# Properties of Z-Transform

Z-Transform has following properties:

1.Linearity Property

If $x(n) \overset{Z.T}{\longleftrightarrow} X(Z)$

and $y(n) \overset{Z.T}{\longleftrightarrow} Y(Z)$

Then linearity property states that

$$a\,x(n) + b\,y(n) \overset{Z.T}{\longleftrightarrow} a\,X(Z) + b\,Y(Z)$$

**Linearity**: The z-transform is a linear transformation, which implies

$$Z(ax_1(n) + bx_2(n)) \;=\; aZ(x_1(n)) + bZ(x_2(n))$$

$a$ and $b$ are the arbitrary constants.

2.Time Shifting Property:

If $x(n) \overset{Z.T}{\longleftrightarrow} X(Z)$

Then Time shifting property states that

$$x(n-m) \overset{Z.T}{\longleftrightarrow} z^{-m}X(Z)$$

## 3.Multiplication by Exponential Sequence Property:

If $x(n) \overset{\text{Z.T}}{\longleftrightarrow} X(Z)$

Then multiplication by an exponential sequence property states that

$$a^n . x(n) \overset{\text{Z.T}}{\longleftrightarrow} X(Z/a)$$

## 4.Time Reversal Property:

If $x(n) \overset{\text{Z.T}}{\longleftrightarrow} X(Z)$

Then time reversal property states that

$$x(-n) \overset{\text{Z.T}}{\longleftrightarrow} X(1/Z)$$

## 5. Multiplication by n Property:

If $x(n) \overset{\text{Z.T}}{\longleftrightarrow} X(Z)$

Then multiplication by n or differentiation in z-domain property states that

$$n^k x(n) \overset{\text{Z.T}}{\longleftrightarrow} [-1]^k z^k \frac{d^k X(Z)}{dZ^K}$$

## EXAMPLE 5.4

Find the z-transform of the sequence defined by

$$x(n) = u(n) - (0.5)^n u(n)$$

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**Solution:**

Applying the linearity of the z-transform discussed above, we have

$$X(z) = Z(x(n)) = Z(u(n)) - Z(0.5^n(n))$$

$$Z(u(n)) = \frac{z}{z-1}$$

$$Z(0.5^n u(n)) = \frac{z}{z-0.5}$$

$$X(z) = \frac{z}{z-1} - \frac{z}{z-0.5}$$

**Shift theorem**: Given X($z$), the z-transform of a sequence $x(n)$, the z-transform of $x(n-m)$, the time-shifted sequence, is given by

$$Z(x(n-m)) = z^{-m}X(z)$$

Ex) Find Z-Transform

$$y(n) = (0.5)^{(n-5)} \cdot u(n-5)$$

where $u(n-5) = 1$ for $n \geq 5$ and $u(n-5) = 0$ for $n<5$.

**Solution:**

We first use the shift theorem to obtain

$$Y(z) = Z\left[(0.5)^{n-5}u(n-5)\right] = z^{-5}Z[(0.5)^{n}u(n)]$$

$$Y(z) = z^{-5} \cdot \frac{z}{z-0.5} = \frac{z^{-4}}{z-0.5}$$

## Linearity

**Example-3: Find the z-transform of the sequence defined by**

$$x[n] = u[n] - (0.5)^{n}u[n]$$

**Solution**

**Applying the linearity of the z-transform, we have**

$$X(z) = Z\{x[n]\} = Z\{u[n] - (0.5)^{n}u[n]\}$$

$$= Z\{u[n]\} - Z\{(0.5)^{n}u[n]\}$$

$$= \frac{z}{z-1} - \frac{z}{z-0.5}$$

19

## Linearity

**Example-4: Find the z-transform of the sequence defined by**

$$x[n] = [3(2)^n - 4(3)^n]u[n]$$

**Solution**

**Applying the linearity of the z-transform, we have**

As we know $Z\{(a)^n u[n]\} = \frac{1}{1-az^{-1}}$

Applying the linearity of the z-transform, we have

$$X(z) = 3\frac{1}{1-2z^{-1}} - 4\frac{1}{1-3z^{-1}}$$

$$= 3\frac{z}{z-2} - 4\frac{z}{z-3}$$

20

## Linearity

**Example-5: Find the z-transform of the signal x[n] defined by**

$$x[n] = \cos(\omega_0 n)\, u[n]$$

**Solution**

- $\cos(\omega_0 n) = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2} = \frac{1}{2}e^{j\omega_0 n} + \frac{1}{2}e^{-j\omega_0 n}$
- $Z\{e^{j\omega_0 n}u[n]\} = \frac{1}{1-e^{j\omega_0}z^{-1}}$

**Applying the linearity of the z-transform, we have**

$$X(z) = \frac{1}{2}\frac{1}{1-e^{j\omega_0}z^{-1}} + \frac{1}{2}\frac{1}{1-e^{-j\omega_0}z^{-1}}$$

$$= \frac{1 - z^{-1}\cos\omega_0}{1 - 2z^{-1}\cos\omega_0 + z^{-2}}$$

21

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## Time Shifting/Shift Theorem

**Example-6: Find the z-transform of the signal x[n] defined by**

$$x[n] = (0.5)^{n-5}u[n-5]$$

**Solution**

$$Z\{(0.5)^n u[n]\} = \frac{1}{1-0.5z^{-1}}$$

**Applying the time shifting property of the z-transform, we have**

$$X(z) = z^{-5}Z\{(0.5)^n u[n]\}$$

$$= z^{-5}\frac{1}{1-0.5z^{-1}} = \frac{z^{-4}}{z-0.5}$$

24

## Time Reversal

**Example-7: Find the z-transform of the signal x[n] = u[-n]**

**Solution**

$$Z\{u[n]\} = \frac{1}{1-z^{-1}}$$

**Applying the time reversal theorem of the z-transform, we have**

$$Z\{u[-n]\} = \frac{1}{1-z}$$

26