

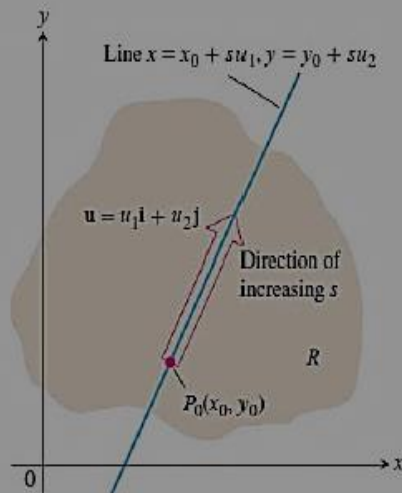


3.12 Directional derivatives

Suppose that the function $f(x, y)$ is defined throughout a region R in the xy – plane, that $P_o(x_o, y_o)$ is a point in R and that $u = u_1\mathbf{i} + u_2\mathbf{j}$ is a unit vector. Thus the equations:

$$x = x_o + su_1, \quad y = y_o + su_2$$

Parameterize the line through P_o parallel to u . if the parameter s measures arc length from P_o in the direction of u , we find the rate of change of f at P_o in the direction of u by calculating $\frac{df}{ds}$ at P_o at



Gradient vector

$$f(x, y, z)$$

$$P_o(x_o, y_o, z_o)$$

$$\nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$$



el f).

nd u is
is:

$$(DuF)|_{p_0} = \nabla f|_{p_0} \cdot u$$

Which is the scalar product of the gradient of F at P₀ and u

" $(DuF)|_{p_0} \rightarrow$ The derivative of F at P₀ in the direction of u"

Example: Find the direction derivative of the function

$F(x,y,z)=x^2 + y^2 + z^2$ at point $p_0(1,1,1)$ in the direction of

Vector $v=i+j+k$.

Solution //

$$(DuF)|_{p_0} = \nabla f|_{p_0} \cdot u$$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k = 2x i + 2y j + 2z k$$



$$\text{at point } p_0(1, 1, 1) \rightarrow \nabla f|_{p_0} = 2i + 2j + 2k$$

$$u = \frac{v}{|v|} = \frac{i+j+k}{\sqrt{1+1+1}} = \frac{i+j+k}{\sqrt{3}}$$

$$(DuF)|_{p_0} = \nabla f|_{p_0} \cdot u = 2i + 2j + 2k \cdot \frac{i+j+k}{\sqrt{3}} = \frac{6}{\sqrt{3}}$$

Example: find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $v = 3i - 4j$

Solution: the direction of v is the unit vector obtained by dividing v by its length:

$$u = \frac{v}{|v|}$$

$$|v| = \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$u = \frac{3i - 4j}{5} = \frac{3}{5}i - \frac{4}{5}j$$

$$\nabla f = \frac{\partial f}{\partial x}i + \frac{\partial f}{\partial y}j = (e^y - y \sin xy)i + (xe^y - x \sin xy)j$$

$$\text{at point } p_0(2, 0) \rightarrow \nabla f|_{p_0} = i + 2j$$

$$(DuF)|_{p_0} = \nabla f|_{p_0} \cdot u = i + 2j \cdot \frac{3i-4j}{5} = \frac{-5}{5} = -1$$



$$\nabla f = \frac{\partial f}{\partial x}i + \frac{\partial f}{\partial z}j = (2y)i + (2x - 6y)j$$

$$\text{at point } p_0(5, 5) \rightarrow \nabla f|_{p_0} = 10i - 20j$$

$$\begin{aligned} (DuF)|_{p_0} &= \nabla f|_{p_0} \cdot u = 10i - 20j \cdot \frac{4i + 3j}{5} = \frac{-20}{5} \\ &= -4 \end{aligned}$$

Exercises:

1. find the derivative of the function $f(x, y, z) = xy + yz + zx$,at the point $P_0(1, -1, 2)$ in the direction of $A = 3i + 6j - 2k$
2. find the derivative of the function $g(x, y, z) = 3e^x \cos yz$,at the point $P_0(0, 0, 0)$ in the direction of $A = 2i + j - 2k$