Lecture 10 - Flexure

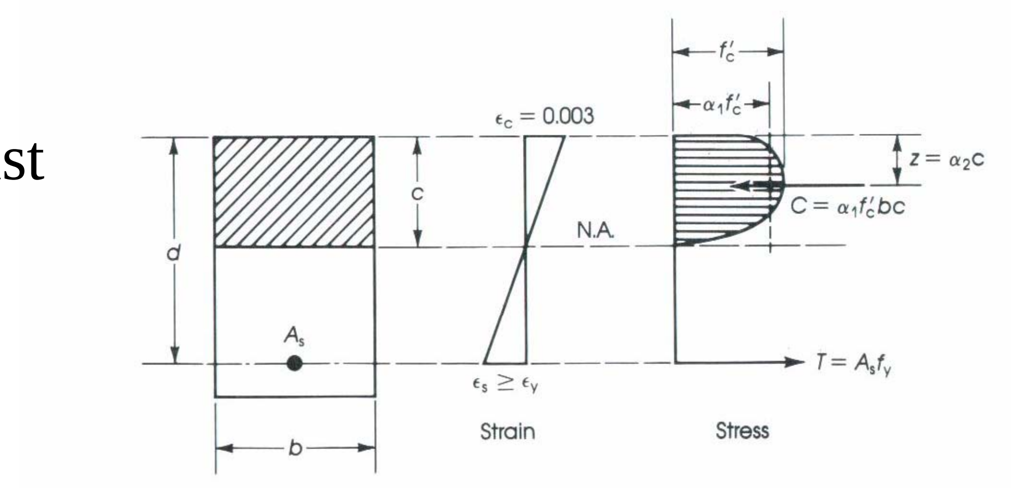
Lecture Goals

- Doubly Reinforced beams

Analysis of Doubly Reinforced Sections

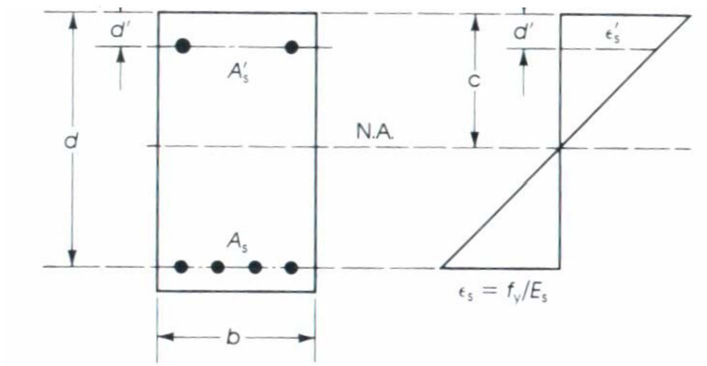
Effect of Compression Reinforcement on the Strength and Behavior

Less concrete is needed to resist the T and thereby moving the neutral axis (NA) up.



