Al-Mustaqbal University

College of Engineering and Technology

Department of Biomedical Engineering

Stage: Second

Electric circuit II

2023-2024

Lecture (11): NODAL ANALYSIS

CHAPTER TEN

SINUSOIDAL STEADY-STATE ANALYSIS

10.1 INTRODUCTION

In Chapter 9, we learned that the forced or steady-state response of circuits to sinusoidal inputs can be obtained by using phasors. We also know that Ohm's and Kirchhoff's laws are applicable to ac circuits. In this chapter, we want to see how nodal analysis, mesh analysis, Thevenin's theorem, Norton's theorem, superposition, and source transformations are applied in analyzing ac circuits. Since these techniques were already introduced for dc circuits, our major effort here will be to illustrate with examples.

Analyzing ac circuits usually requires three steps.

Steps to Analyze ac Circuits:

1. Transform the circuit to the phasor or frequency domain.

2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).

3. Transform the resulting phasor to the time domain.

Step 1 is not necessary if the problem is specified in the frequency domain. In step 2, the analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved. Having read Chapter 9, we are adept at handling step 3.

10.2 NODAL ANALYSIS

The basis of nodal analysis is Kirchhoff's current law. Since KCL is valid for phasors, as demonstrated in Section 9.5, we can analyze ac circuits by nodal analysis. The following examples illustrate this.

Example 10.1: Find i x in the circuit of Fig. 10.1 using nodal analysis.

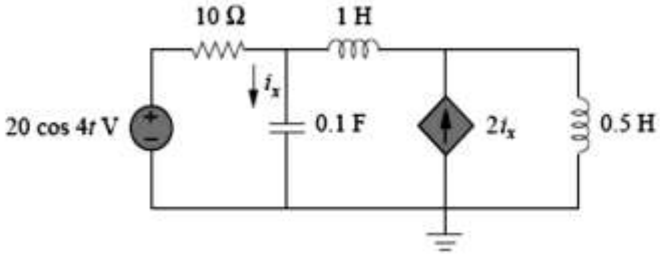

Fig. 10.1

Solution:

We first convert the circuit to the frequency domain:

20 cos 4t ⇒ 20 ∠0°, ω = 4 rad/s

1H ⇒ jωL = j 4

0.5H ⇒ jωL = j 2

0.1F ⇒ $\frac{1}{j\omega C} = -j2.5$

Thus, the frequency-domain equivalent circuit is as shown in Fig. 10.2.

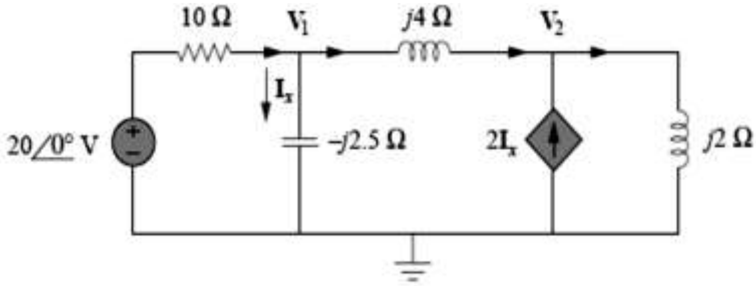

Figure 10.2 Frequency-domain equivalent of the circuit in Fig. 10.1.

Applying KCL at node 1,

$$\frac{20 - V1}{10} = \frac{V1}{-j2.5} + \frac{V1 - V2}{j4}$$

or

$$(1 + j1.5)\,V1 + j2.5\,V2 = 20 \qquad (10.1.1)$$

At node 2,

$$2Ix + \frac{V1 - V2}{j\,4} = \frac{V2}{j2}$$

But Ix = V1 /− j 2.5. Substituting this gives

$$\frac{2V1}{-j2.5} + \frac{V1 - V2}{j4} = \frac{V2}{j\,2}$$

By simplifying, we get

11 V1 + 15 V2 = 0 (10.1.2)

Equations (10.1.1) and (10.1.2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = D = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta 1 = D1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \quad \Delta 2 = D2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -200$$

$$V1 = \frac{D1}{D} = \frac{300}{15 - j5} = 18.97 \angle 18.43° \, V$$

$$V2 = \frac{D2}{D} = \frac{-220}{15 - j5} = 13.91 \angle 198.3° \, V$$

The current Ix is given by

$$Ix = \frac{V1}{-j2.5} = \frac{18.97 \angle 18.43°}{2.5\angle -90°} = 7.59 \angle 108.4° \, A$$

