SOLUTION The shear areas are

$$A_{gv} = \frac{3}{8}(7.5) = 2.813 \text{ in.}^2$$

and, since there are 2.5 hole diameters,

$$A_{nv} = \frac{3}{8}\left[7.5 - 2.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 1.875 \text{ in.}^2$$

The tension area is

$$A_{nt} = \frac{3}{8}\left[1.5 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.3750 \text{ in.}^2$$

(The factor of 0.5 is used because there is one-half of a hole diameter in the tension section.)

Since the block shear will occur in an angle, $U_{bs} = 1.0$, and from AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$
$$= 0.6(58)(1.875) + 1.0(58)(0.3750) = 87.00 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(2.813) + 1.0(58)(0.3750) = 82.51 \text{ kips}$$

The nominal block shear strength is therefore 82.51 kips.

ANSWER a. The design strength for LRFD is $\phi R_n = 0.75(82.51) = 61.9$ kips.

b. The allowable strength for ASD is $\dfrac{R_n}{\Omega} = \dfrac{82.51}{2.00} = 41.3$ kips.

3.6 DESIGN OF TENSION MEMBERS

The design of a tension member involves finding a member with adequate gross and net areas. If the member has a bolted connection, the selection of a suitable cross section requires an accounting for the area lost because of holes. For a member with a rectangular cross section, the calculations are relatively straightforward. If a rolled shape is to be used, however, the area to be deducted cannot be predicted in advance because the member•s thickness at the location of the holes is not known.

A secondary consideration in the design of tension members is slenderness. If a structural member has a small cross section in relation to its length, it is said to be *slender*. A more precise measure is the slenderness ratio, L/r, where L is the member length and r is the minimum radius of gyration of the cross-sectional area. The minimum radius of gyration is the one corresponding to the minor principal axis of the cross section. This value is tabulated for all rolled shapes in the properties tables in Part 1 of the *Manual*.

Although slenderness is critical to the strength of a compression member, it is inconsequential for a tension member. In many situations, however, it is good practice to limit the slenderness of tension members. If the axial load in a slender tension member is removed and small transverse loads are applied, undesirable vibrations or deflections might occur. These conditions could occur, for example, in a slack bracing rod subjected to wind loads. For this reason, the user note in AISC D1 suggests a maximum slenderness ratio of 300. It is only a recommended value because slenderness has no structural significance for tension members, and the limit may be exceeded when special circumstances warrant it. This limit does not apply to cables, and the user note explicitly excludes rods.

The central problem of all member design, including tension member design, is to find a cross section for which the required strength does not exceed the available strength. For tension members designed by LRFD, the requirement is

$$P_u \le \phi_t P_n \quad \text{or} \quad \phi_t P_n \ge P_u$$

where P_u is the sum of the factored loads. To prevent yielding,

$$0.90 F_y A_g \ge P_u \quad \text{or} \quad A_g \ge \frac{P_u}{0.90 F_y}$$

To avoid fracture,

$$0.75 F_u A_e \ge P_u \quad \text{or} \quad A_e \ge \frac{P_u}{0.75 F_u}$$

For allowable strength design, if we use the allowable *stress* form, the requirement corresponding to yielding is

$$P_a \le F_t A_g$$

and the required gross area is

$$A_g \ge \frac{P_a}{F_t} \quad \text{or} \quad A_g \ge \frac{P_a}{0.6 F_y}$$

For the limit state of fracture, the required effective area is

$$A_e \ge \frac{P_a}{F_t} \quad \text{or} \quad A_e \ge \frac{P_a}{0.5 F_u}$$

The slenderness ratio limitation will be satisfied if

$$r \ge \frac{L}{300}$$

where r is the minimum radius of gyration of the cross section and L is the member length.

EXAMPLE 3.11

A tension member with a length of 5 feet 9 inches must resist a service dead load of 18 kips and a service live load of 52 kips. Select a member with a rectangular cross section. Use A36 steel and assume a connection with one line of ⅞-inch-diameter bolts.

LRFD SOLUTION

$$P_u = 1.2D + 1.6L = 1.2(18) + 1.6(52) = 104.8 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{P_u}{0.90F_y} = \frac{104.8}{0.90(36)} = 3.235 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{P_u}{0.75F_u} = \frac{104.8}{0.75(58)} = 2.409 \text{ in.}^2$$

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.235}{1} = 3.235 \text{ in.}$$

Try a $1 \times 3\frac{1}{2}$ cross section.

$$A_e = A_n = A_g - A_{\text{hole}}$$

$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.409 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

$$I_{\min} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$

$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$

$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

ANSWER Use a PL $1 \times 3\frac{1}{2}$.

