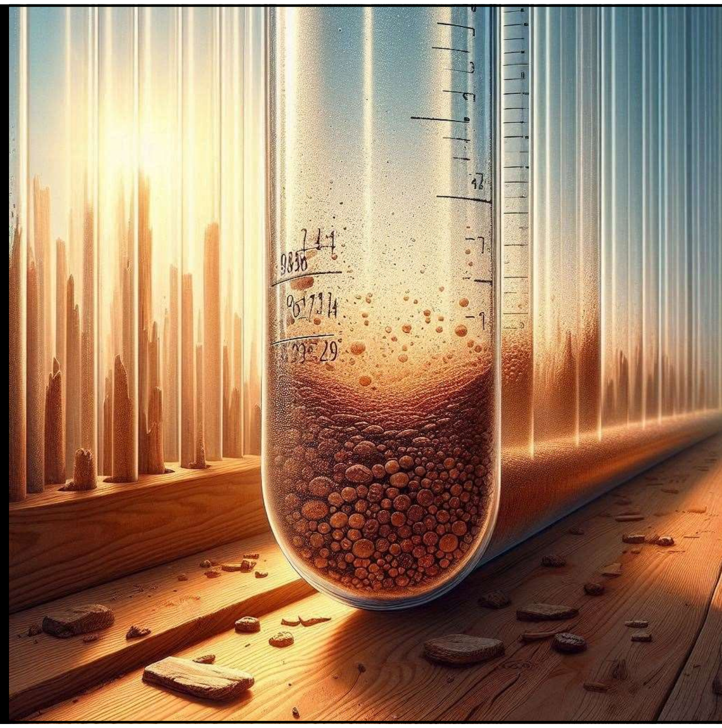
Mass Transfer

Fall 2024

Lecture #3

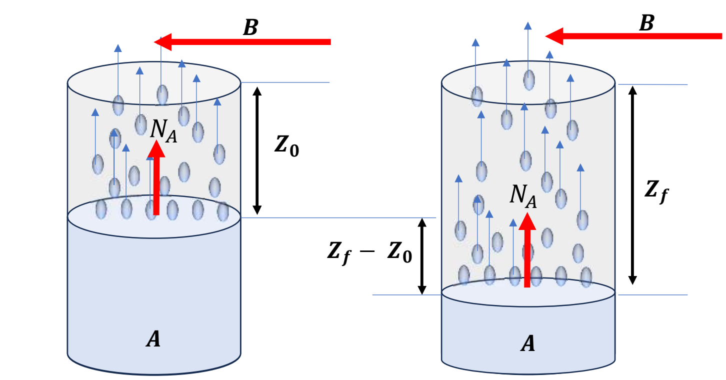
Dr. Haydar Aljaafari



Molecular Diffusion

c) Diffusion through varying path length

A is diffusing through a constant cross-sectional area, component B is non-diffusing, and A level is changing with time.



At t = 0 sec

At t = t_f sec

الحالات

- **الزمن** اللازم لتبخير مقدار معين من السائل
- **مقدار الانخفاض بمستوى السائل** خلال زمن معلوم
- **الانتشارية** لزمن معلوم ومقدار انخفاض معلوم

Notes

- A diffuse in Stagnant B ($N_B = 0$)
- The Cross section area is constant
- The surface level of is changing with time
- The diffusion is starting from the surface
- $P_{A1} = P_A^0$ (Point 1 is the Surface)
- $P_{A2} = 0$

Molecular Diffusion

c) Diffusion through varying path length

For Non-diffusing (B) $(N_B = 0)$

$$N_A = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).\frac{P_{A2}-P_{A1}}{(Z_2-Z_1)}$$

Z is changing with time (t)

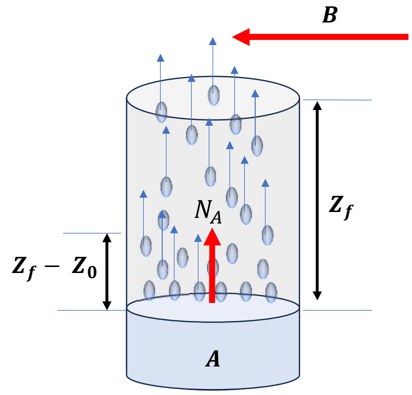
Time can be found from N_A

$N_A = C_A \cdot u_A$

$C_A = \frac{P_A}{R.T}$ When A is Gases ; $C_A = \frac{\rho_A}{MWt_A}$ When A is Liquids

u_A = is the velocity of the molecules of A = $\frac{dZ}{dt}$

$$N_A = \frac{\rho_A}{MWt_A}\cdot\frac{dZ}{dt} = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).\frac{P_{A2}-P_{A1}}{(Z_2-Z_1)}$$