$$T = A_s f_y$$

$$C = T$$

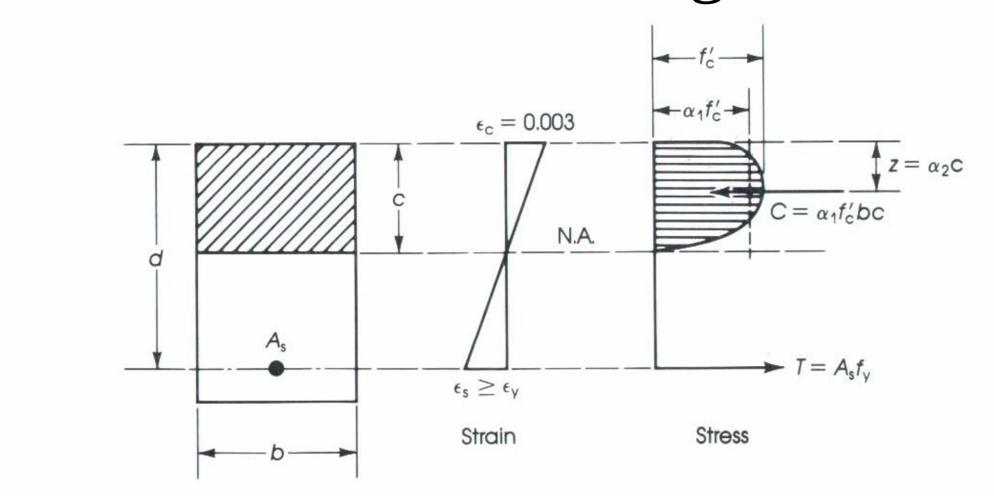


Analysis of Doubly Reinforced Sections

Effect of Compression Reinforcement on the Strength and Behavior

Singly Reinforced $\Rightarrow$

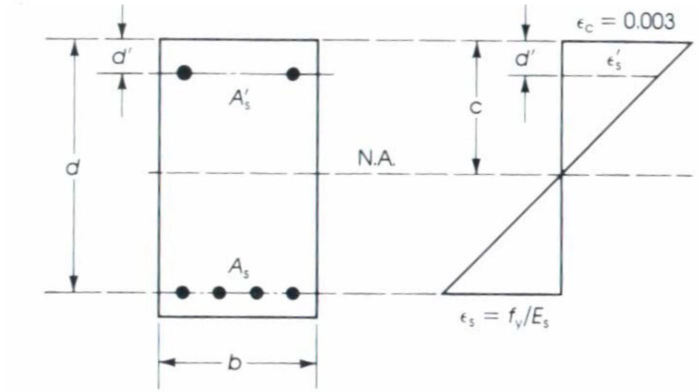
$$C = C_{\mathrm{c}} \;\; ; M_n = A_s f_y \left(d - \frac{a_1}{2} \right)$$



Doubly Reinforced $\Rightarrow$

$$C = C_{\mathrm{c}} + \mathrm{C}'_{\mathrm{s}} \;\; ; M_n = A_s f_y \left(d - \frac{a_2}{2} \right)$$

and $(a_2 < a_1)$



Reasons for Providing Compression Reinforcement

- Reduced sustained load deflections.
  - Creep of concrete in compression zone
  - transfer load to compression steel
  - reduced stress in concrete
  - less creep
  - less sustained load deflection

Reasons for Providing Compression Reinforcement

Effective of compression reinforcement on sustained load deflections.

