Electrical Engineering Fundamentals

EE1

1 Basic Concepts & Basic Laws

1.1 Basic Concepts

1.1.1 System of Units

The basic SI units

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

The SI prefixes

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Examples

$10\ MHz \Rightarrow 10 \times 10^{6}\ Hz$

$2\ mA = 2 \times 10^{-3} = 0.002\ A$

$5\ \mu s = 5 \times 10^{-6}\ s$

1.1.2 Charge and Current

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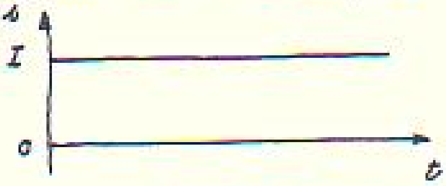
The electric charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C). The charge of an electron is (-1.602×10^{-19} C).

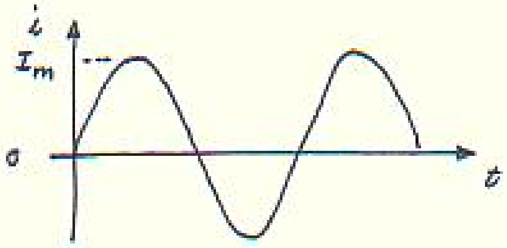
Electric Current: is the rate of change of charge, measured in amperes (A). The current (I) is defined mathematically as:

$$\boxed{i = \frac{dq}{dt}}$$



d.c current

$$\therefore \; q = \int_{t_1}^{t_2} i \, dt$$



a.c current

* A direct current (dc) is current that remains constant with time. The symbol (I) is usually used to represent such a constant current.

* An alternating current (ac) is a current that is varying sinusoidally with time. A time varying current is represented by the symbol (i).

Example: Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s, if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$q = \int_{t_1}^{t_2} i \, dt$$

$$= \int_1^2 (3t^2 - t) \, dt = \left(t^3 - \frac{t^2}{2} \right) \Big|_1^2$$

$$= (8 - 2) - \left(1 - \frac{1}{2}\right) = \underline{\underline{5.5 \text{ C}}}$$

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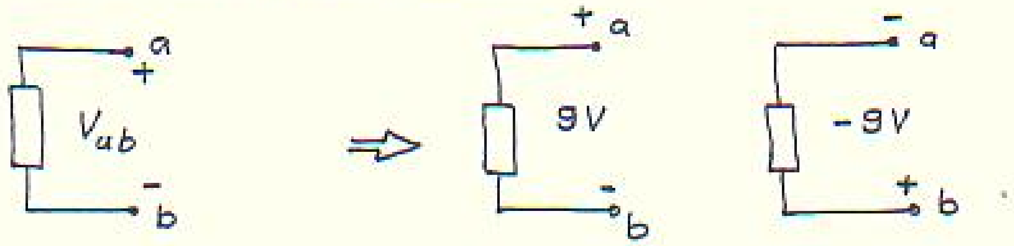
1.1.3 Voltage

: The voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).



Polarity of Voltage V_{ab}

* For the voltage V_{ab} $\Rightarrow$ This means that the potential of point $\underline{\underline{a}}$ is higher than that of point $\underline{\underline{b}}$

$$\boxed{V_{ab} = V_a - V_b}$$

1.1.4 Power and Energy

:

* Power : is the time rate of expending or absorbing energy, measured in watts (W)

$$\Rightarrow \quad p = \frac{dw}{dt}$$

where p is the power in watts (W), w is the energy in joules (J), and t is the time in seconds (s)

We have; $p = \dfrac{dw}{dt} \quad \Longrightarrow \quad p = \dfrac{dw}{dt} \cdot \dfrac{dq}{dq}$

$$\Rightarrow p = \frac{dw}{dq} \cdot \frac{dq}{dt} = v.i$$

$$\boxed{\therefore \; p = vi}$$

* The energy absorbed or supplied by an element

from time t_o to time t is :

$$\boxed{w = \int_{t_o}^{t} p\,dt = \int_{t_o}^{t} vi\,dt}$$

Energy is the capacity to do work, measured in joules (J)

* The electric power utility companies measure energy in watt-hour (Wh), where

$$\boxed{1\ \text{Wh} = 3{,}600\ \text{J}}$$

Example : How much energy does a 100 W electric bulb consume in 2 hours ?

Solution :

$$w = pt = 100 \times 2 = 200\ \text{Wh}$$

or

$$w = pt = (100\,\text{W})(2 \times 60 \times 60)$$
$$= 720000\ \text{J}$$
$$= 720\ \text{kJ}$$

which is the same result (if you convert from joules to watts or vise-versa).

1.1.5 Circuit Elements

: An electric circuit is an interconnection of electrical elementic.

* Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

Circuit Elements
- Passive Elements (can not generate energy) ⇒ Resistors, capacitors, and inductors
- Active Elements (can generate energy) ⇒ Generators, batteries, operational amplifiers

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* The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them.

Voltage (or current) sources
- Independent sources
- Dependent sources

* An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables.

* Dependent sources (or controlled sources) are active elements in which the source quantity is controlled by another voltage or current. (It will be discussed later).

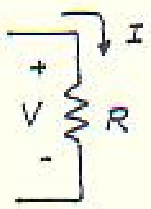
1.2 Basic Laws :

1.2.1 Ohm's Law : (1826)

Ohm's Law states that the voltage V across a resistor is directly proportional to the current I flowing through the resistor.

$V \propto I$

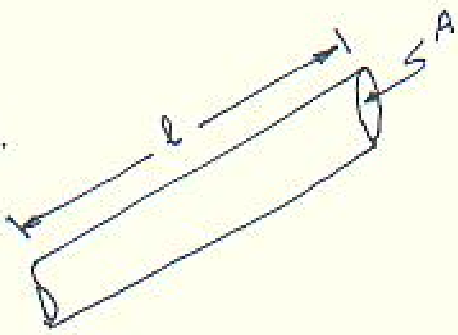
$\Rightarrow \boxed{V = IR}$ قانون أوم



where R is the resistance. The resistance R denotes the ability of an element to resist the flow of electric current, it is measured in ohms (Ω).

For any material, the resistance R depends on its physical dimensions as follows :

$$R = \rho \frac{l}{A}$$



where ρ is the resistivity of the material.

$\Rightarrow$ Good conductors have low resistivities (such copper, aluminum, etc..)

$\Rightarrow$ Insulators have high resistivities (such as mica, paper, etc...)

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Resistivities of Common Materials

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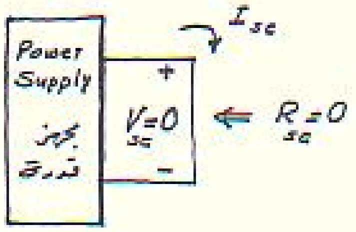
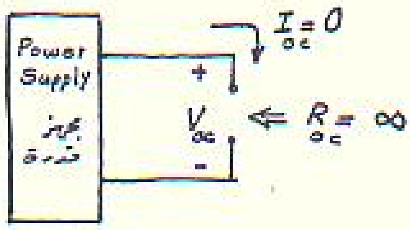
Material	Resistivity ($\Omega.m$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	"
Aluminum	2.80×10^{-8}	"
Gold	2.45×10^{-8}	"
Carbon	4.00×10^{-5}	Semiconductor
Germanium	47.0×10^{-2}	"
Silicon	6.40×10^{-2}	"
Paper	10^{10}	Insulator
Mica	5×10^{11}	"
Glass	10^{12}	"
Teflon	3×10^{12}	"

* The resistance of a short circuit element is approaching zero

* The resistance of an open circuit is approaching infinity.



Short circuit with $R_{sc}=0$
$V_{sc}=0$



Open circuit with $R_{oc}=\infty$
$I_{oc}=0$

* Conductance (G)

A useful quantity in circuit analysis is the reciprocal of resistance (R), is called the conductance (G);

$$G = \frac{1}{R} = \frac{i}{v}$$

The conductance can be explained as the ability of an element to conduct electric current, it is measured in mhos (℧) or in siemens (S).

$$\therefore \quad i = Gv$$

and:

$$p = vi = i^2R = \frac{v^2}{R} \qquad \text{watts} \quad (W)$$

OR

$$p = vi = v^2G = \frac{i^2}{G}$$

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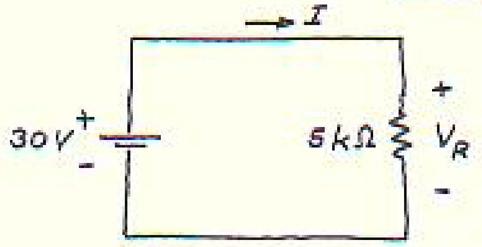
Example: In the circuit shown, calculate the current I, the conductance G, and the power P



$V_R = V$

Solution:

- the current $\quad I = \dfrac{V_R}{R} = \dfrac{30}{5\times10^3} = 6\times10^{-3} = \underline{6\ mA}$

- the conductance $\quad G = \dfrac{1}{R} = \dfrac{1}{5\times10^3} = 0.2\times10^{-3} = 0.2\ mS$

- the power $\quad P = V_R I = 30(6\times10^{-3}) = \underline{180\ mW}$

or $\quad P = I^2R = (6\times10^{-3})^2(5\times10^3) = \underline{180\ mW}$

or $\quad P = V_R^2 G = (30)^2(0.2\times10^{-3}) = \underline{180\ mW}$

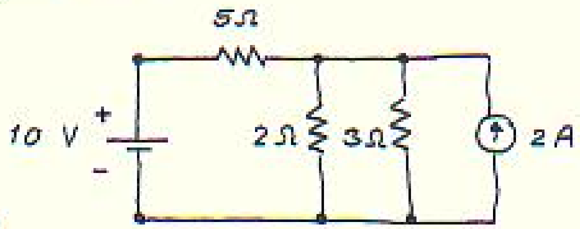
1.2.2 Nodes, branches and loops

* A branch represents a single element in the electric circuit, such as a voltage source or a resistor etc...

* A node represents the point of connection between two or more branches.

* A loop is any closed path in a circuit.

Example: For the circuit shown, determine the number of branches, nodes and the independent loops.



Solution

Since there are 5 elements

$\Rightarrow$ Number of branches = $\underline{5}$ $\qquad$ 10V, 5Ω, 2Ω, 3Ω, and 2A

Number of nodes = $\underline{3}$ (as shown in the figure).

3 عقد

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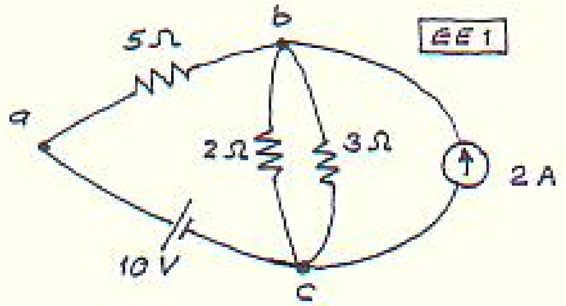
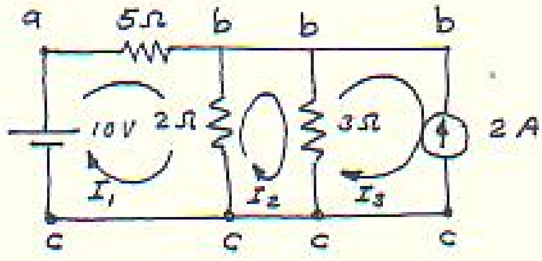


⇒ There are 3 nodes :
a , b and C

* The number of the <u>independent</u> loops = 3



⇒ loop 1 or loop abc :
contains (10V, 5Ω, 2Ω)

⇒ loop 2 or loop bcb :
contains (2Ω, 3Ω)

& ⇒ loop 3 or loop bcb :
contains (3Ω, 2A)

<u>Notes</u> : There are more than 3 (dependent) loops in this example, we had only calculated the INDEPENDENT loops which are only 3.

IN GENERAL; Any circuit with b branches, n nodes and ℓ independent loops, the following fundamental theorem of network topology:

$$\boxed{b = \ell + n - 1}$$

* Two or more elements are in **SERIES** if they are cascaded sequentially and consequently carry the SAME current.

* Two or more elements are in PARALLEL if they are connected to the same two nodes and have consequently the same VOLTAGE across them.

1.2.3 Kirchoff's Laws (1847) :

قانون كيرشوف للتيار * Kirchoff's current law (KCL); states that the algebraic sum of all current entering a node is zero or: The sum of currents entering a node is equal to the sum of curents leaving that node.

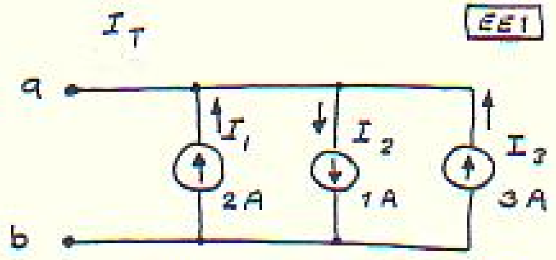
$$\sum_{n=1}^{N} I_n = 0$$

or

$$\boxed{\sum_{m=1}^{M} I_{mi} = \sum_{n=1}^{N} I_{no}}$$

where I_{mi} are the currents entering the node and I_{no} are the currents leaving the node.

Example: For the network shown, calculate the total current I_T



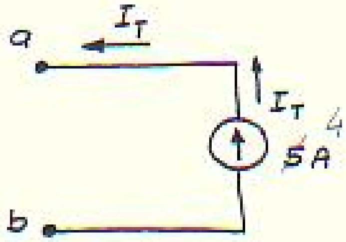
Current Sources in Parallel

Solution

According to KCL;

$$I_T = I_1 - I_2 + I_3 = 2 - 1 + 3 = \underline{\underline{5\ A}}$$

∴ The equivalent circuit for the network can be as shown ⇒



* Kirchoff's Voltage Law (KVL); states that the algebraic sum of all voltages around a closed path (or loop) is zero.

