**Republic of Iraq**

**Ministry of Higher Education and Scientific Research**

**Al-Mustaqbal University College**

**Chemical Engineering and Petroleum Industries Department**

# Subject: Materials Science and Engineering

# 2nd Class

# Lecture five

# Stress and Strain

## Stress (σ)

When a material is subjected to an external force, a resisting force is set up within the component. The internal resistance force per unit area acting on a material or intensity of the forces distributed over a given section is called the stress at a point

• It uses original cross section area of the specimen and also known as engineering stress or conventional stress.

$$\sigma = \frac{F}{A}$$

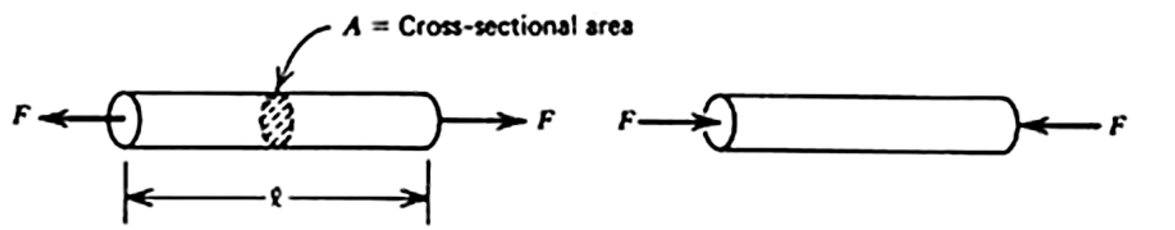
• P is expressed in Newton (N) and A, original area, in square meters (m2), the stress σ will be expresses in N/ m2. This unit is called Pascal (Pa).

• As Pascal is a small quantity, in practice, multiples of this unit is used.

1 kPa = $10^3$ Pa = $10^3$ N/ m2 (kPa = Kilo Pascal)

1 MPa = $10^6$ Pa = $10^6$ N/ m2 = 1 N/mm2 (MPa = Mega Pascal)

1 GPa = $10^9$ Pa = $10^9$ N/ m2 (GPa = Giga Pascal)




a) Tensile stress applied to a rod

b) Compressional stress applied to a rod

## Types of Stress

### 1-Tensile stress ($\sigma_t$)

If $\sigma > 0$ the stress is tensile. i.e. The fibres of the component tend to elongate due to the external force. A member subjected to an external force tensile P and tensile stress distribution due to the force is shown in the given figure.

### 2- Compressive stress ($\sigma_c$)

If $\sigma < 0$ the stress is compressive. i.e. The fibres of the component tend to shorten due to the external force. A member subjected to an external compressive force P and compressive stress distribution due to the force is shown in the given figure.

### 3-Shear stress ($\sigma_s$)

When forces are transmitted from one part of a body to other, the stresses developed in a plane parallel to the applied force are the shear stress. **Shear stress acts parallel to plane of interest.** Forces P is applied transversely to the member AB as shown. The corresponding internal forces act in the plane of section C and are called shearing

# Strain

The displacement per unit length ***(dimensionless)*** is known as strain.

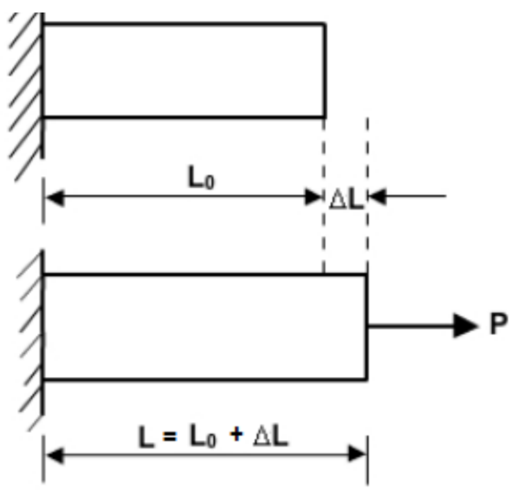
- **Tensile strain ($\varepsilon_t$)**

  The elongation per unit length as shown in the figure is known as tensile strain.

  $$\varepsilon_t = \Delta L / L_o$$

  It is engineering strain or conventional strain.