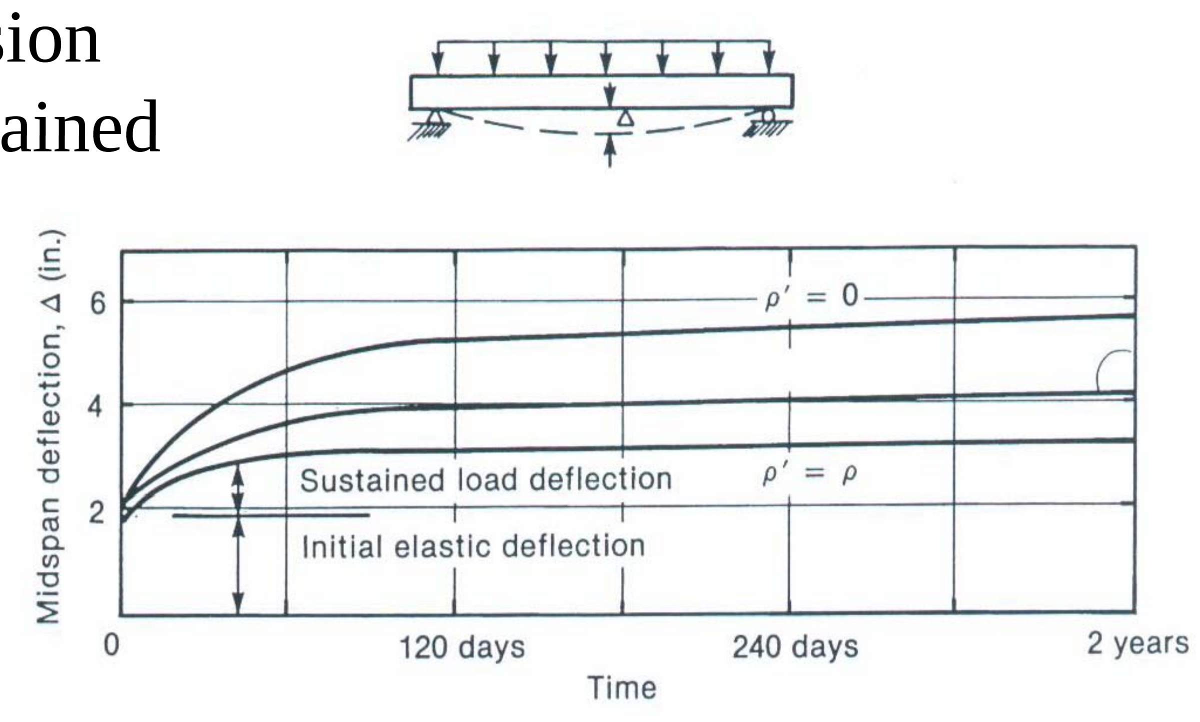


Fig 5-14 MacGregor

Reasons for Providing Compression Reinforcement

- Increased Ductility

  reduced stress block depth $\longrightarrow$ increase in steel strain larger curvature are obtained.

Reasons for Providing Compression Reinforcement

Effect of compression reinforcement on strength and ductility of under reinforced beams.

$\rho < \rho_b$

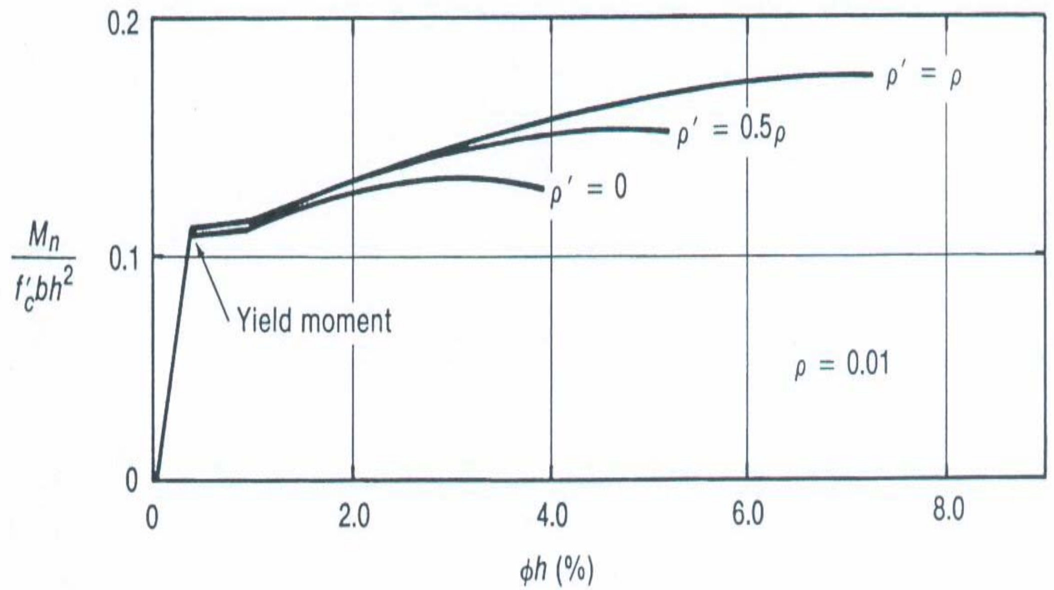


Fig 5-15 MacGregor

Reasons for Providing Compression Reinforcement

- Change failure mode from compression to tension. When $\rho > \rho_{bal}$, addition of As strengthens.

  Compression zone $\longrightarrow$ allows tension steel to yield before crushing of concrete.