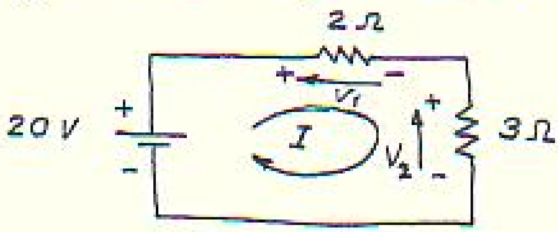
قانون كرشوف للفولتية

∴ Mathematically KVL states that:

$$\boxed{\sum_{m=1}^{M} V_m = 0}$$

where M is the number of voltages in the loop (or the number of branches in the loop), and V_m is the mth voltage.

Example: For the circuit shown, find the voltages V_1 and V_2



Solution:

$$V_1 = 2I$$
$$V_2 = 3I$$

From KVL:

$$\Sigma V = 0 \Rightarrow 20 - V_1 - V_2 = 0 \Rightarrow 20 = 3I + 2I$$

$$\Rightarrow 5I = 20 \Rightarrow I = 4\ A$$

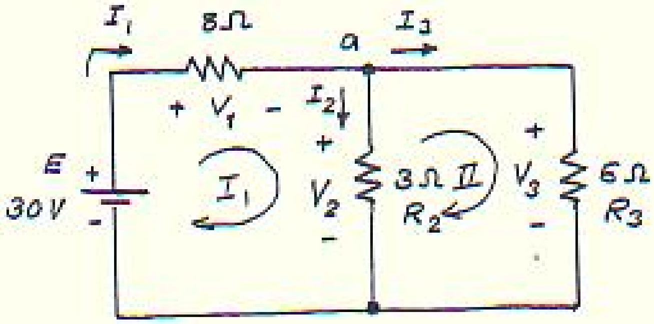
$$\therefore V_1 = 2I = \underline{\underline{8V}} \quad \text{and} \quad V_2 = 3I = \underline{\underline{12V}}$$

Tutorial Sheet № 1

Basic Concepts & Basic Laws

TS1

Example Using Kirchoff's laws, find the currents and voltages in the circuit shown.



Solution:

ملاحظة : المطلوب في المثال ايجاد كل من :
I_1 , I_2 , I_3 , V_1 , V_2 & V_3
باستخدام قانوني كيرشوف

* Using Ohm's law:

$$V_1 = I_1 R_1 = 8 I_1$$
$$V_2 = I_2 R_2 = 3 I_2$$
$$V_3 = I_3 R_3 = 6 I_3$$

* Applying KCL at node a:

$$I_1 = I_2 + I_3 \Rightarrow \boxed{I_1 - I_2 - I_3 = 0} \quad Eq(1)$$

* Applying KVL to loop 1;

$$E - V_1 - V_2 = 0 \Rightarrow 30 - V_1 - V_2 = 0$$

$$\Rightarrow 30 - 8I_1 - 3I_2 = 0$$

$$\therefore \boxed{I_1 = \frac{30 - 3I_2}{8}} \quad Eq(2)$$

* Applying KVL to loop 2;

$$V_2 - V_3 = 0 \Rightarrow V_2 = V_3$$

وهذا واضح لانه
R_2 & R_3 متوازيتان

$$\therefore 6I_3 = 3I_2$$

$$\boxed{\therefore I_3 = \frac{I_2}{2}} \quad Eq(3)$$

$\Rightarrow$ From Eq(1), Eq(2) & Eq(3)

$$\frac{30 - 3I_2}{8} - I_2 - \frac{I_2}{2} = 0 \Rightarrow I_2 = \underline{\underline{2\ A}}$$

$$\text{and } I_1 = \frac{30 - 3I_2}{8} = \frac{30 - 3(2)}{8} = \underline{\underline{3\ A}}$$

$$I_3 = \frac{I_2}{2} = \frac{2}{2} = 1\ A$$

TS1

$\Rightarrow \because V_1 = 8I_1 \quad \Rightarrow \quad \therefore V_1 = 8(3) = 24\ V$

Similarly $V_2 = 3I_2 = 3(2) = 6\ V$

and $V_3 = 6I_3 = 6(1) = 6\ V$

للتأكد من صحة الحل :

$I_1 = I_2 + I_3$

$3 = 2 + 1 \Rightarrow 3 = \underline{\underline{3}}$

كذلك فإن تطبيق قانون كيرشوف للفولتية على المسار المغلق رقم 1 ينتج ما يأتي :

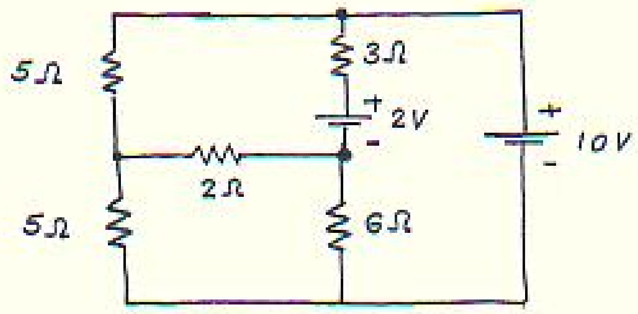
$E = V_1 + V_2$

$\therefore 30 = 24 + 6$

$\therefore 30 = 30$

∴ الحل صحيح

Example: Determine the number of branches, nodes and independent loops in the circuit shown.



Solution:

* There are $\underline{\underline{7}}$ element $\Rightarrow$ no. of branches $= \underline{\underline{7}}$

$\Rightarrow \underline{b = 7}$

a, b, c, d, e $\Leftarrow$ * There are 5 nodes as shown in the figure :

$\Rightarrow n = 5$

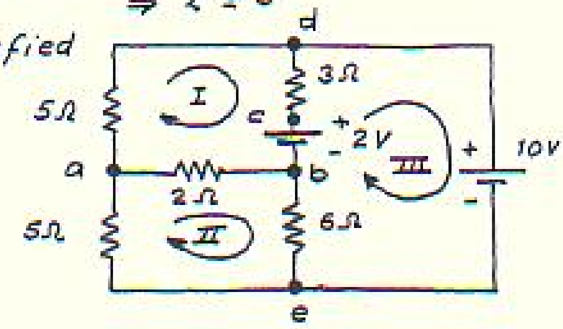
I, II, III $\Leftarrow$ * There are 3 independent loops :

$\Rightarrow \ell = 3$

$\therefore b = \ell + n - 1$ is satisfied

since $7 = 3 + 5 - 1$

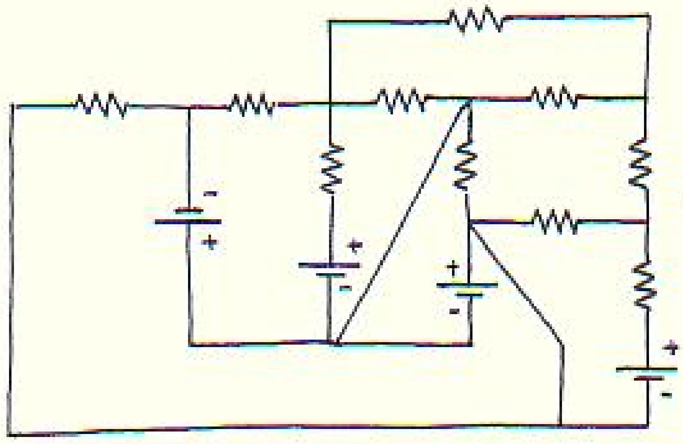
$\Rightarrow 7 = 7$



TS1

Practice Problem

: Identify all nodes, branches and independent loops in the circuit shown in the figure.



Answer

: no. of nodes = 8 $\quad n = 8$

no. of branches = 14 $\quad b = 14$

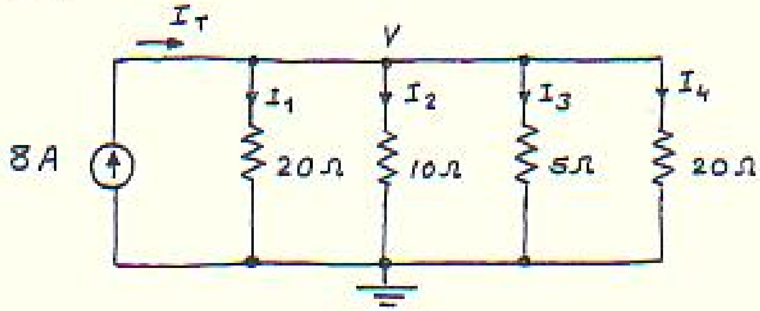
no. of independent loops = 7 $\quad \ell = 7$

Check. Does this satisfy the fundamental theorem of network topology?

$$b = \ell + n - 1 = 7 + 8 - 1 = 14 \quad \underline{\text{YES}}$$

Example

: Determine all currents and voltages in the circuit of the figure shown.



Solution

: KCL ⇒

$$I_T = I_1 + I_2 + I_3 + I_4 \Rightarrow 8 = I_1 + I_2 + I_3 + I_4$$

Ohm's law

$$\begin{aligned} V &= 20 I_1 & \Rightarrow \quad I_1 &= V/20 \\ &= 10 I_2 & I_2 &= V/10 \\ &= 5 I_3 & I_3 &= V/5 \\ &= 20 I_4 & I_4 &= V/20 \end{aligned}$$

Substituting in the current equation;

$$\Rightarrow \quad 8 = \frac{V}{20} + \frac{V}{10} + \frac{V}{5} + \frac{V}{20}$$

$$\therefore \quad 160 = V + 2V + 4V + V$$

$$160 = 8V$$

$$\therefore \quad \underline{V = 20 \text{ Volts}}$$

$$\therefore \quad I_1 = \frac{V}{20} = \frac{20}{20} = 1\ A$$

$$I_2 = \frac{V}{10} = \frac{20}{10} = 2\ A$$

$$I_3 = \frac{V}{5} = \frac{20}{5} = 4\ A$$

$$I_4 = \frac{V}{20} = \frac{20}{20} = 1\ A$$

<u>Check</u> $\quad I_T = I_1 + I_2 + I_3 + I_4$

$$8 = 1 + 2 + 4 + 1$$

$$8 = 8 \quad \checkmark$$

Fundementals of Electrical Engineering

1. Circuit Variables & Circuit Elements

1.1_ System of Units

- The English System
- The metric System — MKS System / CGS System
- The International System of Units (SI)

The (SI) system is the commonly used by all the major engineering societies.

The (SI) system of units

Quantity	Basic unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Degree Kelvin	K
Luminous intensity	Candela	cd

Some scientific notations: In many cases, the SI units is either too small or too large to use conveniently. Standard prefixes correspond to the power of 10 are applied to the basic unit as shown in the table.

Prefix	Symbol	Power
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}

Examples

$$1000000\ W = 10^{6}\ W = 1\ MW$$
$$0.000001\ F = 10^{-6}\ F = 1\ \mu F$$

1.2 Voltage and Current

★ Voltage

In general the potential difference between two points is defined as:

$$\boxed{V = \frac{W}{Q}} \quad \text{volts (V)}$$

where: Q is the charge moving between the two points in Coulombs (C)
W is the energy expended to do this motion in Joules (J)

From the above equation, we have:

$$\boxed{W = QV} \quad \text{joules (J)}$$

and

$$\boxed{Q = \frac{W}{V}}$$

Example

: Determine the energy expended in moving a charge of 50 μC through a potential difference of 6 V.

Solution:

$$W = QV = (50 \times 10^{-6})(6)$$
$$= 300 \times 10^{-6} \text{ J}$$
$$= 300 \ \mu\text{J}$$

★ Current

It is the rate of charge moving in a certain metal, and can be given by:

$$I = \frac{\Delta Q}{\Delta t}$$

or $$\boxed{I = \frac{Q}{t}}$$ amperes (A)

then $$\boxed{Q = It}$$ in Coulombs (C)

and $$\boxed{t = \frac{Q}{I}}$$ in seconds (s)

Example

: A charge of 0.16C is flowing every 64 ms. Determine the resulting current.

Solution

: $$I = \frac{Q}{t} = \frac{0.16}{64 \times 10^{-3}}$$

$$= 2.5 \text{ A}$$

1.3 Circuit Elements

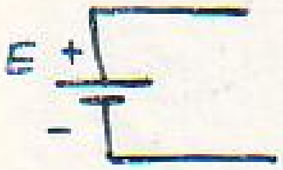
- Voltage sources
- Current sources
- Resistors
- Capacitors
- Inductors

1.4 Voltage and Current Sources

– Voltage Source

: It is a device which supplies a fixed terminal voltage even though the current drain may vary

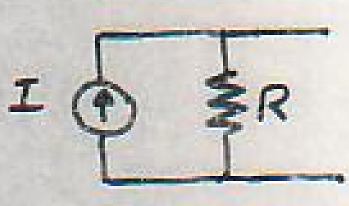
voltage source



Ideal
V.S (with R=0)

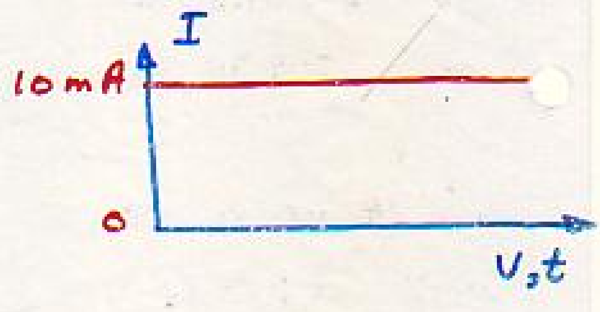
– Current Source

It is a device which supplies a constant current to a load even though there will be variations in the terminal voltage as determined by the load.