At t = t_f sec

Molecular Diffusion

c) Diffusion through varying path length

For Non-diffusing (B) $(N_B = 0)$

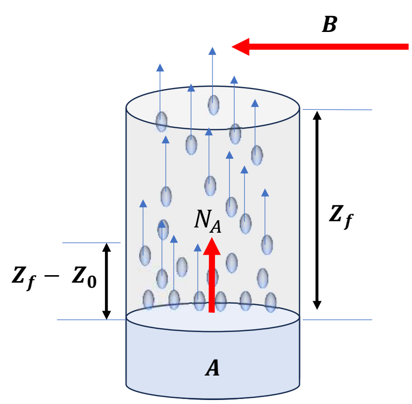
$$N_A = \frac{\rho_A}{MWt_A}\cdot\frac{dZ}{dt} = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).\frac{P_{A2}-P_{A1}}{(Z_2-Z_1)}$$

lets put $Z_2 - Z_1 = Z$

$$\frac{\rho_A}{MWt_A}\cdot\frac{dZ}{dt} = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).\frac{P_{A2}-P_{A1}}{Z}$$

$$\frac{\rho_A}{MWt_A}\cdot\int_{Z_o}^{Z_f} Z\,dZ = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).P_{A2}-P_{A1}\cdot\int_0^{t_f} dt$$

This equation is used to determine the time required to drop the level of liquid to a certain height

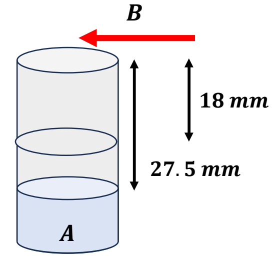


At t = t_f sec

Molecular Diffusion

Example 8: A small diameter tube closed at one end was filled with acetone to within 18 mm of the top and maintained at 290 K with a gentle stream of air blowing across the top. After 15000 sec, the liquid level fell to 27.5 mm, the vapor pressure of acetone was 21.95 kPa, and the atmospheric pressure was 99.75 kPa. **Calculate** the diffusivity of acetone in air. Given: the density of acetone is 790 kg/m^3 and the molecular weight of acetone is 58 kg/kmol.

Solution



$$\frac{\rho_A}{MWt_A} \cdot \int_{Z_o}^{Z_f} Z\, dZ = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).P_{A2} - P_{A1} \cdot \int_0^{t_f} dt$$

$$\frac{\rho_A}{MWt_A} \cdot \left(\frac{Z_f^2}{2} - \frac{Z_o^2}{2}\right) = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).P_{A2} - P_{A1} \cdot t_f$$

$$D_{AB} = \frac{\rho_A}{MWt_A} \cdot \left(\frac{Z_f^2}{2} - \frac{Z_o^2}{2}\right).\left(-\frac{R.T.P_{BM}}{t_f.P_t.(P_{A2} - P_{A1})}\right)$$

Molecular Diffusion

Example 8

$$D_{AB} = \frac{\rho_A}{MWt_A} \cdot \left(\frac{Z_f^2}{2} - \frac{Z_o^2}{2}\right).\left(-\frac{R.T.P_{BM}}{t_f.P_t.(P_{A2} - P_{A1})}\right)$$

Mwt= 58 kg/kmol ; $\rho_A = 790$ kg/m^3

$P_t = 99.75$ kPa; T = 290 K

$R = 8.314 \frac{kPa * m3}{kmol * K}$

Z_o=18 mm = 0.018 m; Z_f = 27.5 mm =0.0275m

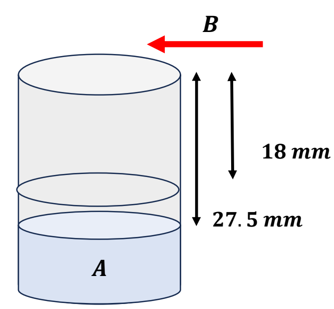
$t_f = 15000\ sec$

$P_{A1} = 21.95$ kPa; $P_{A2} = 0$ kPa;

$P_{B1} = P_t - P_{A1}$; $P_{B2} = P_t - P_{A2}$;

$P_{BM} = \frac{P_{B2} - P_{B1}}{\ln\frac{P_{B2}}{P_{B1}}} = 88.321$ kPa

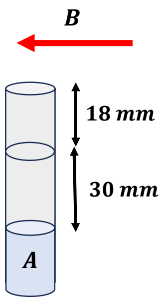
$D_{AB} = 1.9 * 10^{-5}$ m^2/sec



Molecular Diffusion

Example 9: A Metal tube of (3mm) diameter is filled with toluene up to (18mm) from the top. Toluene diffusing in air which passes across the top of the tube. The temperature is 40 °C and the pressure is (0.95 atm). The vapor press of toluene is 200 mm Hg. Calculate the time required drop the toluene level by 3 cm. Given: Toluene Mwt is 92 kg/kmol, toluene density is 0.86 gm/cm^3, $\boldsymbol{D_{AB}}$ = 0.086 cm^2/sec.

