



## Differential Equation

A differential equation is an equation which contains one or more terms and the derivatives of one variable (i.e., dependent variable) with respect to the other variable (i.e., independent variable)  $dy/dx = f(x)$  Here “x” is an independent variable and “y” is a dependent variable.

### Types of Differential Equations

1. Ordinary Differential Equations.
2. Partial Differential Equations.
3. Linear Differential Equations.
4. Nonlinear differential equations.
5. Homogeneous Differential Equations.
6. Nonhomogeneous Differential Equations.

### Applications of Differential Equations

Differential Equations are used to calculate the movement or flow of electricity, motion of an object to and fro like a pendulum, to explain thermodynamics concepts. Also, in medical terms, they are used to check the growth of diseases in graphical representation.

# Differential Equations

## 1. Definition of Differential Equations

**Differential Equation:** is any equation which contains one derivative or more. The derivative may be either ordinary derivative or partial derivative.

المعادلة التفاضلية: هي المعادلة التي تحتوي على مشتقة واحدة على الأقل أو أكثر، وتكون المشتقة إما مشتقة اعتيادية أو مشتقة جزئية.

$$\dot{y} + 4y - 3y = 2x \quad \text{or} \quad \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 3y = 2x$$

**Solution of the Differential Equation** is finding an equation without any derivatives, and when substituted in the differential equation achieved.

حل المعادلة التفاضلية هو إيجاد معادلة خالية من المشتقات وإذا عوضت في المعادلة التفاضلية تحققها.

### 1.1 Classification of Differential equation:

Differential equation divided into:

**1) Ordinary Differential Equation:** independent variable = 1 المعادلة التفاضلية الاعتيادية

Ordinary Differential Equation (ODE): is an equation containing one independent variable.

المعادلة التفاضلية الاعتيادية: وهي المعادلة التي تحتوي على متغير مستقل واحد.

**For Example:**

$$y = x^2$$

$$\frac{dy}{dx} = 2x \quad \text{or} \quad \dot{y} = 2x \quad \rightarrow$$

$$y = f(x)$$

where:

y: is dependent variable متغير معتمد

x: is independent variable متغير مستقل

## 2) Partial Differential Equation: independent variable $> 1$ المعادلة التفاضلية الجزئية

Partial Differential Equation (PDE): is an equation containing more than one independent variable.

المعادلة التفاضلية الجزئية: وهي المعادلة التي تحتوي على أكثر من متغير مستقل.

**For Example:**

$$z = x^2 + y^2$$

$$\frac{dz}{dx} = 2x \quad \text{or} \quad \dot{z} = 2x \quad \rightarrow \quad z = f(x, y)$$

$$\frac{dz}{dy} = 2y \quad \text{or} \quad \dot{z} = 2y \quad \rightarrow \quad z = f(x, y)$$

where:

$z$ : is dependent variable

$x$  &  $y$ : is independent variable

## 1.2 Order of differential equation: رتبة المعادلة التفاضلية

The number of highest derivative in a differential equation. A differential equation of order 1 is called First order, order 2 is called Second order ..... etc.

هي أعلى مشتقة موجودة في المعادلة التفاضلية.

**For Example:**

$$\dot{y} + y = x \quad \text{First order}$$

$$\ddot{y} + 2\dot{y} - y = \sin x \quad \text{Second order}$$

## 1.3 Degree of the differential equation: درجة المعادلة التفاضلية

The highest power which is raised to the highest-order derivative existed in differential equation.

هو أعلى أس موجود على أعلى مشتقة في المعادلة التفاضلية.

**For Example:**

$$1) \quad \ddot{y} + 2\dot{y} - y = 0 \quad \text{the degree is 1}$$