Current source

Ideal



1.5 Electrical Resistance

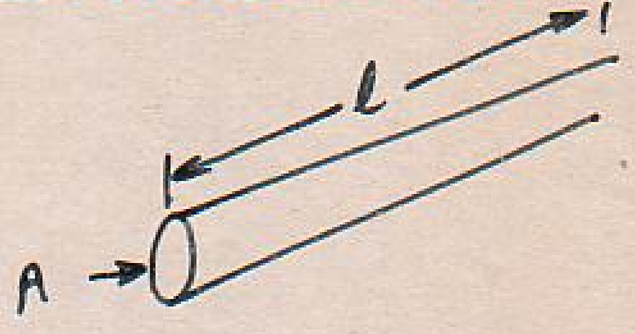
$$\boxed{R = \rho \frac{\ell}{A}}$$

where R is the resistance in Ω

ρ is the resistivity of the material Ω.m

l is the length of the conductor in m

A is the cross-sectional area in m^2



Example

: Determine the resistance of 100 ft of a copper wire if the diameter is 0.0126 in, given that the resistivity of copper is 1.724×10^{-6} Ω.cm.

Solution

$$R = \rho \frac{l}{A}$$

$\rho = 1.724 \times 10^{-6}$ Ω.cm

$l = 100$ ft $= 1200$ in

$= 1200 \times 2.54$

$= 3048$ cm

$$\therefore R = \frac{(1.724 \times 10^{-6})(3048)}{8.04 \times 10^{-4}}$$

$$= \underline{\underline{6.5\ \Omega}}$$

$d = 0.0126$ in

$= 0.032$ cm

$$A = \frac{\pi d^2}{4} = 8.04 \times 10^{-4}\ \text{cm}^2$$

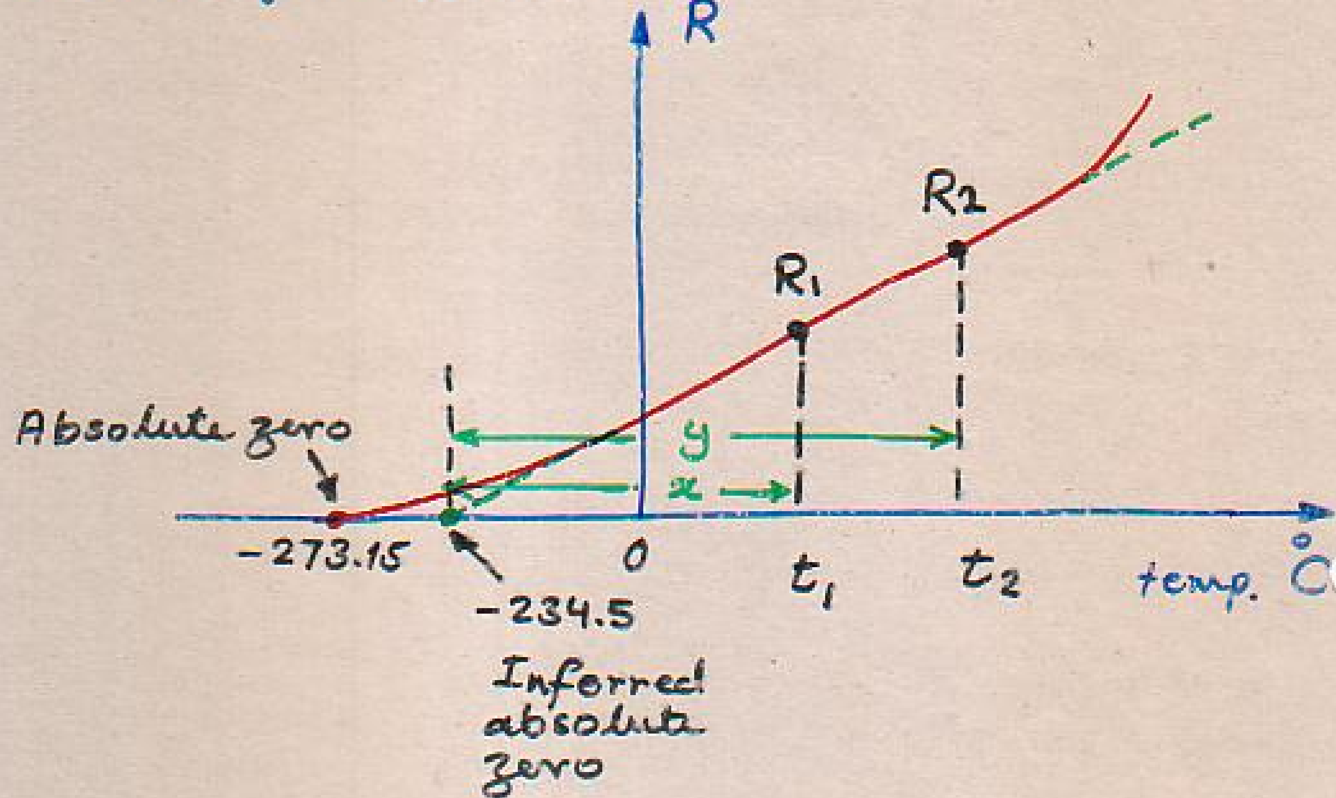
Temperature Effects on Resistance

The effect of rise of temp. is :

- to increase the resistance of pure metals
- to increase the resistance of alloys.
- to decrease R of insulators and partial

conductors such as carbon.

Typical variation of R with temp. is shown below (for copper).



From the graph $\Rightarrow \dfrac{x}{R_1} = \dfrac{y}{R_2}$

or $$\frac{t_1 + 234.5}{R_1} = \frac{t_2 + 234.5}{R}$$

In general, for any material

$$\frac{|T| + t_1}{R_1} = \frac{|T| + t_2}{R_2}$$

where $|T|$ is the inferred absolute zero of the material (°C).

Example: If the resistance of a copper wire is 50 Ω at 20 °C, what is its resistance at 100 °C?

Solution: $$\frac{234.5 + 20}{50} = \frac{234.5 + 100}{R_2}$$

$\Rightarrow R_2 = 65.72\ \Omega$

Another relation for the temp. effect on R is given as

$$\boxed{R_2 = R_1\left[1 + \alpha_1 (t_2 - t_1)\right]}$$

where α_1 is the temperature coefficient of resistance

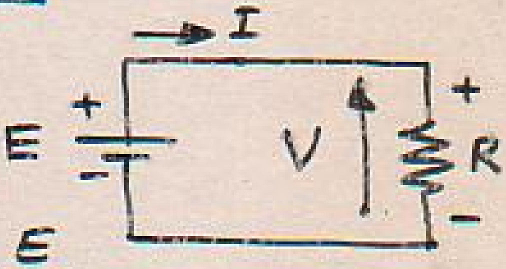
1.6 Ohm's & Kirchoff's Laws

_ Ohm's Law

: It is an equation that establishes a relationship among the current (I), voltage (V) and the resistance (R) of an electrical circuit. It is given by:

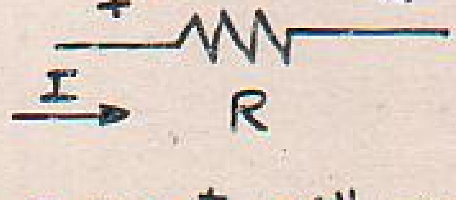
in volts (V)

in amperes (A) $\boxed{I = \frac{V}{R}}$ in Ω



$$I = \frac{E}{R}$$

For resistive element, the polarity of the voltage drop as show for the indicated current direction

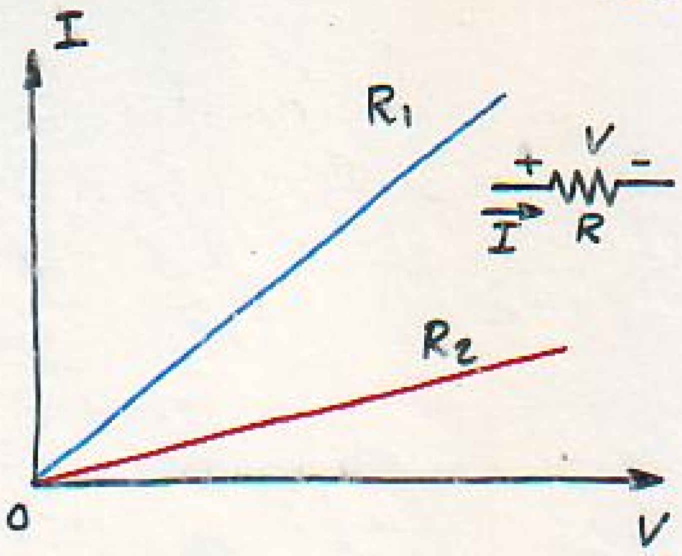


A reversal in current will reverse the polarity.

$$I = \frac{V}{R} \Rightarrow V = IR$$

$$\text{slope} = \frac{1}{R} = \frac{\Delta I}{\Delta V}$$

$$\Rightarrow R = \frac{\Delta V}{\Delta I}$$

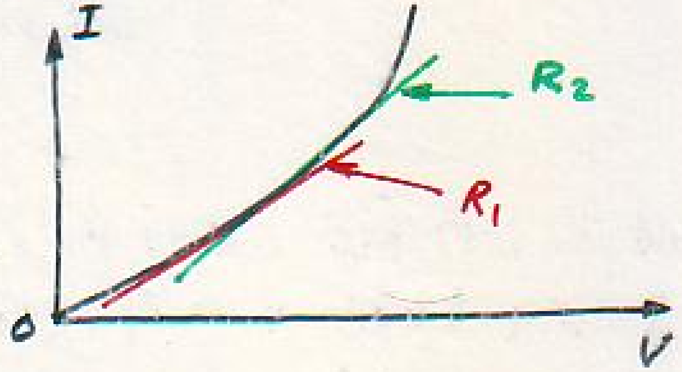


$R_2 > R_1$

For non-linear resistor the resistance can be determined graphically as:

$$R = \frac{\Delta V}{\Delta I}$$

from the I-V graph.

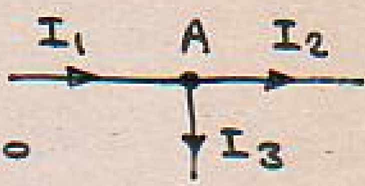


- Kirchoff's Laws

* Kirchoff's Current Law (KCL)

It states that " in any electrical network, the algebraic sum of the currents meeting at a point (or junction) is zero. or in other words; it simply means that the total currents leaving a junction is equal to the total currents entering that junction.

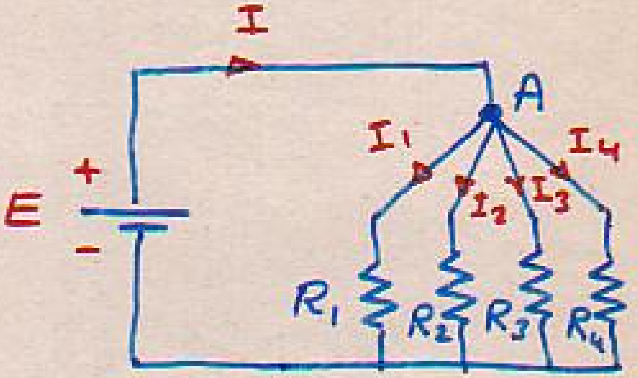
Example



$$I_1 + (-I_2) + (-I_3) = 0$$

$$\Rightarrow \quad I_1 = I_2 + I_3$$

Example



at junction A

$$\Sigma I = 0$$

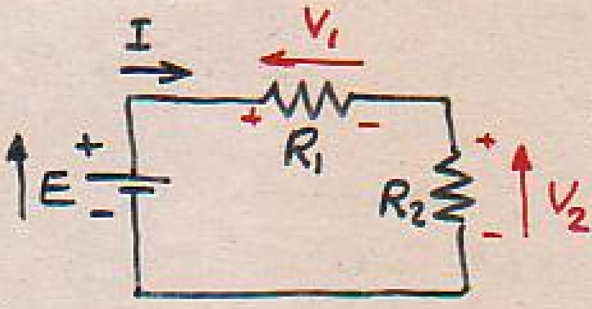
$$I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0$$

$$\Rightarrow \quad I = I_1 + I_2 + I_3 + I_4$$

* Kirchoff's Voltage Law (KVL)

: It states that "the algebraic sum of all potentials around any closed path (loop) is zero, ie,

$$\sum_{0} V = 0$$



Moving CW around the loop shown, then:

$$E + (-V_1) + (-V_2) = 0$$

or

$$E = V_1 + V_2$$

1.7 Power and Energy

* **Power**: It is defined as the rate of doing work, ie,

$$P = \frac{W}{t} \quad \text{Watts (W)}$$

or joules/sec (J/s)

From the earliest analysis, we have:

$$W = QV$$

$$\Rightarrow \quad P = \frac{QV}{t} \qquad \Leftarrow \frac{Q}{t} = I$$

$$\boxed{\therefore \quad P = VI}$$

Since $I = \frac{V}{R}$ Ohm's law $\Rightarrow$ $\boxed{P = \frac{V^2}{R}}$

or $\boxed{P = I^2 R}$

1 Watt = 1 J/s

1 horsepower = 746 W

* **Energy**

It is a quantity represents the product of the power (P) and the period (t), ie

$$W = Pt$$

Watt.sec. W.s

or Joules

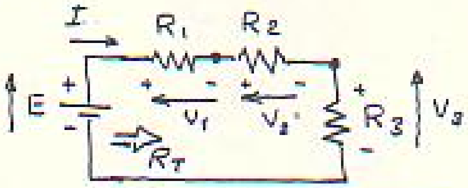
$$\text{Energy (kWh)} = \frac{\text{Power (W)} \times \text{time (h)}}{1000}$$

EE2

2. Circuit Transformations

2.1 Series Circuits

: Two elements are in series if they have only one point in common that is not connected to other current carrying elements of the network.