  Effective reinforcement ratio $= (\rho - \rho')$

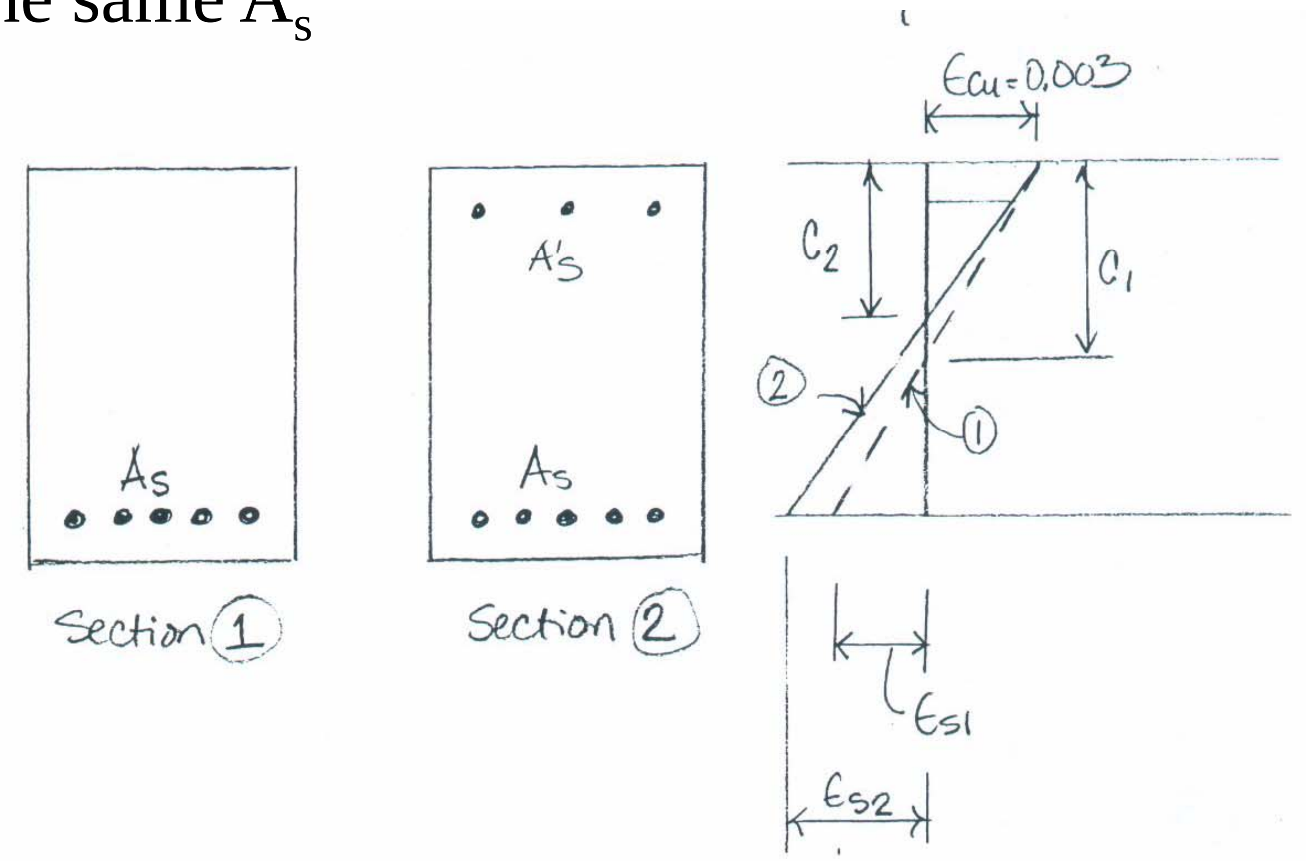
Reasons for Providing Compression Reinforcement

- Eases in Fabrication

  use corner bars to hold & anchor stirrups.

