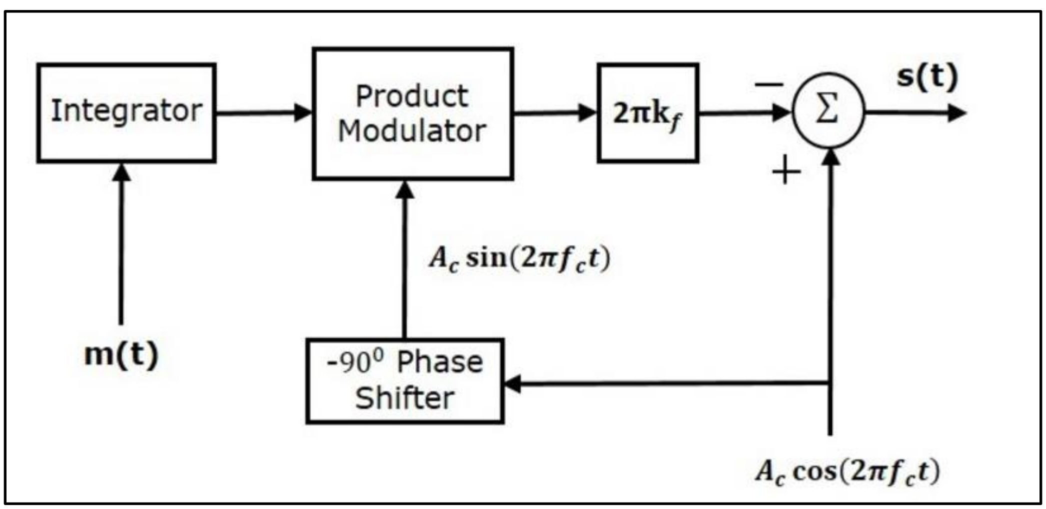

Fig. 4: The block diagram of NBFM modulator.

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Generation of WBFM:

Wideband FM signals can be generated using two primary methods:

Indirect Method:

This approach involves first generating a narrowband FM (NBFM) signal. The modulation index is then increased to the desired range by applying frequency multiplication. This technique is commonly referred to as the indirect method of generating wideband FM signals.

Direct Method:

In this method, the carrier frequency is directly varied in accordance with the modulating signal. This is known as the direct method of generating wideband FM signals.

1- Indirect Method

This approach is referred to as the Indirect Method because a wideband FM (WBFM) signal is generated indirectly. The process begins with the generation of a narrowband FM (NBFM) signal, which is then converted into a WBFM signal using frequency multipliers.

The block diagram for generating a WBFM signal is shown below. It consists of two main stages:

i. **Generation of NBFM:**

In the first stage, an NBFM signal is produced using an NBFM modulator. The block diagram for this modulator was discussed in the previous section. The NBFM signal has a modulation index of less than one ($\beta < 1$).

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ii. Frequency Multiplication:

To achieve the desired modulation index ($\beta > 1$) for a WBFM signal, an appropriate frequency multiplier is used. A frequency multiplier is a nonlinear device that outputs a signal with a frequency that is 'n' times the input signal frequency, where n is the multiplication factor.

By selecting the correct value of n, the modulation index can be increased to meet the requirements for wideband FM.

This revision improves readability and maintains technical accuracy while presenting the information in a structured manner.

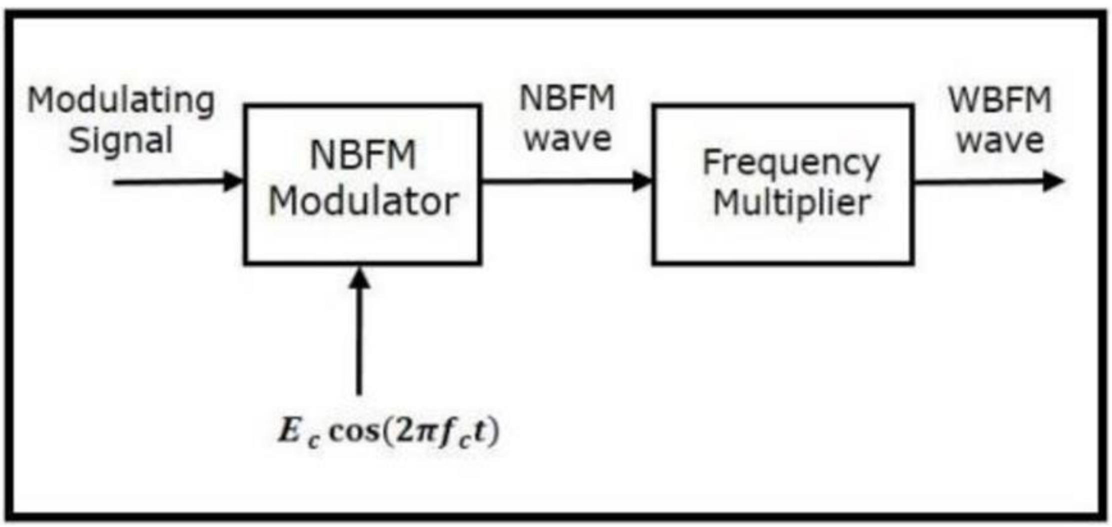

Fig .5: Indirect method for WBFM Generation.