For the series cct. shown, using KVL we have:

$$E = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$= I(R_1 + R_2 + R_3)$$
$$= IR_T$$

$$\Rightarrow \boxed{R_T = R_1 + R_2 + R_3}$$

$$\therefore \boxed{I = \frac{E}{R_T}}$$

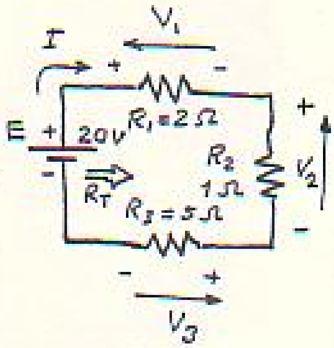
In general, for a series cct. consisting N resistors, then the total resistance of such a cct. R_T is given as

$$R_T = R_1 + R_2 + R_3 + \cdots\cdots R_N$$

Example

: For the cct. shown;

a. Find the total resistance
b. Calculate the current I
c. Determine the voltages V_1, V_2 and V_3



Solution

: a. $R_T = R_1 + R_2 + R_3$
$= 2 + 1 + 5 = \underline{8\,\Omega}$

b. $I = \frac{E}{R_T} = \frac{20}{8} = \underline{2.5\text{ A}}$

c. $V_1 = IR_1 = (2.5)(2) = 5\text{ V}$
$V_2 = IR_2 = (2.5)(1) = 2.5\text{ V}$
$V_3 = IR_3 = (2.5)(5) = 12.5\text{ V}$

يلاحظ
$V_1 + V_2 + V_3 = E$
$5 + 2.5 + 12.5 = 20\text{ V}$
$\therefore 20\text{ V} = 20\text{ V}$

2.1.1 Voltage Sources in Series

: The net voltage will the algebraic sum of all sources that are connected in series.

Example

:

$$E_T = E_1 + E_2 + E_3$$
$$= 10 + 6 + 2 = 18\text{ V}$$

* ملاحظة : لا يمكن ربط مصادر التيار بالتوالي لأنه ذلك يدفع مخالف لقانون كيرشوف للتيار

2.1.2 Voltage Divider Rule

EE2

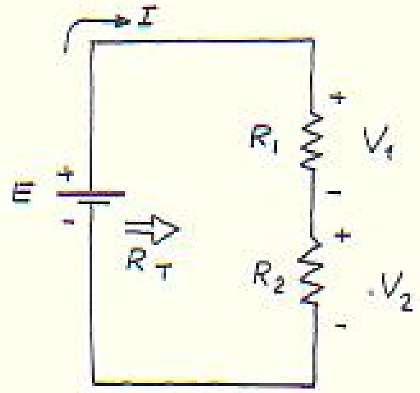
Voltage Divider Rule: Consider the series cct. shown.

we have:

$$R_T = R_1 + R_2$$

and

$$I = \frac{E}{R_T}$$



$$V_1 = IR_1 = \left(\frac{E}{R_T}\right).R_1$$

$$= \frac{E.R_1}{R_T}$$

Similarly;

$$V_2 = IR_2 = \left(\frac{E}{R_T}\right).R_2$$

$$= \frac{E.R_2}{R_T}$$

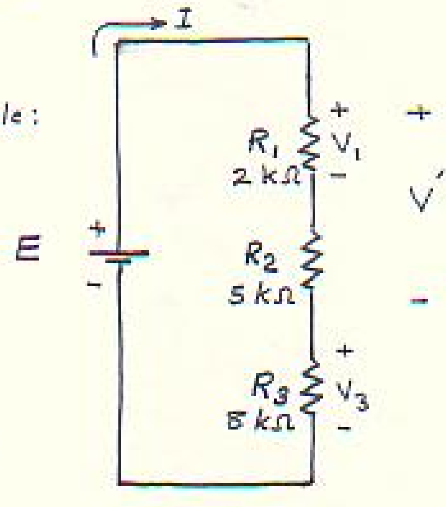
Hence, we can write:

$$\boxed{V_x = \frac{E.R_x}{R_T}}$$ ⇐ Voltage divider rule

This means that "the voltage divider rule" can be understood to state that:

> The voltage across a resistor in a series circuit is equal to the value of that resistor times the the total applied voltage across the series elements divided by the total resistance of the series elements.

Example: Determine the voltages V_1, V_3, and V' for the cct. shown.



Solution: Using the voltage divider rule:

$$V_1 = \frac{E.R_1}{R_T}$$

$$= \frac{(45).(2\times10^3)}{(2+5+8)\times10^3}$$

$$= \underline{6\ V}$$

$$V_3 = \frac{E.R_3}{R_T} = \frac{(45)(8\times10^3)}{(2+5+8)\times10^3}$$

$$= \underline{24\ V}$$

$$V' = \frac{E \cdot R'}{R_T} \Leftarrow R' = (2+5)\times 10^{3}\ \Omega = 7\times 10^{3}\ \Omega$$

$$= \frac{(45)\cdot(7\times 10^{3})}{(2+5+8)\times 10^{3}}$$

$$= \underline{21\ V}$$

نلاحظ انه :

$$E = V_3 + V'$$

$$45\ V = 24V + 21\ V$$

$$45\ V = 45\ V$$

نلاحظ ان فرق الجهد على طرفي اي عنصر من عناصر دائرة التوالي يتناسب مع مقاومة ذلك العنصر ، أي ان المقاومة الكبيرة يقابلها فرق جهد كبير والمقاومة الصغيرة يقابلها فرق جهد صغير ، وفي جميع الاحوال يجب ان يكون مجموع فروق الجهد مساوياً الى فولتية المصدر .

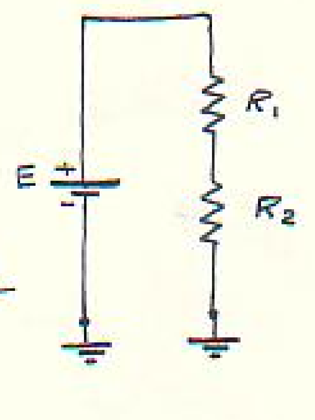
NOTATION

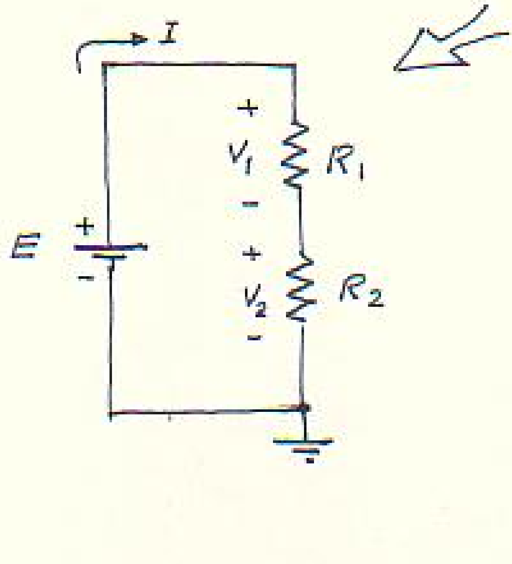
ground potential

It is common, for safety purposes and as a reference to ground electrical and electronic systems. The symbol for the ground connection is :

with its defined potential level (zero volts). As a consequence, the cct. might need to be redrawn in the ordinary form so as to be analyzed.

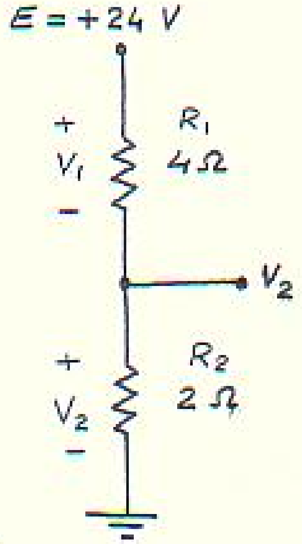
Example : For the cct. shown ; with ground potentials connected for the source and to elements; this cct. can be redrawn to make it easy analyzing it.





EE2

Example: Using the voltage divider rule, determine the voltages V_1 and V_2 for the cct shown:

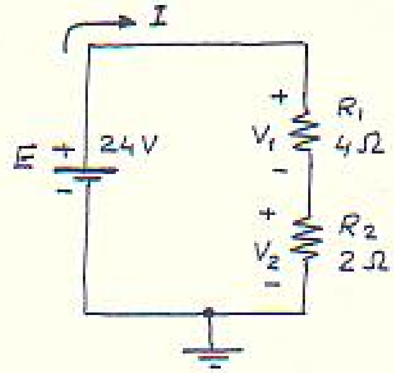


Solution: Redrawing the cct. with the standard battery symbol, then the cct. will be as shown below:

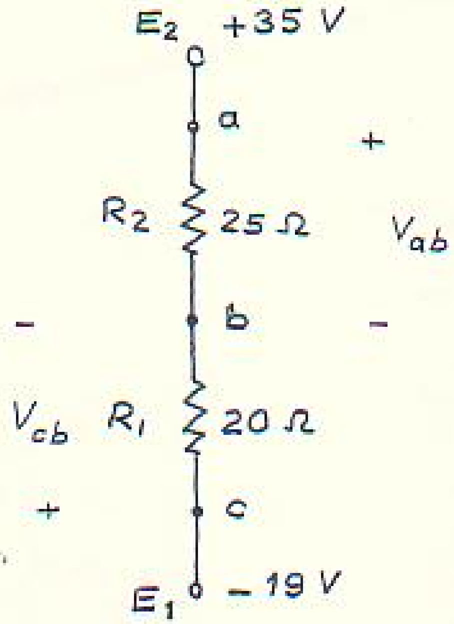
$$\therefore \quad V_1 = \frac{R_1 E}{R_T} = \frac{(4).(24)}{4+2} = \underline{16\ V}$$

and

$$V_2 = \frac{R_2 E}{R_T} = \frac{(4).(24)}{4+2} = \underline{8\ V}$$

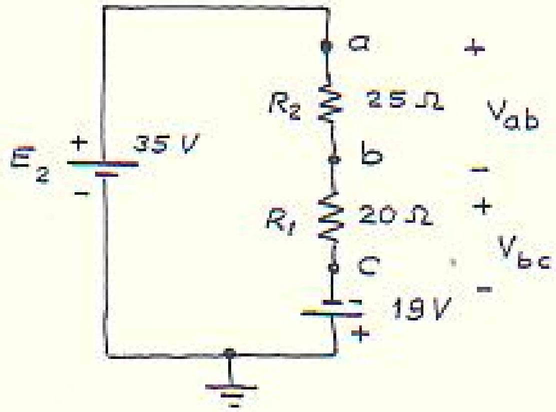


Example: For the cct. shown, determine V_{ab}, V_{cb} and V_b.



EE2

Solution: The cct. is redrawn as shown;



$$\therefore I = \frac{E_1 + E_2}{R_T} = \frac{19 + 35}{20 + 25}$$
$$= 1.2\ A$$

$$V_{ab} = I R_2 = (1.2)(25)$$
$$= 30\ V$$

and $V_{cb} = -V_{bc} = -I R_1$
$$= -(1.2)(20)$$
$$= -24\ V$$

$$V_c = -E_1 = -19\ V$$

ملاحظة ١/
يمكن الحل باستخدام voltage divider rule مع ملاحظة ان $V_{cb} = -V_{bc}$.

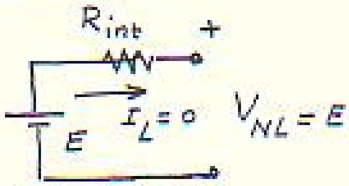
ملاحظة ٢/ : ان V_{ab} تعني فرق جهد النقطة a عن النقطة b ، فاذا كانت V_{ab} موجبة فان هذا يعني ان V_a اكبر من V_b والعكس صحيح .

ملاحظة ٣/ : ان V_a تعني جهد النقطة a بالنسبة الى الارضي .

Internal Resistance of Voltage Sources:

Ideal Voltage source

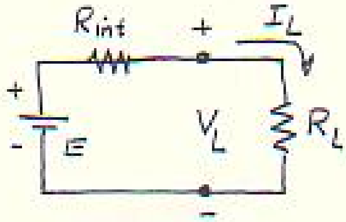
①



$V_{NL} = E$

Actual Voltage source (at no load)

②



Actual Source connected to load R_L

③

- Applying KVL on figure ③ : $E - I_L R_{in} - V_L = 0$
- Applying KVL on figure ② : $E = V_{NL}$

Substituting, then:

$$V_{NL} - I_L R_{in} - V_L = 0$$

$$\boxed{\therefore V_L = V_{NL} - I_L R_{in}}$$

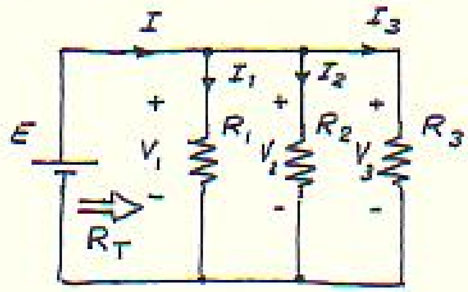
2.2 Parallel Circuits

: Two branches or elements or networks are in parallel if they have two points in common.