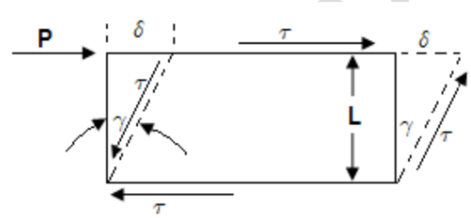
  Here we divide the elongation to original length not actual length ($L_o$ + D L)



- **Compressive strain ($\varepsilon_c$)**

If the applied force is compressive then the reduction of length per unit length is known as compressive strain. It is negative. Then $\varepsilon_c = (-\Delta L) / L_o$

- **Shear Strain ( *g* ):** When a force P is applied tangentially to the element shown. Its edge displaced to dotted line. Where *d* is the lateral displacement of the upper face of the element relative to the lower face and L is the distance between these faces. Then the shear strain is $(g) = \frac{d}{L}$




## <u>Young's modulus (E)</u>

is the modulus of elasticity under tension or compression. In other words, it describes how stiff a material is or how readily it bends or stretches. Young's modulus relates stress (force per unit area) to strain (proportional deformation) along an axis or line.

The basic principle is that a material undergoes elastic deformation when it is compressed or extended, returning to its original shape when the load is removed. More deformation occurs in a flexible material compared to that of a stiff material.

-A low Young's modulus value means a solid is elastic.

-A high Young's modulus value means a solid is inelastic or stiff.

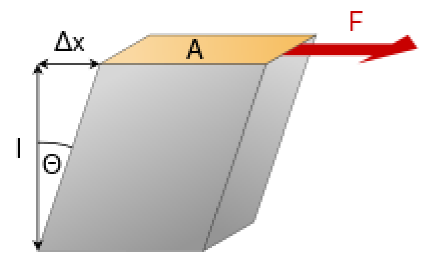
$$\boxed{E = \sigma / \varepsilon = (F/A) / (\Delta L/L_0) = FL_0 / A\Delta L = mgL_0 / \pi r^2 \Delta L}$$

- E is Young's modulus
- σ is the uniaxial stress (tensile or compressive).

- $\varepsilon$ is the strain, which is the change in length per original length
- F is the force of compression or extension
- A is the cross-sectional surface area.

## Shear Modulus

$$\boxed{G = \frac{Fl}{A\Delta x}}$$




## Passion ratio

*$\mu = - \varepsilon_t / \varepsilon_l$ (1)*

*where*

*$\mu$ = Poisson's ratio*

*$\varepsilon_t$ = transverse strain (m/m, ft/ft)*

*$\varepsilon_l$ = longitudinal or axial strain (m/m, ft/ft)*

## Examples

**Example – 1:**

**A wire 2 m long and 2 mm in diameter, when stretched by weight of 8 kg has its length increased by 0.24 mm. Find the stress, strain and Young's modulus of the material of the wire. g = 9.8 m/s²**

**Given:** Initial length of wire = L = 2 m, Diameter of wire = 2 mm, Radius of wire 2/2 = 1 mm = $1 \times 10^{-3}$ m, Weight attached = m = 2 kg, Increase in length = l = 0.24 mm = $0.24 \times 10^{-3}$ m, g = 9.8 m/s².

**To Find:** Stress =? Strain =? Young's modulus of material = Y =?

**Solution:**

$$\text{Stress} = F / A = mg / \pi r^2$$

$$\therefore \text{Stress} = (8 \times 9.8) / (3.142 \times (1 \times 10^{-3})^2)$$

$$\therefore \text{Stress} = (8 \times 9.8) / (3.142 \times 1 \times 10^{-6})$$

$$\therefore \text{Stress} = 2.5 \times 10^{7} \text{ N/m}^2$$

$$\text{Strain} = l / L = 0.24 \times 10^{-3} / 2$$

$$\therefore \text{Strain} = 0.12 \times 10^{-3} = 1.2 \times 10^{-4}$$

Now, Young's modulus of elasticity= Y = Stress / Strain

$$\therefore Y = (2.5 \times 10^{7}) / (1.2 \times 10^{-4})$$

$$\therefore Y = 2.08 \times 10^{11} \text{ N/m}^2$$

**Ans.:** Stress = $2.5 \times 10^{7}$ N/m², Strain = $1.2 \times 10^{-4}$ , Yong's modulus of elasticity= $2.08 \times 10^{11}$ N/m²

Q.1: The thickness of a metal plate is 0.3 inches. We drill a hole of the radius of 0.6 inches on the plate. If, the shear strength is $FA = 4 \times 10^4$ lb square inch, determine the force we need to make the hole.