Transforming this to the time domain,

ix = 7.59 cos(4t + 108.4°) A

<u>Practice problem 10.1:</u> Using nodal analysis, find v1 and v2 in the circuit of Fig. 10.3.

<u>Answer:</u> v1 (t) = 20.96 sin (2t + 58°) V,

v2 (t) = 44.11 sin (2t + 41°) V.

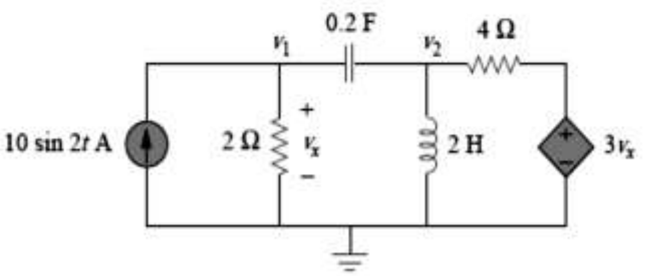

Fig. 10.3

<u>Example 10.2:</u> Compute V1 and V2 in the circuit of Fig. 10.4.

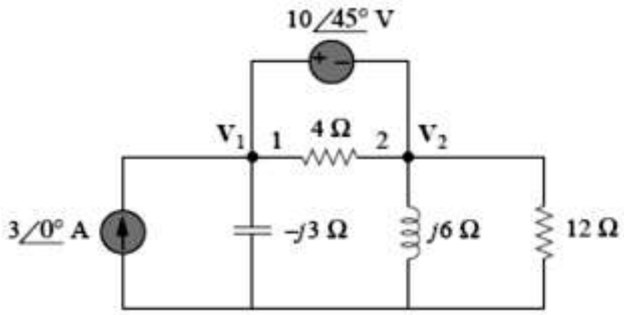

Figure 10.4

Solution: Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{V1}{-j\,3} + \frac{V2}{j6} + \frac{V2}{12}$$

or

$$36 = j4\,V1 + (1 - j2)\,V2 \qquad (10.2.1)$$

But a voltage source is connected between nodes 1 and 2, so that

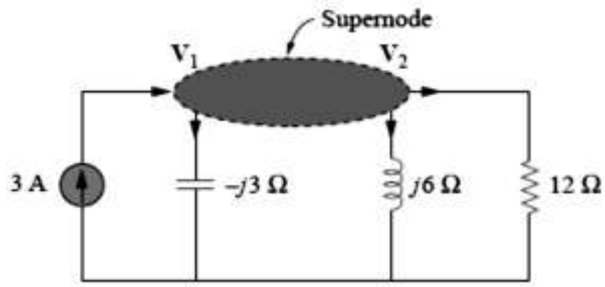

Figure 10.5 A supernode in the circuit of Fig. 10.4.

$$V1 = V2 + 10\ \angle 45^\circ \qquad (10.2.2)$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\ \angle 135^\circ = (1 + j2)\,V2 \quad \Rightarrow \quad V2 = 31.41\ \angle -87.18^\circ\ V$$

From Eq. (10.2.2),

$$V1 = V2 + 10\ \angle 45^\circ = 25.78\ \angle -70.48^\circ\ V$$

Practice problem 10.2: Calculate V1 and V2 in the circuit shown in Fig. 10.6.

Answer: $V1 = 19.36\ \angle 69.67^\circ$ V,

$V2 = 3.376\ \angle 165.7^\circ$ V.