For the parallel cct. shown; using KCL, we have:

$$I = I_1 + I_2 + I_3$$

$$= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$



since $V_1 = V_2 = V_3 = E = V$

$$\therefore \; I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$E = V_1 = V_2 = V_3$$

$$\therefore \; \frac{V}{R_T} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$I = \frac{E}{R_T} = \frac{V}{R_T}$$

$$\therefore \; \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general, for N resistors connected in parallel, then:

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \frac{1}{R_N}}$$

Notation Conductance G

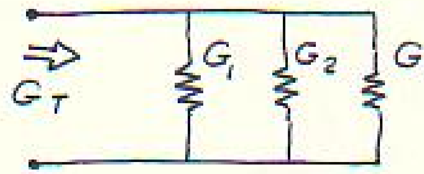
For parallel networks, it is common to use the idea of conductane in the cct. analysis. The conductance (G) is defined as:

$$G = \frac{1}{R} \qquad \text{Siemens} \quad (S)$$

So, we can write the total conductance G_T for the parallel cct shown, as:

$$G_T = G_1 + G_2 + G_3$$

$$\Rightarrow \; R_T = \frac{1}{G_T}$$



<u>NOTE</u> that, the total resistance R_T of parallel resistors is <u>always less than</u> the value of the <u>smallest resistor</u>.

Special Cases

: The general relation for the total resistance of parallel resistors is :

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

*** Case 1**

For equal resistors in parallel , ie, when

$$R_1 = R_2 = R_3 \text{ ---- } R_N = R$$

then ;
$$\frac{1}{R_T} = \underbrace{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots \frac{1}{R}}_{N}$$

$$\therefore \frac{1}{R_T} = N\left(\frac{1}{R}\right) \Longrightarrow \boxed{R_T = \frac{R}{N}}$$

*** Case 2**

: For two resistors in parallel , then R_T is given as :

$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}}$$

This means that, the total resistance of the two parallel resistors is the product of the two divided by their sum.

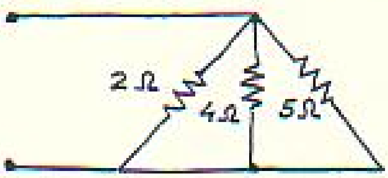
*** Case 3**

: For three resistors in parallel, then R_T is given as :

$$\boxed{R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}}$$

Example

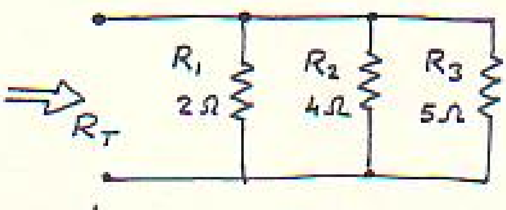
: Determine the total resistance for the network shown :



Solution

Redraw the cct. to be as shown ;

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$
$$= 0.5 + 0.25 + 0.2$$
$$= 0.95$$



$$\Rightarrow \therefore R_T = \frac{1}{0.95} = \underline{1.053\ \Omega}$$

Example: For the parallel network shown;

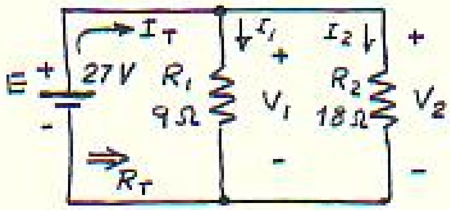
a. Calculate R_T

b. Determine I_T

c. Calculate I_1 and I_2

d. Determine the power to each resistive load.

e. Determine the power delivered by the source and compare it with the total power dissipated by the resistive elements.



Solution:

a. $$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9)(18)}{9+18} = \frac{162}{27} = \underline{6\,\Omega}$$

b. $$I_T = \frac{E}{R_T} = \frac{27}{6} = \underline{4.5\ A}$$

c. $$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27}{9} = \underline{3\ A}$$

and

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27}{18} = \underline{1.5\ A}$$

نلاحظ ان : $I_T = I_1 + I_2$

d. $$P_1 = I_1 V_1 = E I_1 = (27)(3) = \underline{81\ W}$$

$$P_2 = I_2 V_2 = E I_2 = (27)(1.5) = \underline{40.5\ W}$$

e. $$P_s = E I_T = (27)(4.5) = 121.5\ W$$

$$P_s = P_1 + P_2 = 81 + 40.5 = 121.5\ W$$

* نلاحظ ان مجموع القدرة المستهلكة في المقاومتين R_1 و R_2 يساوي القدرة التي يجهزها المصدر E

2.2.1 Current Divider Rule

: *Consider the parallel cct. shown;*

We have; $I = \frac{V}{R_T} \Rightarrow V = I R_T$

and $$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

but $$I_1 = \frac{V}{R_1} = \frac{I R_T}{R_1} = I \cdot \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1}$$

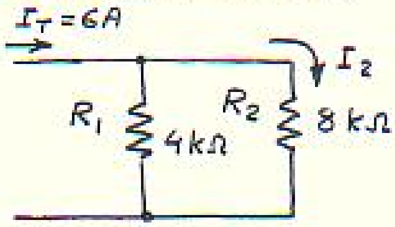
$$\therefore \boxed{I_1 = I \cdot \frac{R_2}{R_1 + R_2}}$$

Similarly, for I_2, we have:

$$I_2 = \frac{V}{R_2} = \frac{I R_T}{R_2} = I . \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_2}$$

$$\therefore \boxed{I_2 = I . \frac{R_1}{R_1 + R_2}}$$

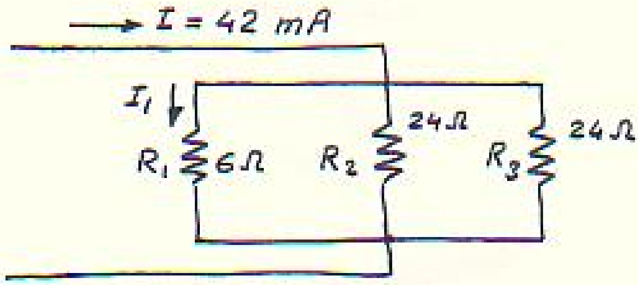
Example: Determine the current I_2 for the network shown;



solution:
$$I_2 = I_T . \frac{R_1}{R_1 + R_2}$$

$$= 6 \frac{4 \times 10^3}{(4+8) \times 10^3} = 6 . \frac{4}{12} = \underline{2 \text{ A}}$$

Example: Find the current I_1 for the network shown;



Solution:

$$I_1 = I . \frac{R_T}{R_1}$$

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

or

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{24} + \frac{1}{24}$$

$$\Rightarrow R_T = 4 \, \Omega$$

$$\therefore I_1 = 42 \times 10^{-3} \frac{4}{6}$$

$$= 28 \times 10^{-3} = \underline{28 \text{ mA}}$$

2.2.2 Voltage Sources in Parallel :

EE2

$E_1 = E_2 = E$ and $I_S = I_1 + I_2$

To increase the current rating of the source, two or more batteries in parallel of the same terminal voltage would be used.

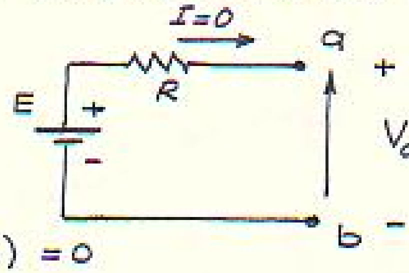
2.3 Open and Short Circuits

: We often need to apply the open and short circuits in the analysis fof electric networks.

* **Open Circuit** : An open circuit is simply two isolated terminals not connected by an element of any kind.

Consider the circuit shown ; with open circuit terminals a and b.

$V_{\text{open circuit}} = V_{oc} = V_{ab} = E$

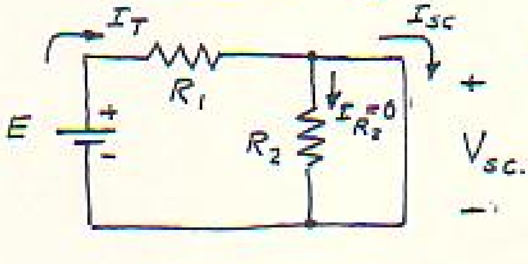


since I (in the open circuit) = 0

In general : An open circuit CAN HAVE a potential difference (voltage) across its terminals but the current is always ZERO.

* **Short Circuit** : A short circuit is a direct connection of zero ohms across an element or combination of elements.

Consider the circuit shown , with a short circuit across the resistor R_2

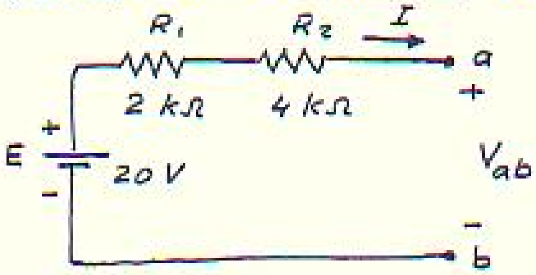


$I_{sc} = I_T = \frac{E}{R_1}$ since $I_{R_2} = 0$

$V_{\text{short cct.}} = V_{sc} = \underline{\underline{0}}$

In general: A short circuit CAN CARRY a current of any level but the potential difference (voltage) across its terminals is always ZERO.

Examples: (a). For the network shown, determine V_{ab}.

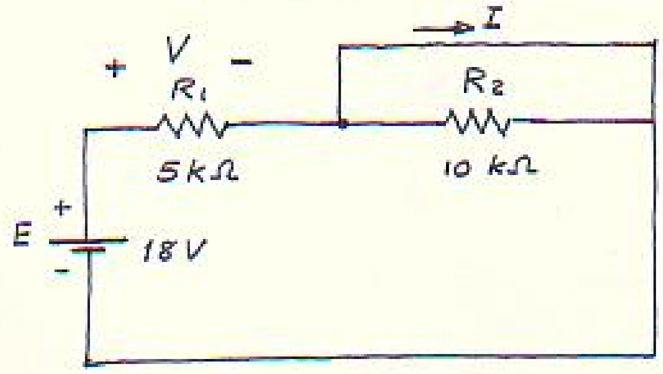


Solution: - We have an open cct across the terminal a and b, the

$I = 0 \Rightarrow V_1 = 0$ and $V_2 = 0$

- Applying KVL $\Rightarrow V_{ab} = E = 20\,V$

(b). Calculate I and V for the network shown;



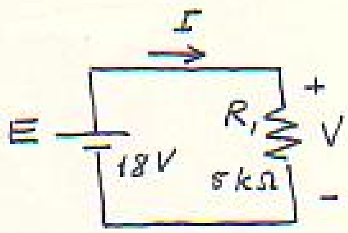
Solution: - We have a short cct. across R_2

$\Rightarrow$ No current through R_2

$$\Rightarrow I = \frac{E}{R_T} = \frac{E}{R_1 + 0} = \frac{18}{5\,k\Omega}$$

$$= \underline{3.6\ mA}$$

Note: the cct can be redrawn to be as shown



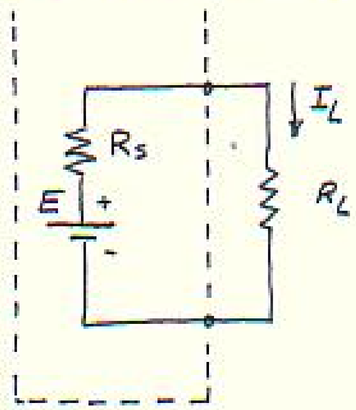
$$\therefore V = I R_1 = E = \underline{18\,V}$$

2.4 Source Transformation :

EE2

It is often necessary or convenient to have a voltage source rather than a current source or a current source rather than a voltage source.

In the cct. shown, we have a voltage source connected to a load resistance R_L



Voltage Source

We have :

$$I_L = \frac{E}{R_T} = \frac{E}{R_s + R_L}$$

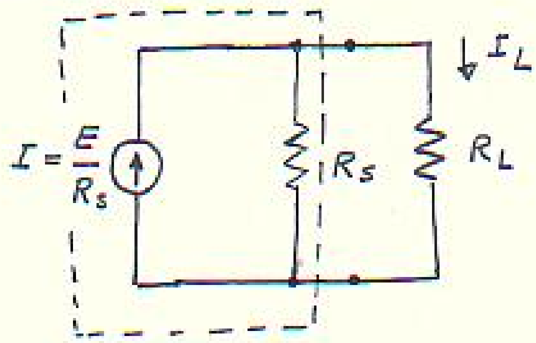
Multiplying the numerator by $(R_s/R_s = 1)$, we have :

$$I_L = \frac{(R_s/R_s)E}{R_s + R_L} = \frac{R_s(E/R_s)}{R_s + R_L}$$

$I = \frac{E}{R_s}$

$$\therefore\ I_L = \frac{R_s \cdot I}{R_s + R_L}$$

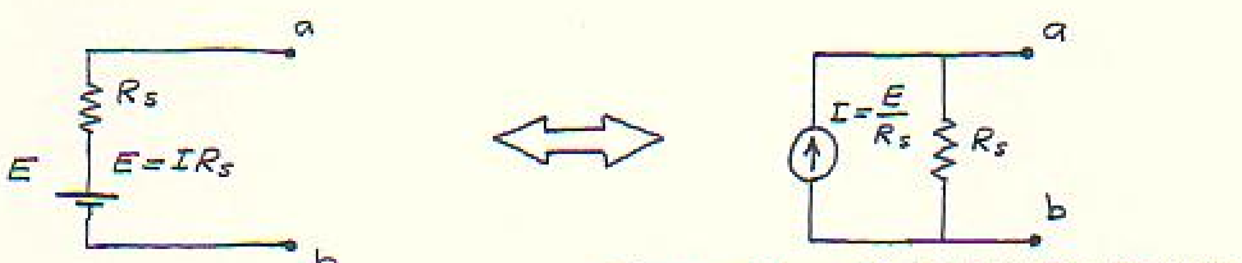
This is a current divider equation, which can be represented by the circuit below; which is the equivalent cct. of the voltage source.