Effect of Compression Reinforcement

Compare the strain distribution in two beams with the same A_s



Effect of Compression Reinforcement

Section 1:

$$T = A_s f_s$$

$$T = C_{c1} = 0.85 f_c' ba = 0.85 f_c' b\beta_1 c_1$$

$$c_1 = \frac{A_s f_s}{0.85 f_c' b\beta_1}$$

Section 2:

$$T = A_s f_s$$

$$T = C_s' + C_{c1}$$

$$= A_s' f_s' + 0.85 f_c' ba_2$$

$$= A_s' f_s' + 0.85 f_c' b\beta_1 c_2$$

$$c_2 = \frac{A_s f_s - A_s' f_s'}{0.85 f_c' b\beta_1}$$

Addition of A'_s strengthens compression zone so that less concrete is needed to resist a given value of T. ⟶ NA goes up ($c_2 < c_1$) and ε_s increases ($\varepsilon_{s2} > \varepsilon_{s1}$).

Doubly Reinforced Beams

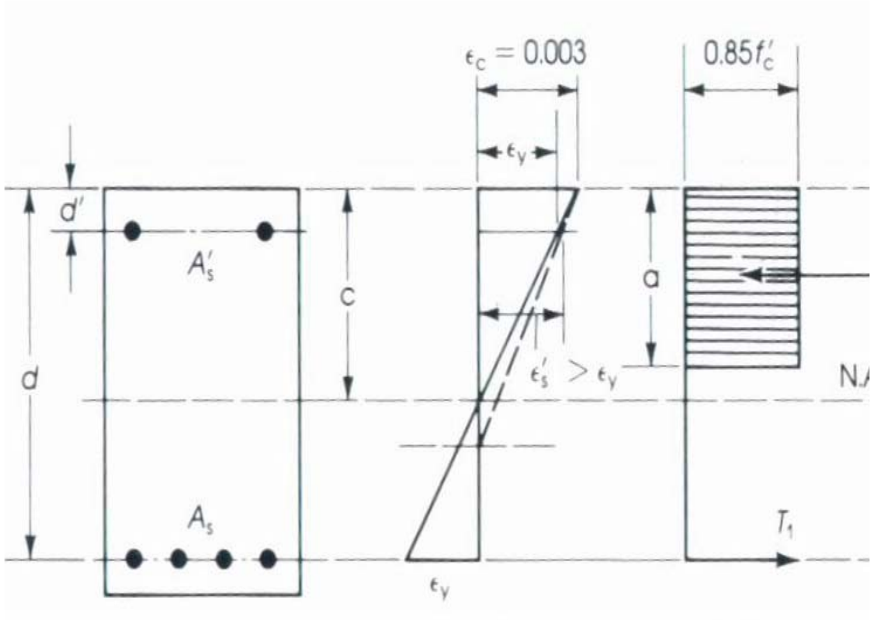
Four Possible Modes of Failure

- Under reinforced Failure
  - (Case 1) Compression and tension steel yields
  - (Case 2) Only tension steel yields
- Over reinforced Failure
  - (Case 3) Only compression steel yields
  - (Case 4) No yielding Concrete crushes

Analysis of Doubly Reinforced Rectangular Sections

<u>Strain Compatibility Check</u>

Assume ε_s' using similar triangles



$$\frac{\varepsilon_s}{(c-d')} = \frac{0.003}{c} \Rightarrow$$

$$\varepsilon_s = \frac{(c-d')*0.003}{c}$$

Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

Using equilibrium and find a

$$T = C_c' + C_s' \Rightarrow a = \frac{\left(A_s - A_s'\right) f_y}{0.85 f_c' b}$$

$$c = \frac{a}{\beta_1} = \frac{\left(A_s - A_s'\right) f_y}{\beta_1 \left(0.85 f_c' b\right)} = \frac{\left(\rho - \rho '\right) d \; f_y}{\beta_1 \left(0.85 f_c'\right)}$$