ASD SOLUTION

$$P_a = D + L = 18 + 52 = 70.0 \text{ kips}$$

For yielding, $F_t = 0.6F_y = 0.6(36) = 21.6$ ksi, and

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{70}{21.6} = 3.24 \text{ in.}^2$$

For fracture, $F_t = 0.5F_u = 0.5(58) = 29.0$ ksi, and

$$\text{Required } A_e = \frac{P_a}{F_t} = \frac{70}{29.0} = 2.414 \text{ in.}^2$$

(The rest of the design *procedure* is the same as for LRFD. The numerical results may be different)

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.241}{1} = 3.241 \text{ in.}$$

Try a 1 × 3 ½ cross section.

$$A_e = A_n = A_g - A_{\text{hole}}$$
$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.414 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

$$I_{\min} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$
$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$
$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

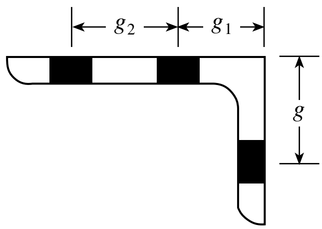
ANSWER Use a PL 1 × 3½.

Example 3.11 illustrates that once the required area has been determined, the procedure is the same for both LRFD and ASD. Note also that in this example, the required areas are virtually the same for LRFD and ASD. This is because the ratio of live load to dead load is approximately 3, and the two approaches will give the same results for this ratio.

The member in Example 3.11 is less than 8 inches wide and thus is classified as a bar rather than a plate. Bars should be specified to the nearest ¼ inch in width and to the nearest ⅛ inch in thickness (the precise classification system is given in Part 1 of the *Manual* under the heading "Plate Products"). It is common practice to use the PL (Plate) designation for both bars and plates.

If an angle shape is used as a tension member and the connection is made by bolting, there must be enough room for the bolts. Space will be a problem only when there are two lines of bolts in a leg. The usual fabrication practice is to punch or drill holes in standard locations in angle legs. These hole locations are given in Table 1-7A in Part 1 of the *Manual*. This table is located at the end of the dimensions and properties table for angles. Figure 3.24 presents this same information. Gage distance g applies when there is one line of bolts, and g_1 and g_2 apply when there are two lines. Figure 3.24 shows that an angle leg must be at least 5 inches long to accommodate two lines of bolts.

FIGURE 3.24



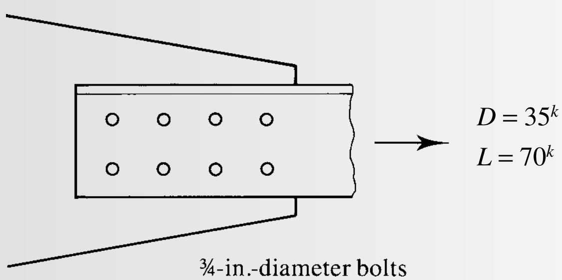
Usual Gages for Angles (inches)

Leg	8	7	6	5	4	3½	3	2½	2	1¾	1½	1⅜	1¼	1
g	4½	4	3½	3	2½	2	1¾	1⅜	1⅛	1	⅞	⅞	¾	⅝
g_1	3	2½	2¼	2										
g_2	3	3	2½	1¾										

EXAMPLE 3.12

Select an unequal-leg angle tension member 15 feet long to resist a service dead load of 35 kips and a service live load of 70 kips. Use A36 steel. The connection is shown in Figure 3.25.

FIGURE 3.25



LRFD SOLUTION

The factored load is

$$P_u = 1.2D + 1.6L = 1.2(35) + 1.6(70) = 154 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{154}{0.90(36)} = 4.75 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{154}{0.75(58)} = 3.54 \text{ in.}^2$$

The radius of gyration should be at least

$$\frac{L}{300} = \frac{15(12)}{300} = 0.6 \text{ in.}$$

To find the lightest shape that satisfies these criteria, we search the dimensions and properties table for the unequal-leg angle that has the smallest acceptable gross area and then check the effective net area. The radius of gyration can be checked by inspection. There are two lines of bolts, so the connected leg must be at least 5 inches long (see the usual gages for angles in Figure 3.24). Starting at either end of the table, we find that the shape with the smallest area that is at least equal to 4.75 in.^{2} is an L6 × 4 × ½ with an area of 4.75 in.^{2} and a minimum radius of gyration of 0.864 in.

Try L6 × 4 × ½.

$$A_n = A_g - A_{\text{holes}} = 4.75 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 3.875 \text{ in.}^2$$

Because the length of the connection is not known, Equation 3.1 cannot be used to compute the shear lag factor U. Since there are four bolts in the direction of the load, we will use the alternative value of $U = 0.80$.

$$A_e = A_n U = 3.875(0.80) = 3.10 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)}$$

Try the next larger shape from the dimensions and properties tables.

Try L5 × 3½ × ⅝ ($A_g = 4.93$ in.^{2} and $r_{\min} = 0.746$ in.)

$$A_n = A_g - A_{\text{holes}} = 4.93 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 3.836 \text{ in.}^2$$

$$A_e = A_n U = 3.836(0.80) = 3.07 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)}$$

(Note that this shape has slightly more gross area than that produced by the previous trial shape, but because of the greater leg thickness, slightly more area is deducted for the holes.) Passing over the next few heavier shapes,

Try L8 × 4 × ½ ($A_g = 5.80$ in.^{2} and $r_{\min} = 0.863$ in.)

$$A_n = A_g - A_{\text{holes}} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_n U = 4.925(0.80) = 3.94 \text{ in.}^2 > 3.54 \text{ in.}^2 \qquad \text{(OK)}$$

*The notation N.G. means "No Good."