Example 2: The electromechanical energy conversion device shown in Figure 10 is in equilibrium when the gap distance g is at 0.5 mm and the current intake is i = 1 A. The torque produced by the system on the arm is 30 nm. If the cross-sectional area of the pole faces is 4 × 4 cm^2 and the reluctance of the core can be neglected, calculate the length l of the arm.

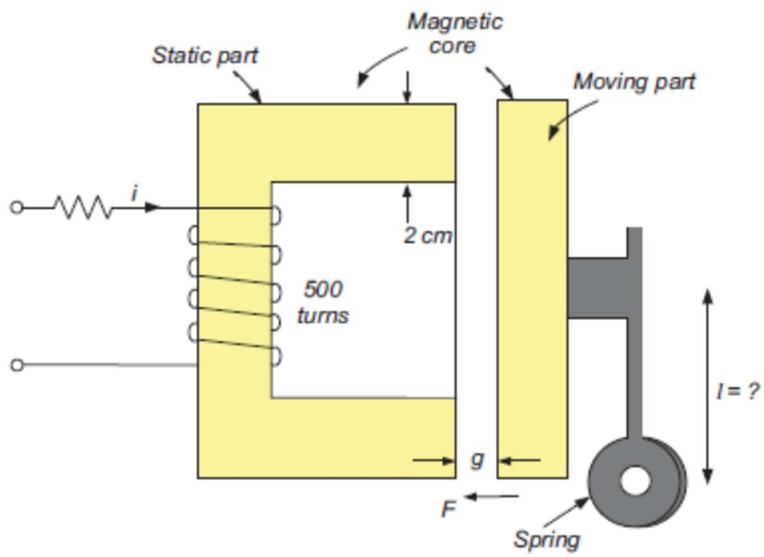

Fig. 10 electromechanical energy conversion device.

Solution

$$Ni = H_g \times 2g = \frac{B_g}{\mu_0} \times 2g \Longrightarrow B_g = \frac{Ni\mu_0}{2g}$$

W = Magnetic energy stored in the air gap $= \frac{B_g{}^2}{2\mu_0} \times$ (Volume of the air gap)

$$F = \frac{dW}{dx} = \frac{B_g{}^2}{2\mu_0} \times 2A = \left(\frac{Ni\mu_0}{2g}\right)^2 \times \frac{A}{\mu_0}$$

$$\frac{500^2 \times 1^2 \times 4\pi \times 10^{-7} \times 4 \times 4 \times 10^{-4}}{4 \times 0.5^2 \times 10^{-6}} = 502.4 \text{ N}$$

6.3 Singly Excited Rotating Actuator

The singly excited linear actuator mentioned above becomes a singly excited rotating actuator if the linearly movable plunger is replaced by a rotor, as illustrated in the diagram on the right-hand side. Through a derivation similar to that for a singly excited linear actuator, one can readily obtain that the torque acting on the rotor can be expressed as the negative partial derivative of the energy stored in the magnetic field against the angular displacement or as the positive partial derivative of the coenergy against the angular displacement, as summarized in the following table.

Energy	Coenergy
$dWf = id\lambda - Td\theta$	$dWf' = id\lambda + Td\theta$
$Wf(\lambda,\theta) = \int_0^{\lambda} i(\lambda,\theta)d\lambda$	$W'f(\lambda,\theta) = \int_0^{i} \lambda(i,\theta)di$
$Wf(\lambda,\theta) = \frac{\lambda^2}{2L(\theta)}$	$W'f(\lambda,\theta) = \frac{i^2 L(\theta)}{2}$
$i = \frac{\partial Wf(\lambda,\theta)}{\partial \lambda}$	$\lambda = \frac{\partial Wf'(i,\theta)}{\partial i}$
$T = \frac{\partial Wf(\lambda,\theta)}{\partial \theta}$	$T = \frac{\partial Wf'(i,\theta)}{\partial \theta}$
$T = \frac{1}{2}\left[\frac{\lambda}{L(\theta)}\right]^2 \frac{dL(\theta)}{d\theta} = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$	$T = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$