Solution: The shear stress is exerted over the surface of the cylindrical shape.

Therefore, the area of the cylindrical surface,

$= 2\pi rh = 2 \times 3.14 \times 0.06 \times 0.30$

= 0.11304 square inch

Given, $FA = 4 \times 10^4$ lb square inch

Thus, to drill the hole, the force needed $= 4 \times 10^4 \times 0.11304$

Force = 4521.6 lb

Example:-

**A mild steel wire of radius 0.5 mm and length 3 m is stretched by a force of 49 N. calculate a) longitudinal stress, b) longitudinal strain c) elongation produced in the body if Y for steel is $2.1 \times 10^{11}$ N/m².**

**Given:** Initial length of wire = L = 3 m, radius of wire = 0.5 mm = $0.5 \times 10^{-3}$ m = $5 \times 10^{-4}$ m,Force applied =49 N, Young's modulus for steel = Y = $2.1 \times 10^{11}$ N/m².

**To Find:** Stress =? Strain =? elongation =?

**Solution:**

$$\text{Stress} = F / A = mg / \pi r^2$$

$$\therefore \text{Stress} = 49 / (3.142 \times (5 \times 10^{-4})^2)$$

$$\therefore \text{Stress} = 49 / (3.142 \times 25 \times 10^{-8})$$

$$\therefore \text{Stress} = 6.238 \times 10^7 \text{ N/m}^2$$

$$\text{Now, } Y = \text{Stress} / \text{Strain}$$

$$\therefore \text{Strain} = \text{Stress} / Y = (6.238 \times 10^7) / (2.1 \times 10^{11})$$

$$\therefore \text{Strain} = 2.970 \times 10^{-4}$$

$$\text{Now, Strain} = l / L$$

$$\therefore l = \text{Strain} \times L$$

$$\therefore l = 2.970 \times 10^{-4} \times 3$$

$$\therefore l = 8.91 \times 10^{-4} \text{ m} = 0.891 \times 10^{-3} \text{ m} = 0.891 \text{ mm}$$

Ans.: Stress = $6.238 \times 10^7$ N/m², Strain = $2.970 \times 10^{-4}$, Elongation = 0.891 mm.

# Stress and Strain/Part 2

## Example

A Steel rod (E=200 GPa) has a circular cross section and is 10m long. Determine the minimum diameter if the rod must hold a 30 kN tensile force without deforming more than 5mm. Assume the steel stays in the elastic region. Note, 1 GPa = $10^9$ Pa.

Solution:

*Knowing the initial length and the change in length permits the calculation of strain.*

$$\varepsilon = \frac{\Delta l}{l_o} = \frac{5mm(\frac{1m}{1000mm})}{10\ m} = 0.0005$$

*In the elastic region, the stress $\sigma$ is directly proportional to the strain $\varepsilon$, by the Modulus of Elasticity, E*

$$\frac{F}{A_o} = \sigma = E\varepsilon$$

*Rearranging, substituting values and converting units,*

$$\sigma = E\varepsilon = (200\ GPa)0.0005 = 0.1\ GPa = 0.1 \times 10^9 Pa = 0.1 \times 10^9\ N/m^2$$

*The definition of stress $\sigma = \frac{F}{A_o}$ can be used to find the required cross section area.*

$$A_o = \frac{F}{\sigma} = \frac{30kN\left(\frac{1000N}{kN}\right)}{0.1 \times 10^9\ N/m^2} = 0.0003m^2(\frac{1000mm}{m})(\frac{1000mm}{m}) = 300mm^2$$