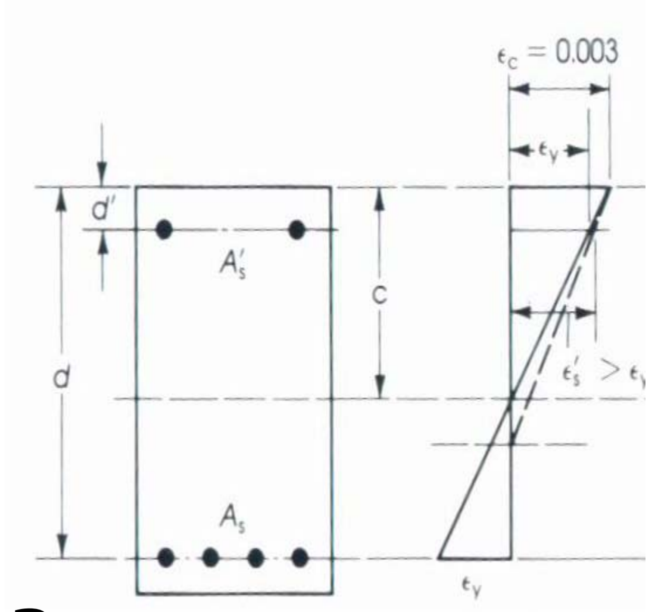
Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

The strain in the compression steel is

$$\varepsilon_s' = \left(1 - \frac{d'}{c}\right)\varepsilon_{cu}$$

$$= \left(1 - \frac{\beta_1\left(0.85 f_c'\right) d'}{\left(\rho - \rho'\right) d\ f_y}\right) 0.003$$

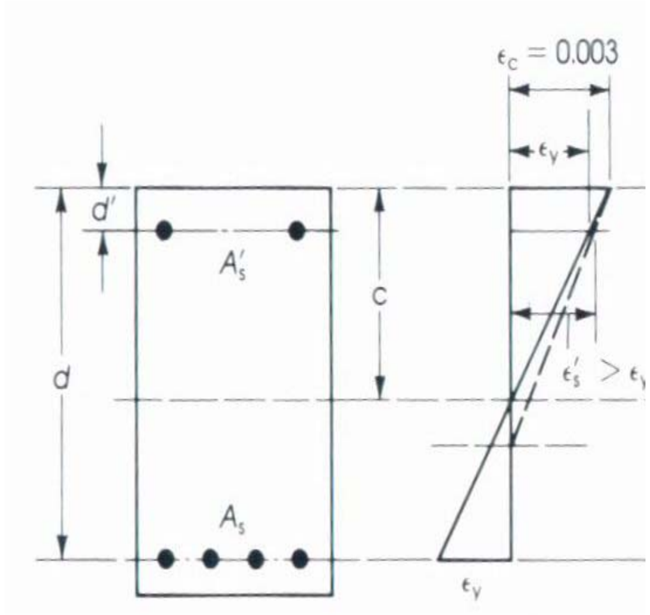


Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

Confirm

$$\varepsilon_s' \geq \varepsilon_y = \frac{f_y}{E_s}; \qquad \varepsilon_s \geq \varepsilon_y$$



$$\varepsilon_s' = \left(1 - \frac{\beta_1 \left(0.85 f_c'\right) d'}{\left(\rho - \rho'\right) d\ f_y}\right) 0.003 \geq \frac{f_y}{E_s} = \frac{f_y}{29 \text{ x } 10^6}$$

Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

Confirm

$$-\frac{\beta_1\left(0.85 f_c'\right)d'}{\left(\rho-\rho'\right)d\ f_y} \geq \frac{f_y - 87000}{87000}$$

$$\left(\rho-\rho'\right) \geq \left(\frac{\beta_1\left(0.85 f_c'\right)d'}{d\ f_y}\right)\left(\frac{87000}{87000 - f_y}\right)$$

Analysis of Doubly Reinforced Rectangular Sections

If the statement is true than

$$M_n = \left(A_s - A_s'\right) f_y \left(d - \frac{a}{2}\right) + A_s' f_y \left(d - d'\right)$$

else the strain in the compression steel

$$f_s = E\varepsilon_s$$

Analysis of Doubly Reinforced Rectangular Sections

<u>Strain</u>

Compute the stress in the compression steel.

$$f_s' = 29 \text{ x } 10^6 \left(1 - \frac{\beta_1 \left(0.85 f_c' \right) d'}{\left(\rho - \rho' \right) d \; f_y} \right) 0.003$$