Current source

In general

A voltage source with voltage E and series resistor R_s can be replaced by a current source with a current I and parallel resistor R_s as shown :

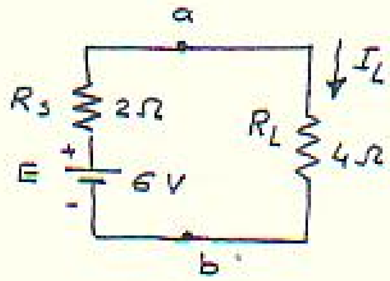


← current to voltage source

voltage source to current source →

EE2

Example: Convert the voltage source, in the cct below, to a current source, then calculate the current through the load for each source.

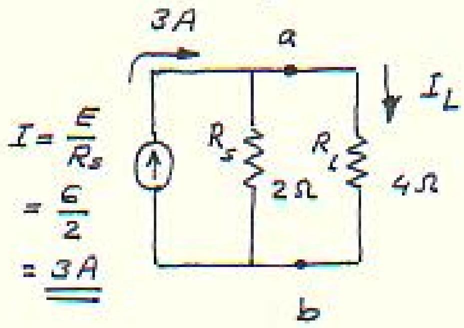


Solution:

* For the voltage source cct;

$$I_L = \frac{E}{R_s + R_L} = \frac{6}{2+4}$$

$$= \underline{\underline{1\ A}}$$



current source

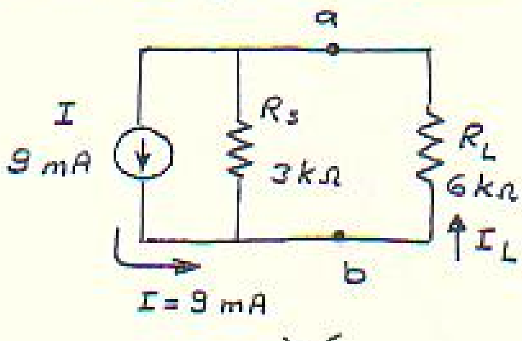
* For the current source cct;

$$I_L = \frac{I R_s}{R_s + R_L} = \frac{(3)(2)}{2+4}$$

$$= \underline{1\ A}$$

نلاحظ ان I_L متساوي في الحالتين وهذا صحيح.

Example: Convert the current source, in the cct. shown, to a voltage source and determine I_L for each source.



Solution:

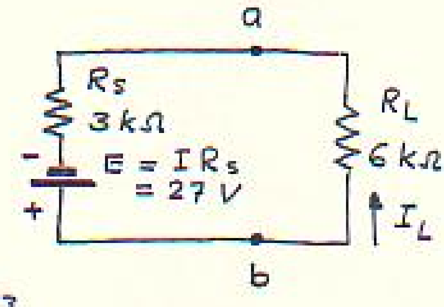
* For the current source cct;

current divider rule ⇒

$$I_L = \frac{I \cdot R_s}{R_s + R_L} = \frac{(9 \times 10^{-3})(3 \times 10^{3})}{(3 + \ \ ) \times 10^{3}}$$

$$= 3 \times 10^{-3} = \underline{3\ mA}$$

* For the voltage source cct;



Voltage source

$$I_L = \frac{E}{R_T} = \frac{E}{R_s + R_L} = \frac{27}{(3+6) \times 10^{3}}$$

$$= 3 \times 10^{-3} = \underline{3\ mA}$$

2.3 Series-Parallel Circuits :

EE2

Example : For circuit shown,

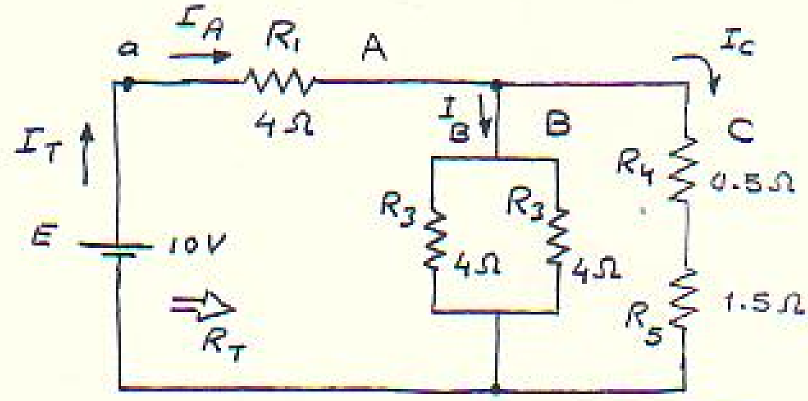


Fig (1)

Solution :

$$R_1 = R_A = 4\,\Omega$$

$$R_B = R_2 // R_3 = \frac{R_N}{N} = \frac{4}{2} = 2\,\Omega .$$

$$R_C = R_4 + R_5 = 0.5 + 1.5 = 2\,\Omega$$

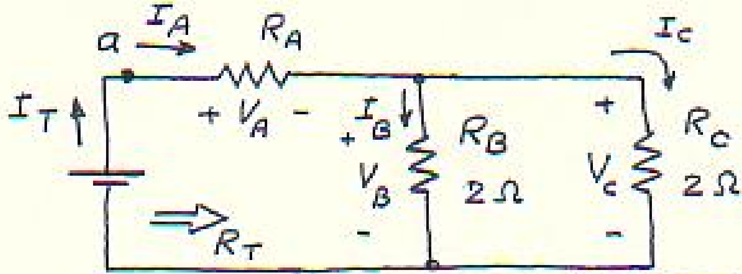


Fig (2)

$$R_{B//C} = \frac{R_N}{N} = \frac{2}{2} = 1\,\Omega$$

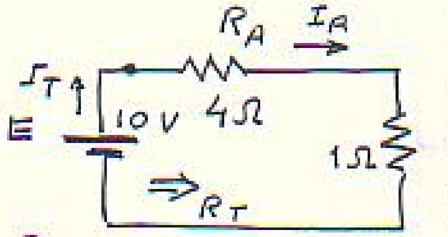


Fig (3)

From Fig. 3 $\Rightarrow \therefore R_T = R_A + R_{B//C} = 4 + 1 = 5\,\Omega$

$$\Rightarrow \therefore I_T = \frac{E}{R_T} = \frac{10}{5} = 2\ A$$

From Fig (2) $\quad I_B = I_C = \dfrac{I_A}{2} = \dfrac{2}{2} = 1\ A$

From Fig (1) $\Rightarrow I_{R_4} = I_{R_5} = I_C = 1\ A$

From fig (2) $\Rightarrow$ $V_A = I_A R_A = (2)(4) = 8\,V$

$V_B = I_B R_B = (1)(2) = 2\,V$

$V_C = I_C R_C = (1)(2) = 2\,V$

For check

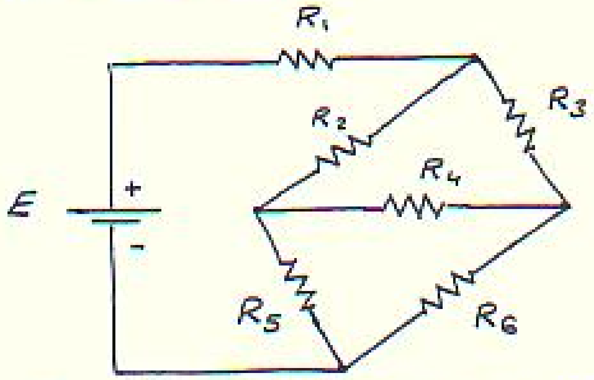
KVL $\quad \Sigma_0 V = 0 \Rightarrow E - V_A - V_B = 0 \Rightarrow 10 = 8 + 2$

$\therefore 10\,V = 10\,V$

Wye - Delta Transformation

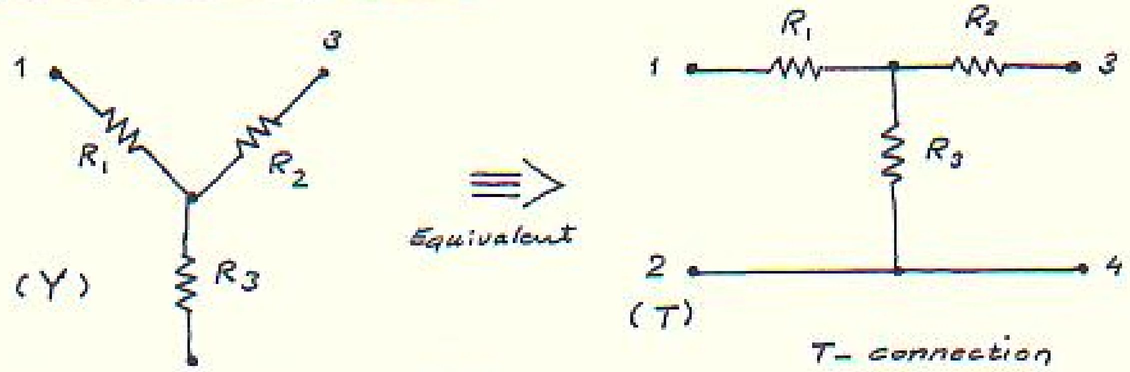
EE2

: There are some cases often arises in circuit analysis, when the resistors are neither in parallel nor in series. For example, consider the circuit shown:



In this circuit, R_1, R_2, R_3 ... R_6 are neither in series nor in parallel

* Wye (Y or star) connection

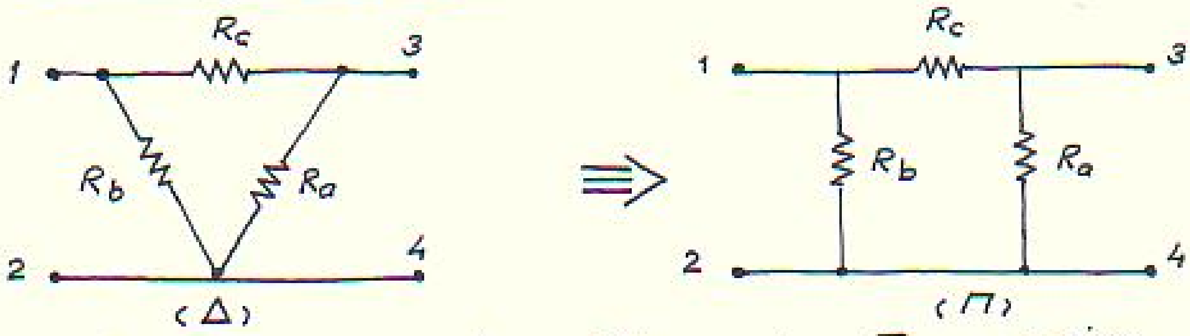


Y or star circuit connection

T- connection

Y and T connections

* Delta (Δ or π) circuit connection



Delta circuit connection & its equivalent Π- connection

EE2

Δ $\qquad$ Y

* Delta to Wye Transformation :

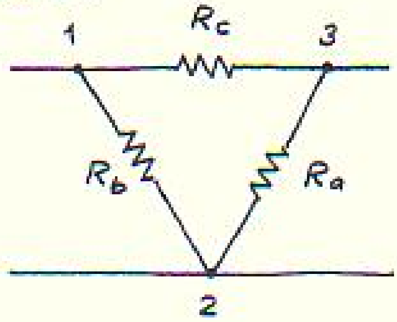
- We have Δ and want to get its equivalent star circuit

- Consider the Δ circuit shown; to be transformed into its equivalent star shown below :

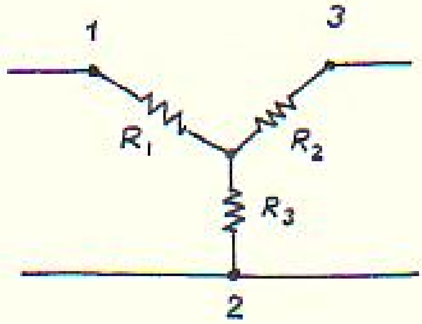
$$R_{12} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$R_{13} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_{23} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$



Delta



$$R_{12} = R_1 + R_3$$

$$R_{13} = R_1 + R_2$$

$$R_{23} = R_2 + R_3$$

$$\therefore \quad R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \quad \cdots (1)$$

and

$$R_1 + R_2 = \frac{R_c(R_b + R_c)}{R_a + R_b + R_c} \quad \cdots (2)$$

$$R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \quad \cdots (3)$$

Subtraction Eq.(3) from Eq.(1) and adding the resulting equation to Eq.(1) results in :

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}}$$

Similarly ;

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}}$$

and ;

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

IN GENERAL

> Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three resistors

* Wye to Delta Transformation

- We have Y connected circuit and want to get its equivalent Δ.
- Consider the Y circuit shown, its equivalent Δ is shown below ;

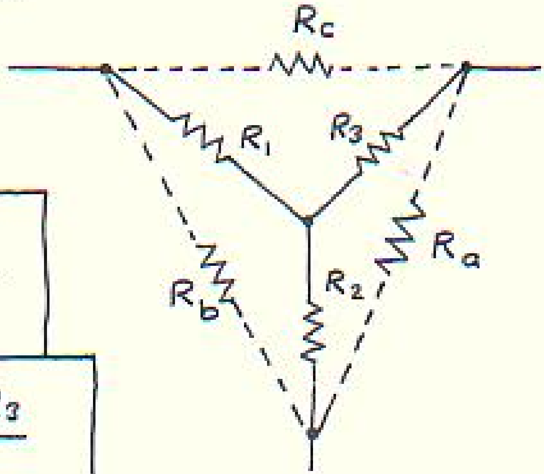
Using the previous sets of equations, then we have :

$$\boxed{R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}}$$

$$\boxed{R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}}$$

and

$$\boxed{R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}}$$



IN GENERAL

> Each resistor in the <u>Δ</u> network is the sum of all possible products of <u>Y</u> resistors taken two at a time, divided by the opposite <u>Y</u> resistor.