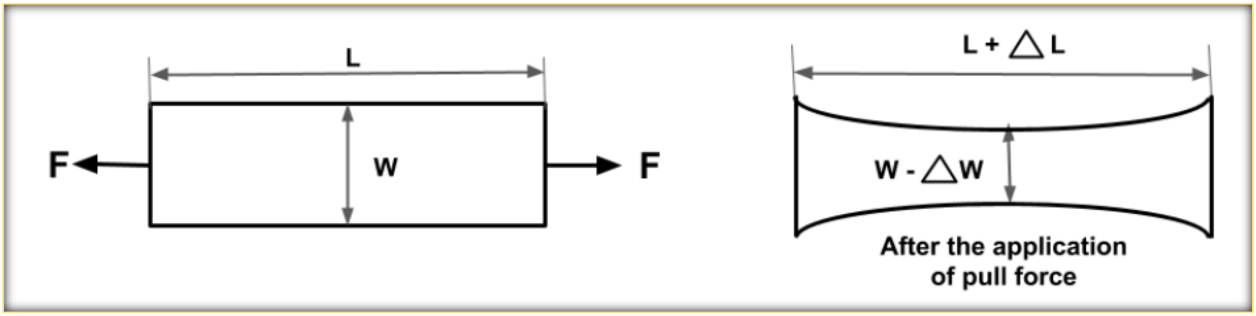
*The diameter, $d_o$ is solved from the area of a circle*

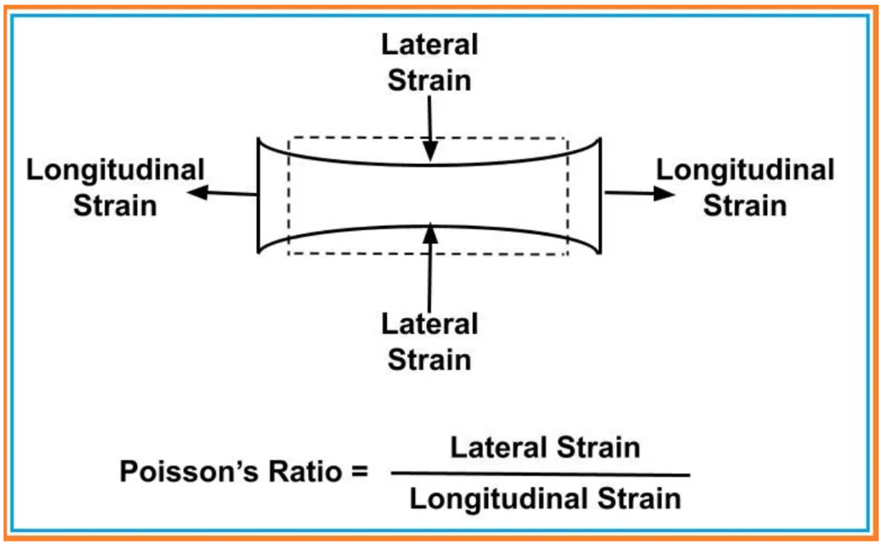
$$A_o = \frac{\pi d_o^2}{4}$$

$$d_o^2 = \frac{A_o 4}{\pi}$$

$$d_o = \sqrt{\frac{A_o 4}{\pi}} = \sqrt{\frac{300mm^2 * 4}{3.14}} = 19.5mm$$

# <u>Poisson Ratio</u>

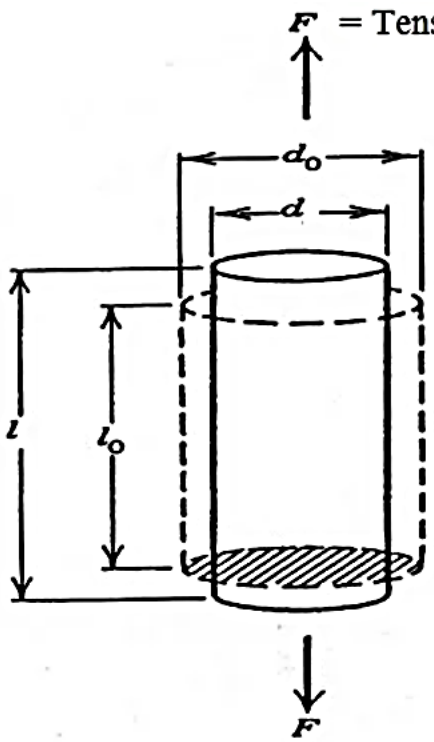



$$\nu = \text{Poisson's Ratio} = \frac{\text{Strain in the Direction perpendicular to load}}{\text{Strain in the Direction of load}}$$