When the angular displacement θ between stator magnetic axes and the rotor magnetic axis is zero, the effective air gap length (g) is a minimum and the cross-sectional area of the poles A is a maximum. Hence at this instant, the reluctance of the magnetic circuit is a minimum.

$$\mathcal{R} = \frac{2lg}{\mu_0 A} \qquad\qquad L = \frac{N^2}{\mathcal{R}}$$

Consequently, the inductance of the magnetic circuit, would be a maximum. When the magnetic axes of the rotor and the stator are at right angles to each other, the reluctance R is maximum, leading to a minimum inductance L.

Email (osamah.jaber.ghayyib@uomus.edu.iq)

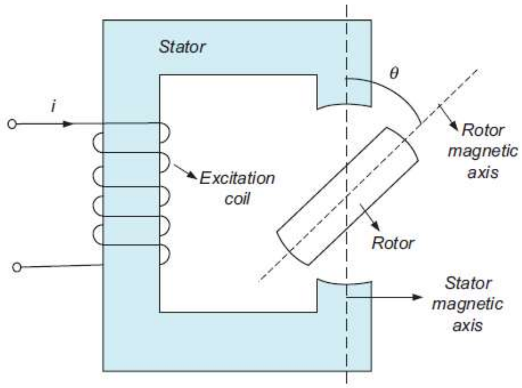

Fig. 11 General structure of a rotating machine.

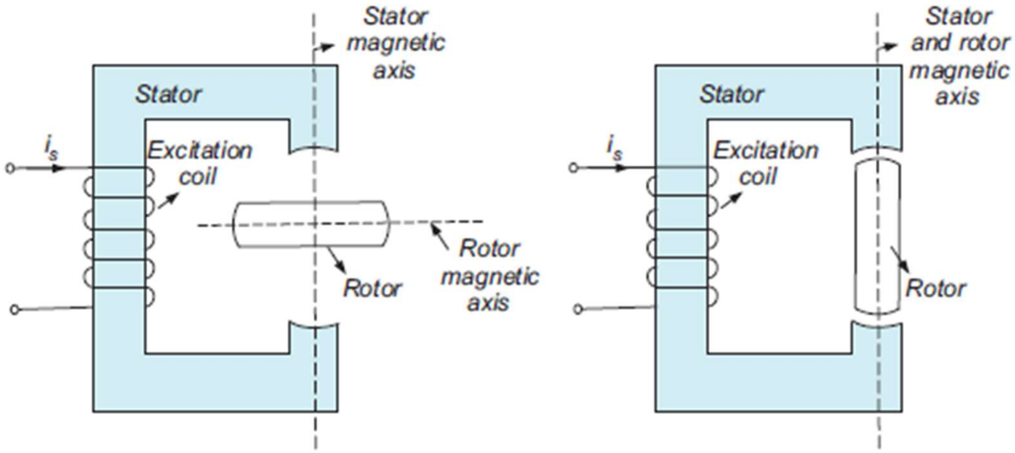

Fig. 12 Singly excited rotating machine at different angular degree.

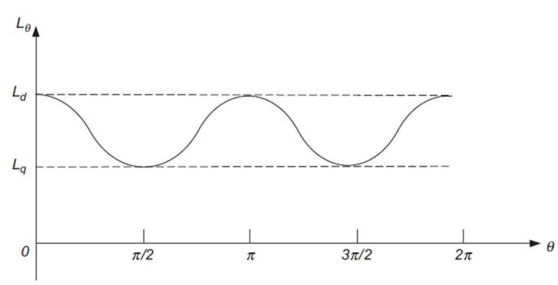

Fig. 13 Variation of inductance of a reluctance motor with displacement

When the rotor is rotating with an angular velocity of ω_m, the inductance of the magnetic assembly varies between its maximum and minimum values as shown in Figures 12.