Notes :

* The Y and Δ networks are said to be **balanced** when :

$$R_1 = R_2 = R_3 = R_Y$$

and

$$R_a = R_b = R_c = R_\Delta$$

* Under **balance** condition, the conversion equations become :

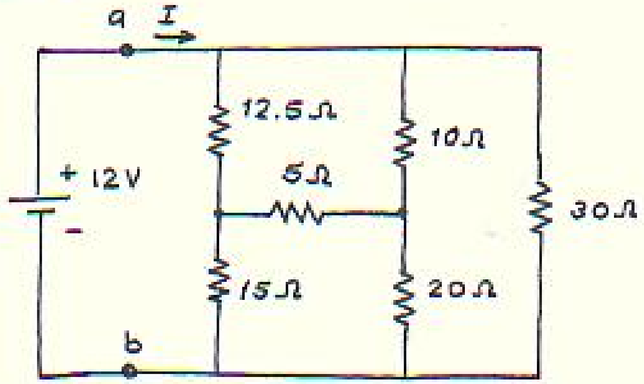
$$\boxed{R_Y = \frac{R_\Delta}{3}}$$

or

$$\boxed{R_\Delta = 3R_Y}$$

Example

Obtain the quivalent resistance R_{ab} for the circuit shown and use it to find the current I



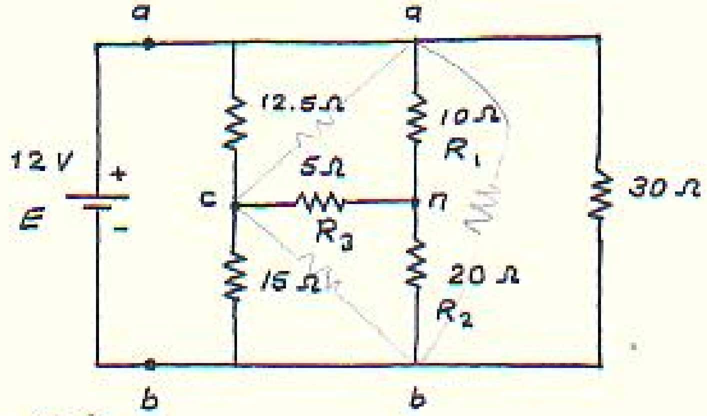
Solution

* We can't use the relations of series connected or parallel connected resistors to obtain R_{ab}.

* We try to use Δ–Y transformations or Y–Δ to get R_{ab}.

* If we transform the Y consisting of:

$$R_1 = 10\,\Omega$$
$$R_2 = 20\,\Omega$$
and $$R_3 = 5\,\Omega$$



∴ the equivalent Δ circuit contains:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$= \frac{10\times20 + 20\times5 + 5\times10}{10} = \frac{350}{10}$$

$$= 35\,\Omega$$

Similarly;

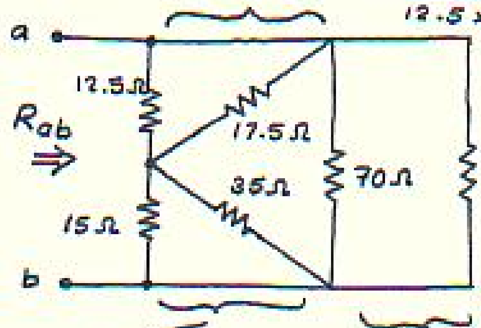
$$R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{20} = \frac{350}{20} = 17.5\,\Omega$$

and:

$$R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{5} = \frac{350}{5} = 70\,\Omega$$

12.5 // 17.5

$$= \frac{12.5\times17.5}{12.5\times17.5} = 7.3\,\Omega$$



15 // 35

$$= \frac{15\times35}{15+35} = 10\,\Omega$$

$$70 \,//\, 30 = \frac{70\times30}{70+30} = 21\,\Omega$$

$$\therefore R_{ab} = (7.3 + 10.5) \,//\, 30$$

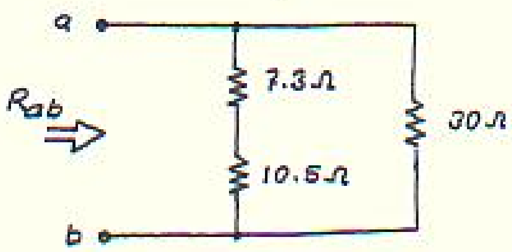
$$= \frac{17.8\times21}{17.8+21}$$

$$\Rightarrow R_{ab} = \underline{9.632\,\Omega}$$

$$\therefore I = \frac{E}{R_{eq}} = \frac{E}{R_{ab}}$$

$$= \frac{12}{9.632}$$

$$\Rightarrow I = \underline{1.246\ A}$$



Tutorial Sheet № 2

TS2

ملاحظة : جميع اسئلة الكتاب والامثلة المحلولة فيه مطلوبة

Example 1: Three resistors are connected in series across a 12 V battery. The first resistor has a value of 1Ω, the second has a voltage drop of 4 V, and the third has a power dissipation of 12 W.
Calculate the value of the circuit current.

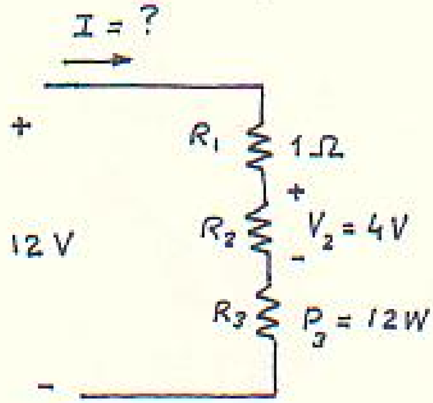
Solution:

We have

$$P_3 = I^2 R_3 = 12\ W \quad \text{—} ①$$

and

$$V_2 = I R_2 = 4\ V \quad \text{—} ②$$



From ②

$$\therefore\ I = \frac{4}{R_2}$$

From ① & ②

$$\left(\frac{4}{R_2}\right)^2 R_3 = 12$$

$$\therefore\ R_3 = \frac{3}{4} R_2^2$$

$R_2 = ?$
$R_3 = ?$

From the circuit shown, we have:

$$12 = I(R_1 + R_2 + R_3) = I(1 + R_2 + R_3)$$

Substituting for I and R_2, we have:

$$12 = \frac{4}{R_2}\left(1 + R_2 + \frac{3}{4}R_2^2\right)$$

$$\therefore\ 3R_2^2 - 8R_2 + 4 = 0$$

$$\therefore\ R_2 = \frac{8 \pm \sqrt{64-48}}{6} = 2\Omega \text{ or } \frac{2}{3}\Omega$$

$$\therefore\ R_3 = \frac{3}{4}R_2^2 \Longrightarrow R_3 = 3\Omega \text{ or } \frac{1}{3}\Omega$$

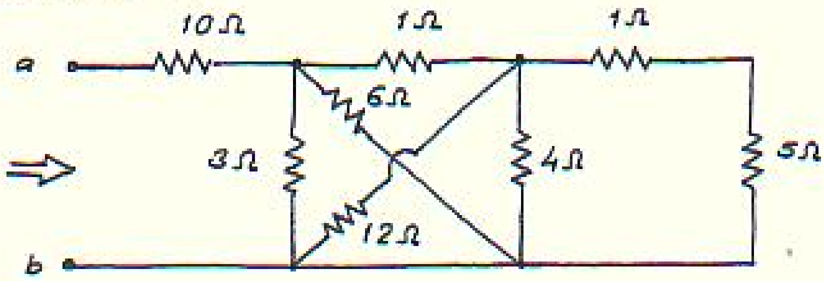
$$\therefore\ I = \frac{12}{R_1 + R_2 + R_3} = \frac{12}{1+2+3} = \underline{2\,A}$$

or

$$I = \frac{12}{1 + (\frac{2}{3}) + (\frac{1}{3})} = \underline{6\,A}$$

Practice Problem

: Calculate the equivalent resistance R_{ab} in the circuit shown.

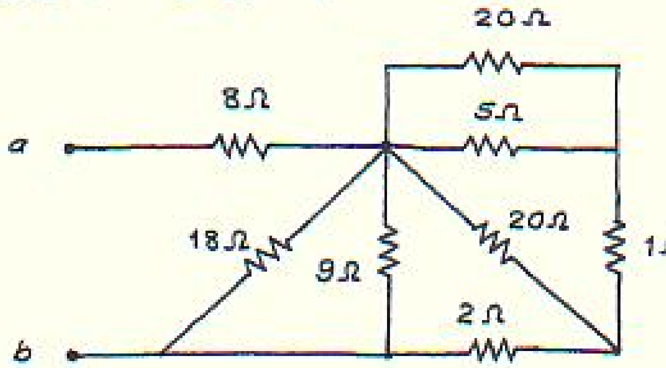


Answer

: $R_{ab} = 11.2\ \Omega$

Practice Problem

: Find R_{ab} for the circuit shown:

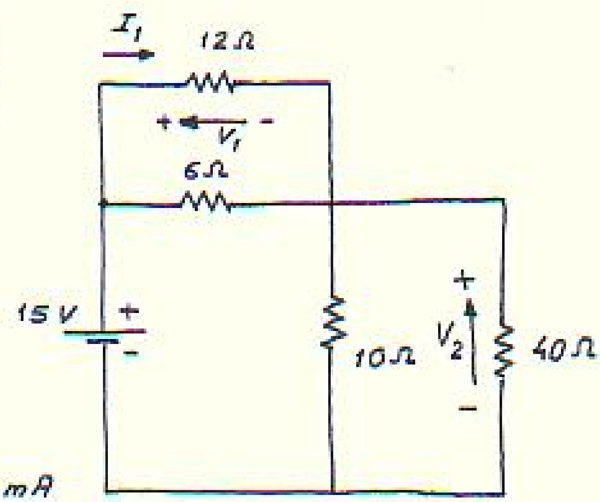


Answer

: $R_{ab} = 11\ \Omega$

Practice Problem

: Find V_1 and V_2 in the circuit shown. Also calculate I_1 and I_2 and the power dissipated in the 12 Ω and 40 Ω resistors.



Answer

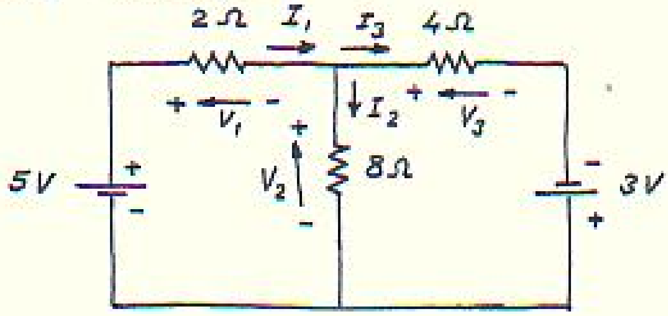
:

$V_1 = 5V$, $V_2 = 10V$

$I_1 = 416.7\ mA$, $I_2 = 250\ mA$

$P_1 = 2.083\ W$, $P_2 = 2.5\ W$

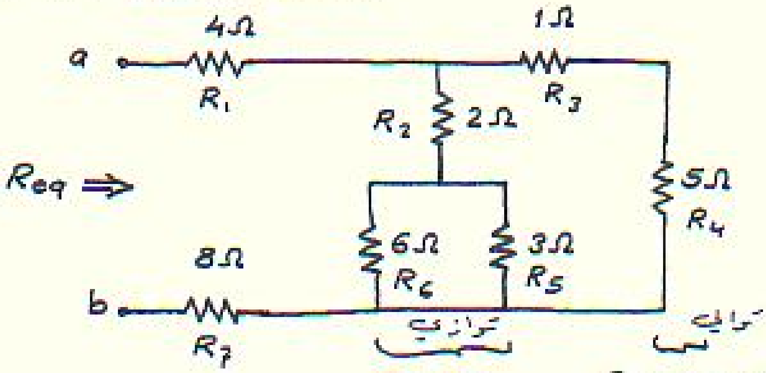
Practice Problem: Find the currents and voltages in the circuit shown, using Kirchoff's laws.



Answer: $V_1 = 3V$, $V_2 = 2V$, $V_3 = 5V$

$I_1 = 1.5 A$, $I_2 = 0.25 A$, $I_3 = 1.25 A$

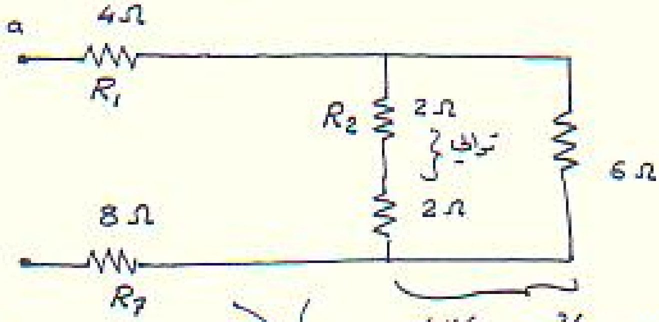
Example: Find R_{eq} for the circuit shown.