Analysis of Doubly Reinforced Rectangular Sections

Go back and calculate the equilibrium with f_s'

$$T = C_c' + C_s' \Rightarrow a = \frac{\left(A_s f_y - A_s' f_s\right)}{0.85 f_c' b}$$

$$c = \frac{a}{\beta_1}$$

Iterate until the c value is adjusted for the f_s'

$$f_s' = \left(1 - \frac{d'}{c}\right) 87000$$

Analysis of Doubly Reinforced Rectangular Sections

Go back and calculate the moment capacity of the beam

$$M_{n} = \left(A_{s} f_{y} - A'_{s} f'_{s} \right)\left(d - \frac{a}{2} \right) + A'_{s} f'_{s} \left(d - d' \right)$$

Limitations on Reinforcement Ratio for Doubly Reinforced beams

Lower limit on ρ (ACI 10.5)

same as for single reinforce beams.

$$\rho_{\min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}$$

Example: Doubly Reinforced Section

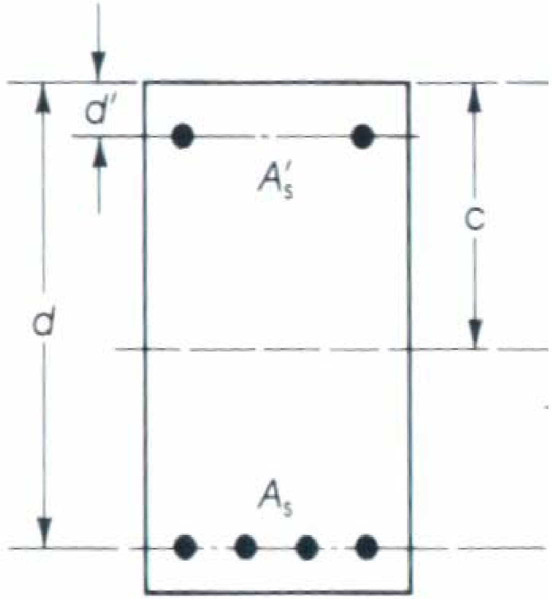
Given:

f'_c= 4000 psi f_y = 60 ksi

A'_s = 2 #5 A_s = 4 #7

d'= 2.5 in. d = 15.5 in

h=18 in. b =12 in.



Calculate M_n for the section for the given compression steel.

Example: Doubly Reinforced Section

Compute the reinforcement coefficients, the area of the bars #7 (0.6 in^2) and #5 (0.31 in^2)

$$A_s = 4\left(0.6 \text{ in}^2\right) = 2.4 \text{ in}^2$$

$$A_s' = 2\left(0.31 \text{ in}^2\right) = 0.62 \text{ in}^2$$

$$\rho = \frac{A_s}{bd} = \frac{2.4 \text{ in}^2}{\left(12 \text{ in.}\right)\left(15.5 \text{ in.}\right)} = 0.0129$$

$$\rho' = \frac{A_s'}{bd} = \frac{0.62 \text{ in}^2}{\left(12 \text{ in.}\right)\left(15.5 \text{ in.}\right)} = 0.0033$$

Example: Doubly Reinforced Section

Compute the effective reinforcement ratio and minimum ρ

$$\rho_{eff} = \rho - \rho' = 0.0129 - 0.0033 = 0.00957$$

$$\rho = \frac{200}{f_{\mathrm{y}}} = \frac{200}{60000} = 0.00333 \text{ or } \frac{3\sqrt{f_{\mathrm{c}}}}{f_{\mathrm{y}}} = \frac{3\sqrt{4000}}{60000} = 0.00316$$

$$\rho \geq \rho_{\mathrm{min}} \Rightarrow 0.0129 \geq 0.00333 \text{ OK!}$$

Example: Doubly Reinforced Section

Compute the effective reinforcement ratio and minimum ρ

$$(\rho - \rho') \geq \left(\frac{\beta_1 (0.85 f_c') d'}{d\ f_y} \right) \left(\frac{87000}{87000 - f_y} \right)$$

$$\geq \left(\frac{0.85(0.85(4\text{ ksi}))(2.5\text{ in.})}{60\text{ ksi}(15.5\text{ in.})} \right) \left(\frac{87}{87 - 60} \right) = 0.0398$$

$0.00957 \not\geq 0.0398$ Compression steel has not yielded.