Solution



$$\frac{\rho_A}{MWt_A} \cdot \int_{Z_o}^{Z_f} Z\, dZ = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).P_{A2} - P_{A1} \cdot \int_0^{t_f} dt$$

$$\frac{\rho_A}{MWt_A} \cdot \left(\frac{Z_f^2}{2} - \frac{Z_o^2}{2}\right) = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).P_{A2} - P_{A1} \cdot t_f$$

$$t_f = \frac{\rho_A}{MWt_A} \cdot \left(\frac{Z_f^2}{2} - \frac{Z_o^2}{2}\right).\left(-\frac{R.T.P_{BM}}{D_{AB}.P_t.(P_{A2}-P_{A1})}\right)$$

DR. ALJAAFARI

7

Molecular Diffusion

Example 9

$$t_f = \frac{\rho_A}{MWt_A} \cdot \left(\frac{Z_f^2}{2} - \frac{Z_o^2}{2}\right).\left(-\frac{R.T.P_{BM}}{D_{AB}.P_t.(P_{A2}-P_{A1})}\right)$$

Mwt= 92 kg/kmol ; $\rho_A = 860$ kg/m^3

$P_t = 0.95$ atm; T = 313 K

$R = 0.082\ \frac{atm*m3}{kmol*K}$

Z_o=18 mm = 0.018 m; Z_f = 18+30 mm =0.048 m

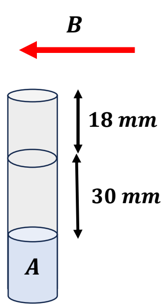
$P_{A1} = \frac{200}{760} = 0.26$ atm; $P_{A2} = 0$ atm;

$P_{B1} = P_t - P_{A1}$; $P_{B2} = P_t - P_{A2}$;

$P_{BM} = \frac{P_{B2}-P_{B1}}{\ln\frac{P_{B2}}{P_{B1}}} = 0.81$ atm

$\boldsymbol{D_{AB}} = 0.086$ cm^2/sec = 0.0000086 m^2/sec

$t_f = 90486$ sec



DR. ALJAAFARI

8

Molecular Diffusion

d) Diffusion through varying area

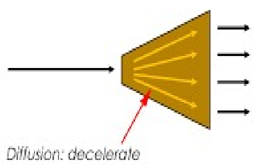
The cross – sectional area varies with Z (the direction of diffusion) such as sphere sublimation, diffusion in conical container.

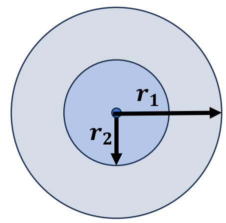
الحالات

- تسامي كرة صلبة
- انتشار خلال مخروط

In the previous cases N_A was assumed constant because the area of diffusion is constant. For the varying cross-section area, we will use the following.

$N_A = \frac{\overline{N_A}}{area}$ Where N_A is not constant; and $\overline{N_A}$ is constant at steady state





Molecular Diffusion

d) Diffusion through varying area

- Diffusion in a conical container

Diffusion of A in non-diffusing B

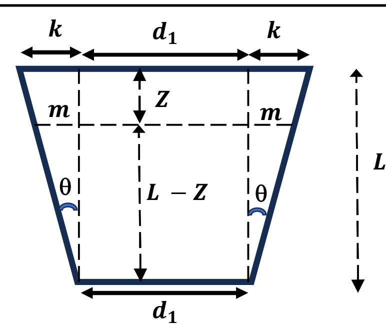
$$N_A = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).\frac{dP_A}{dZ}$$

$$\frac{\overline{N_A}}{area} = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).\frac{dP_A}{dZ}$$

$$\overline{N_A}\int_{Z_1}^{Z_2}\frac{dZ}{area} = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).\int_{P_{A1}}^{P_{A2}} dP_A$$

Area is changing with Z, so we need to correlate them

$$area = \frac{\pi . d^2}{4}$$



$$\tan\theta = \frac{K}{L} = \frac{m}{L-Z} \Rightarrow m = \frac{K(L-Z)}{L}$$

total diameter at any heigh

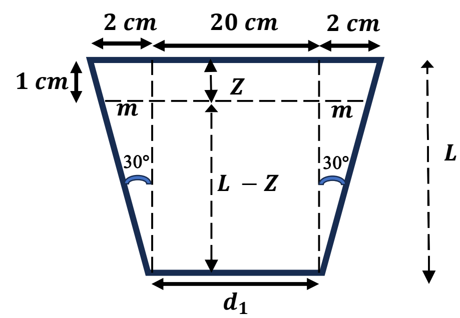
$$d = d_1 + 2m$$

$$d = d_1 + 2\left(\frac{K(L-Z)}{L}\right)$$

$$\overline{N_A}\int_{Z_1}^{Z_2}\frac{dZ}{\frac{\pi}{4}\left(d_1 + \frac{2K(L-Z)}{L}\right)^2} = -\frac{D_{AB}}{R.T}\left(\frac{P_t}{P_{BM}}\right).\int_{P_{A1}}^{P_{A2}} dP_A$$

HW 4: Due date Saturday, Nov 2nd

Q1 An open conical vessel is filled with water up to 1 cm from its top, as shown in the figure below. **Calculate** the time required to drop the level to 2 cm from its top, given that the diffusivity of water in air at 25 °C & 1 atm is 0.256 cm^2 /sec, and the vapor pressure of water at 25 °C is 0.0313 atm.



Q2 Solve Q8 and Q9 and check the final results