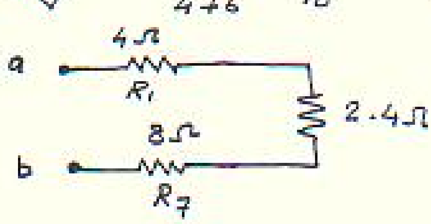
Solution:

$R_5 // R_6 = \frac{6 \times 3}{6+3} = 2\,\Omega$

$R_4 + R_3 = 5 + 1 = 6$



$\frac{4 \times 6}{4+6} = \frac{24}{10} = 2.4\,\Omega$

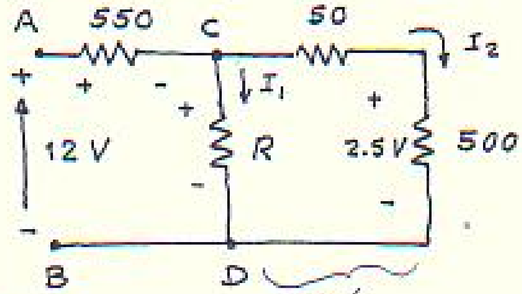


$\therefore R_{eq} = R_{ab} = 4 + 2.4 + 8$

$= \underline{14.4\,\Omega}$

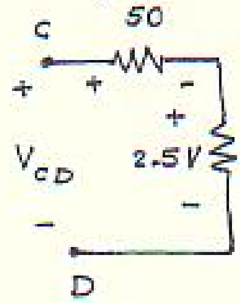
TS2

Example : What is the value of the unknown resistor R in the circuit shown, if the voltage drop across the $500\,\Omega$ resistor is 2.5 V ? All resistors are in Ohms.



Solution

Using the voltage divider rule;



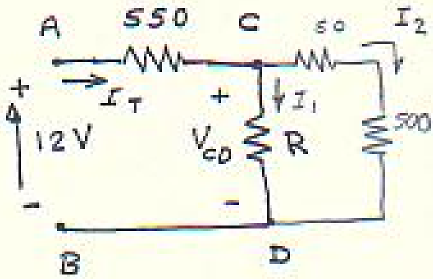
* The voltage drop across the $50\,\Omega$ resistor is :

$$V_{50\,\Omega} = 2.5 \times \frac{50}{500}$$

$$= 0.25\ V$$

$$\therefore\ V_{CD} = 2.5 + 0.25$$

$$= 2.75\ V$$



* $V_{550\,\Omega} = 12 - V_{CD}$

$$= 12 - 2.75 = 9.25\ V$$

$$\therefore\ I_T = \frac{V_{550\,\Omega}}{550} = \frac{9.25}{550} = 0.0168\ A$$

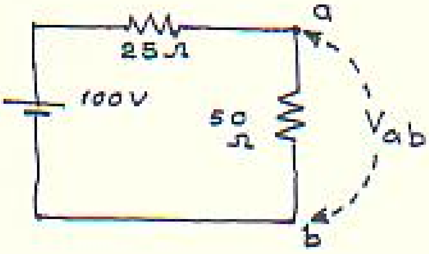
$$I_2 = \frac{V_{500}}{500} = \frac{2.5}{500} = 0.005\ A$$

$$\therefore\ I_1 = I - I_2 = 0.0168 - 0.005 = 0.0118\ A$$

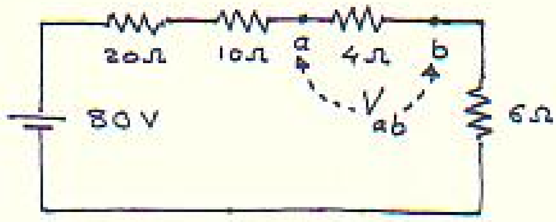
$$\therefore\ R = \frac{V_{CD}}{I_1} = \frac{2.75}{0.0118} = 233\ \Omega$$

$\boxed{TS2}$

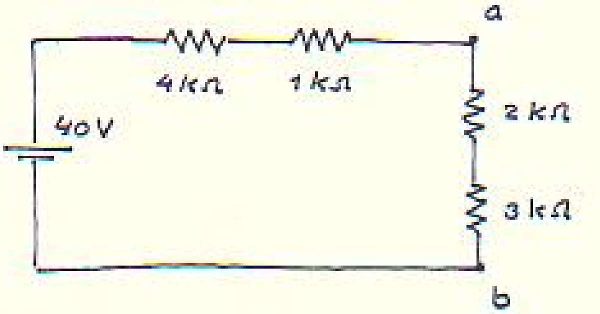
Example: For the networks shown, find V_{ab} (with polarity).



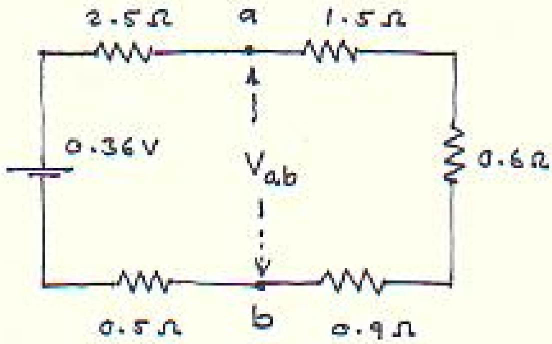
(a)



(b)

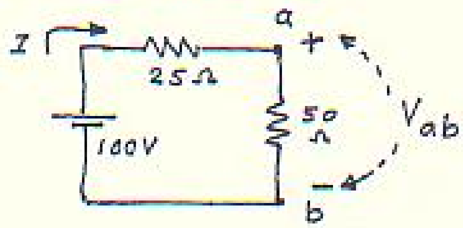


(c)



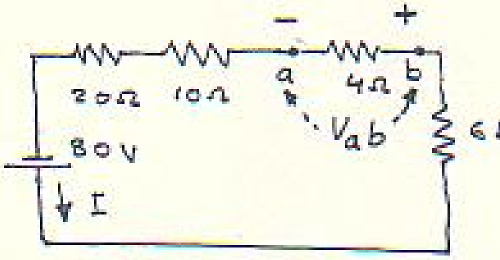
Solution: (a). Using the voltage divider rule; then

$$\therefore \underset{+\,-}{V_{ab}} = 100 \cdot \frac{50}{25+50} = 66.67 \text{ V}$$



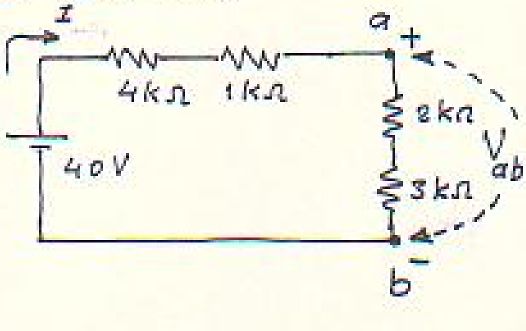
(b). Using the voltage divider rule, then:

$$\therefore \underset{-\,+}{V_{ab}} = 80 \frac{4}{20+10+4+6} = 8 \text{ V}$$



(c). Using the voltage divider rule, then:

$$\underset{+\,-}{V_{ab}} = \frac{(2+3)\times 10^{3}}{(4+1+2+3)\times 10^{3}} = 20 \text{ V}$$

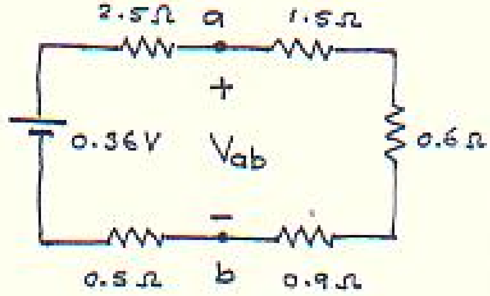


TS2

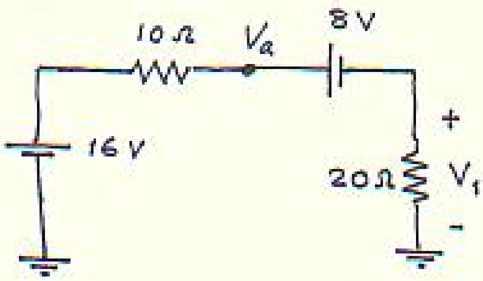
(d). Using the voltage divider rule, then;

$$\underset{+\ -}{V_{ab}} = 0.36\,\frac{(1.5+0.6+0.9)}{2.5+1.5+0.6+0.9+0.5}$$

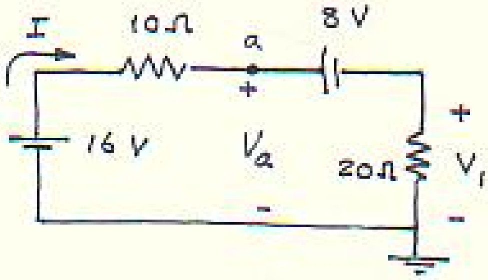
$$= 0.18\ V$$



Example: Determine the voltage V_a and V_1, for the network shown.



Solution: Redraw the circuit to be as shown;



using KVL

$$V_a = 8 + V_1$$
$$= 8 + I(20)$$

$$I = ?$$

$$I = \frac{16-8}{20+10}$$
$$= \frac{8}{30}$$

$$\therefore V_a = 8 + \frac{8}{30}(20) = 8 + \frac{16}{3} = \frac{40}{3}\ V$$
$$= \underline{13.33\ V}$$

OR

using KVL

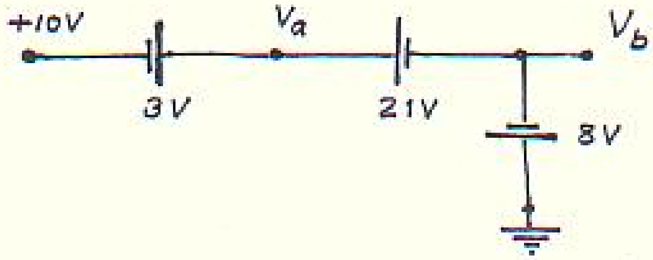
$$16 = I(10) + V_a \Rightarrow 16 = \frac{8}{30}(10) + V_a$$

$$\therefore V_a = 16 - \frac{8}{3} = \frac{40}{3} = \underline{13.33\ V}$$

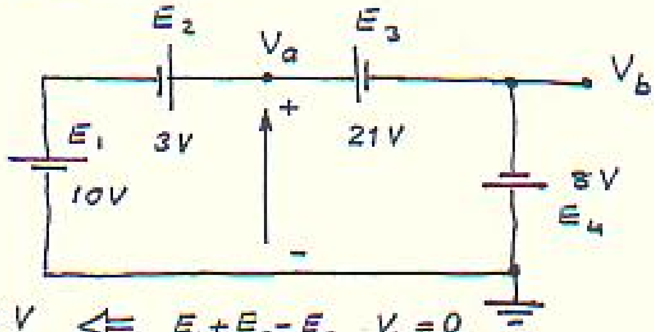
$$V_1 = I(20) = \frac{8}{3}(20) = \frac{16}{3} = \underline{5.33\ V}$$

TS2

Example: Determine the voltage V_{ab} for the network shown;



Solution: Redraw the cct. to be as shown;



$V_b = -8\ V$

$V_b = -E_4$ directly the drop of point $\underline{b}$ with respect to the ground.

$\boxed{OR}$ using KVL

$V_b = 10 + 3 - 21 = -8\ V \Leftarrow E_1 + E_2 - E_3 \quad V_b = 0$

KVL $\quad V_a = 10 + 3 = 13\ V \Leftarrow E_1 + E_2 - V_a = 0$

$\therefore V_{ab} = V_a - V_b$

$= 13 - (-8) = 21\ V$

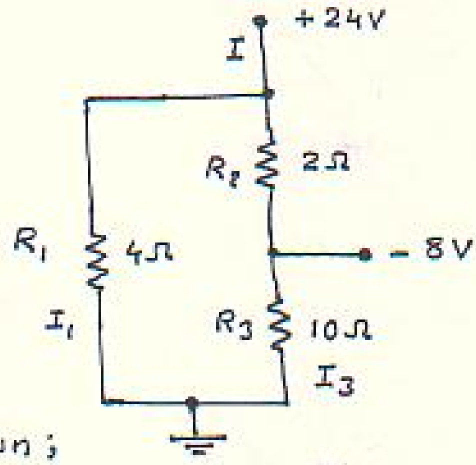
OR

$V_a - E_3 + E_4 = 0$

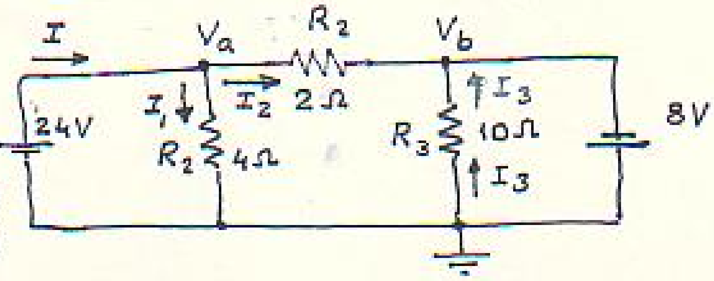
$\therefore V_a = E_3 - E_4$

$= 21 - 8 = 13\ V$

Example: For the network shown, find the currents I, I_1 and I_3 and indicate their directions.



Solution: Redraw the cct to be as shown;



$I_3 = \dfrac{V_b}{R_3} = \dfrac{-8}{10} = -0.8\ A$

$I_1 = \dfrac{V_a}{R_2} = \dfrac{24}{4} = 6\ A$

$I_2 = \dfrac{V_a - V_b}{R_2} = \dfrac{24+8}{2} = 16\ A$

$I = I_1 + I_2 = 6 + 16 = 24 A$