6.4 Doubly-Excited System

A *doubly-excited system* is the type of magnetic system in which two independent coils are used to produce magnetic field. Examples of doubly-excited systems are synchronous machine, separately excited DC machines, loudspeakers, tachometers etc.

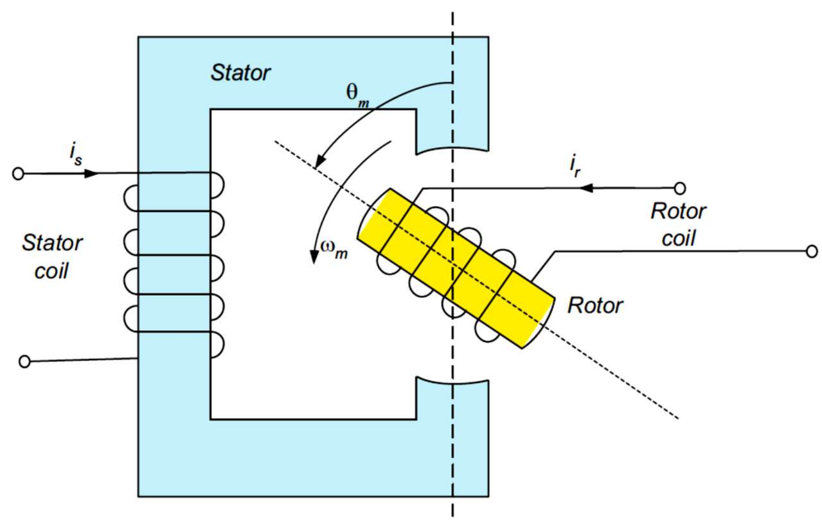

Fig. 13 General structure of a rotating machine.

In these types of energy converters, the rotor is mounted on a shaft and is free to rotate between the poles of the stator. Let us take a general case in which both stator and rotor have windings carrying currents i_s and i_r, as shown in Figure 13. The current is fed into the rotor circuit through fixed brushes and rotor-mounted slip rings.

As we saw before, W_f the magnetic energy deposited in the system, is a state function and how it comes to a certain value is of no consequence. So let us assume the system is static, there is no mechanical output, and W_f is the stored magnetic field energy within the system. Consequently,

$$dW_f = e_s i_s dt + e_r i_r dt \tag{11}$$

$$= i_s d\lambda_s + i_r d\lambda_r \tag{12}$$

For a linear magnetic system,

$$\lambda_s = L_{ss} i_s + L_{sr} i_r \tag{13}$$

$$\lambda_r = L_{rs} i_s + L_{rr} i_r \tag{14}$$

Here,

L_{ss} is the self-inductance of the stator winding.

L_{rr} is the self-inductance of the rotor winding.

L_{rs} and L_{sr} are mutual inductances of the rotor and stator windings.

From equations (12), (13), and (14),

$$dW_f = i_s d(L_{ss} i_s + L_{sr} i_r) + i_r d(L_{sr} i_s + L_{rr} i_r) \tag{15}$$

$$= L_{ss} i_s di_s + L_{rr} i_r di_r + L_{sr} d(i_s i_r) \tag{16}$$

Total field energy will be

$$W_f = L_{ss} \int_0^{i_s} i_s di_s + L_{rr} \int_0^{i_r} i_r di_r + L_{sr} \int_0^{i_s i_r} d(i_s i_r) \tag{17}$$

Similar to the force equation we obtained in previous chapters, in rotational electromechanical energy conversion systems, the torque developed would be

$$T = \left. \frac{\partial W_f'(i, \theta_m)}{\partial \theta_m} \right|_{i = \text{constant}}$$

In a linear magnetic system, the stored magnetic energy W_f would be equal to the coenergy of the system W_f'. Therefore, from the two equations above, we can write

$$T = \frac{1}{2} i_s^2 \frac{dL_{ss}}{d\theta_m} + \frac{1}{2} i_r^2 \frac{dL_{rr}}{d\theta_m} + i_s i_r \frac{dL_{sr}}{d\theta_m}$$