$$\text{Poisson's Ratio} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

***<u>lateral strain</u>**, also known as **transverse strain**, is defined as the ratio of the change in diameter of a circular bar of a material to its diameter due to deformation in the longitudinal direction. It occurs when under the action of a <u>longitudinal stress,</u> a body will extend in the direction of the stress and contract in the transverse or lateral direction (in the case of tensile stress). When put under compression, the body will contract in the direction of the stress and extend in the transverse or lateral direction. It is a dimensionless quantity, as it is a ratio between two quantities of the same dimension.




$$\text{Poisson Ratio} = \frac{\text{Lateral Strain}}{\text{Linear Strain}}$$

$$\text{Linear strain} = \frac{\Delta l}{l} = \frac{l - l_o}{l_o}$$

$$\text{Lateral Strain} = \frac{\Delta d}{d_o} = \frac{d - d_o}{d_o}$$

**Fig. 2: Poisson's ratio explanation**

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Poisson's Ratio=Transverse (Lateral) Strain/Axial (Longitudinal) Strain

Let's understand this philosophy using the example in Fig. 2. In this image, A tensile force (F) is applied in a bar of diameter do and length lo. With the action of this force F, the bar elongates and final length in l. Also, the diameter reduces and the final diameter is d.

**Problem:**

Consider a steel bar of 100 mm length and 50 mm width. If after the application of 50-newton force, steel bar length increases to 102 mm. What will be the change in width?

**Solution:**

Let's consider steel bar width is reduced by dW mm

Poisson Ratio for Steel = 0.3

Longitudinal Strain = (102-100) / 100 = 0.02

According to ***poisson ratio formula***:

$$0.3 = (dW / 50) /0 .02$$

$$(dW / 50) = 0.3 \times 0.02 = 0.006$$

$$dW = 0.3$$

Example

**When a brass rod of diameter 6 mm is subjected to a tension of 5 $\times 10^3$ N, the diameter changes by $3.6 \times 10^{-4}$ cm. Calculate the longitudinal strain and Poisson's ratio for brass given that Y for the brass is $9 \times 10^{10}$ N/m².**

**Given:** Diameter of rod = D = 6 mm, Radius of wire = 6/2 = 3 mm = $3 \times 10^{-3}$ m, Load F = $5 \times 10^3$ N, Change in diameter = d = $3.6 \times 10^{-4}$ cm = $3.6 \times 10^{-6}$ m, Y for the brass is $9 \times 10^{10}$ N/m².

**Solution:**

$$Y = \text{Longitudinal Stress} / \text{Longitudinal Strain}$$

$$\therefore Y = F / (A \times \text{Longitudinal Strain})$$

$$\therefore \text{Longitudinal Strain} = F / (A \times Y)$$

$$\therefore \text{Longitudinal Strain} = F / (\pi r^2 \times Y)$$

$$\therefore \text{Longitudinal Strain} = 5 \times 10^3 / (3.142 \times (3 \times 10^{-3})^2 \times 9 \times 10^{10})$$

$$\therefore \text{Longitudinal Strain} = 5 \times 10^3 / (3.142 \times 9 \times 10^{-6} \times 9 \times 10^{10})$$

$$\therefore \text{Longitudinal Strain} = 1.96 \times 10^{-3}$$

Now, Lateral strain = $d / D = (3.6 \times 10^{-6}) / (6 \times 10^{-3}) = 6 \times 10^{-4}$

$$\text{Poisson's ratio} = \text{Lateral strain} / \text{Longitudinal strain}$$

$$\therefore \text{Poisson's ratio} = (6 \times 10^{-4}) / (1.96 \times 10^{-3}) = 0.31$$

**Ans:** Longitudinal strain is $1.96 \times 10^{-3}$ and Poisson's ratio is 0.31.