2- Direct Method

This method is called as the Direct Method because we are generating a wide band FM wave directly. In this method, Voltage Controlled Oscillator (VCO) is used to generate WBFM.VCO produces an output signal, whose frequency is proportional

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to the input signal voltage. This is similar to the definition of FM wave. The block diagram of the generation of WBFM wave is shown in the following figur**e.**

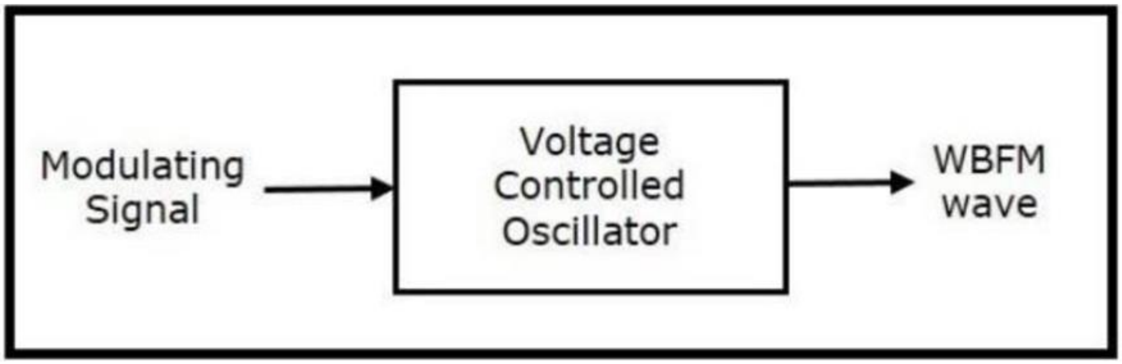

Fig. 6: Direct method for WBFM Generation.

Demodulation of FM

The most popular method for FM demodulation is the Phase Locked Loop (PLL). The working of a PLL FM demodulator is very easy to understand. The input FM signal and the output of the VCO are applied to the phase detector circuit.

The output of the phase detector is filtered using a low pass filter, the amplifier and then used for controlling the VCO. When there is no carrier modulation and the input FM signal is in the center of the pass band (i.e. carrier wave only) the VCO's tune line voltage will be at the center position. When deviation in carrier frequency occurs (that means modulation occurs) the VCO frequency follows the input signal in order to keep the loop in lock. As a result, the tune line voltage to the VCO varies and this variation is proportional to the modulation done to the FM carrier wave. This voltage variation is filtered and amplified in order to get the demodulated signal.

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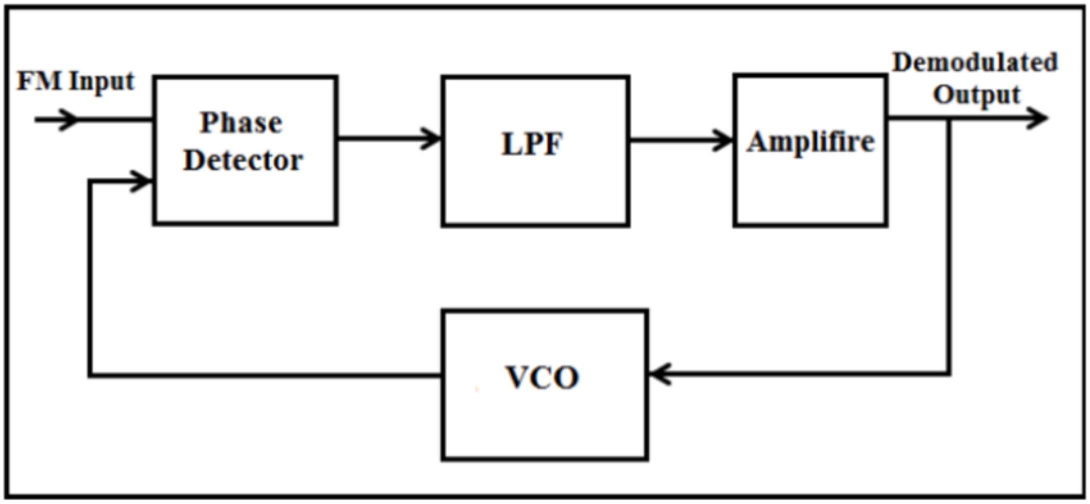

Fig.7: Block diagram of FM Demodulation.

Example: A 10 MHz carrier is frequency modulated by a pure signal tone of frequency 8 kHz. The frequency deviation is 32 KHz. Calculate the bandwidth of the resulting FM waveform.

Solution:

$BW = 2(fm + \Delta f)$

$BW = 2(8 + 32)$

$BW = 80\text{KHz}$

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