Example: Doubly Reinforced Section

Compute the effective reinforcement ratio and minimum ρ

$$f_s' = 29 \text{ x } 10^6 \left(1 - \frac{\beta_1 \left(0.85 f_c'\right) d'}{\left(\rho - \rho'\right) d \ f_y} \right) 0.003$$

$$= 87 \left(1 - \frac{0.85\left(0.85\left(4 \text{ ksi}\right)\right)\left(2.5 \text{ in.}\right)}{\left(0.00957\right)\left(15.5 \text{ in.}\right)\left(60 \text{ ksi}\right)} \right) = 16.37 \text{ ksi}$$

Use an iterative technique to find f'_s

Example: Doubly Reinforced Section

Compute the iterative values (1)

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(16.37 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(0.85\right)\left(12 \text{ in.}\right)}$$

$$c = 3.86 \text{ in.}$$

$$f_s' = E_s \varepsilon_s' = 29 \text{ x } 10^6 \left(1 - \frac{d'}{c}\right) 0.003 = 87\left(1 - \frac{\left(2.5 \text{ in.}\right)}{\left(3.86 \text{ in.}\right)}\right)$$

$$= 30.65 \text{ ksi}$$

Use an iterative technique to find f’$_s$

Example: Doubly Reinforced Section

Compute the iterative values (2)

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(30.65 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(0.85\right)\left(12 \text{ in.}\right)}$$

$$c = 3.604 \text{ in.}$$

$$f_s' = E_s \varepsilon_s' = 29 \text{ x } 10^6 \left(1 - \frac{d'}{c}\right) 0.003 = 87\left(1 - \frac{\left(2.5 \text{ in.}\right)}{\left(3.604 \text{ in.}\right)}\right)$$

$$= 26.66 \text{ ksi}$$

Use an iterative technique to find f'_s

Example: Doubly Reinforced Section

Compute the iterative values (2)

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4\ \text{in}^2\right)\left(60\ \text{ksi}\right) - \left(0.62\ \text{in}^2\right)\left(30.66\ \text{ksi}\right)}{0.85\left(4\ \text{ksi}\right)\left(0.85\right)\left(12\ \text{in.}\right)}$$

$$c = 3.676\ \text{in.}$$

$$f_s' = E_s \varepsilon_s' = 29 \text{ x } 10^6 \left(1 - \frac{d'}{c}\right) 0.003 = 87\left(1 - \frac{\left(2.5\ \text{in.}\right)}{\left(3.676\ \text{in.}\right)}\right)$$

$$= 27.82\ \text{ksi}$$

Use an iterative technique to find f'_s

Example: Doubly Reinforced Section

Compute the iterative values (etc)

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(27.49 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(0.85\right)\left(12 \text{ in.}\right)}$$

$$c = 3.66 \text{ in.}$$

$$f_s' = E_s \varepsilon_s' = 29 \text{ x } 10^6 \left(1 - \frac{d'}{c}\right) 0.003 = 87\left(1 - \frac{\left(2.5 \text{ in.}\right)}{\left(3.66 \text{ in.}\right)}\right)$$

$$= 27.57 \text{ ksi}$$

Use f'_s=27.56 ksi c=3.66 in

Example: Doubly Reinforced Section

Compute the moment capacity of the beam

$$M_n = \left(A_s f_y - A_s' f_s'\right)\left(d - \frac{a}{2}\right) + A_s' f_s' \left(d - d'\right)$$

$$= \left(\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(27.56 \text{ ksi}\right)\right)\left(15.5 \text{ in.} - \frac{0.85\left(3.66 \text{ in.}\right)}{2}\right)$$

$$+ \left(0.62 \text{ in}^2\right)\left(27.56 \text{ ksi}\right)\left(15.5 \text{ in.} - 2.5 \text{ in.}\right)$$

$$= 1991.9 \text{ k-in.} \Rightarrow 166 \text{ k-ft}$$