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LECTURE NO. 9

THIN WALLED PRESSURE VESSELS

Introduction

Introduction

A pressure vessel is a pressurized container, often cylindrical or spherical. The pressure acting on the inner surface is resisted by tensile stresses in the walls of the vessel. If the wall thickness t is **sufficiently small** compared to the inner diameter of the vessel, d_i, these stresses are almost uniform throughout the wall thickness. It can be shown that if $(t/d_i) < (1/20)$, the stresses between the inner and outer surfaces of the wall vary by **less than 5%.** Thin wall pressure vessels are widely used in industry for storage and transportation of liquids and gases when configured as tanks. See Figure 1.

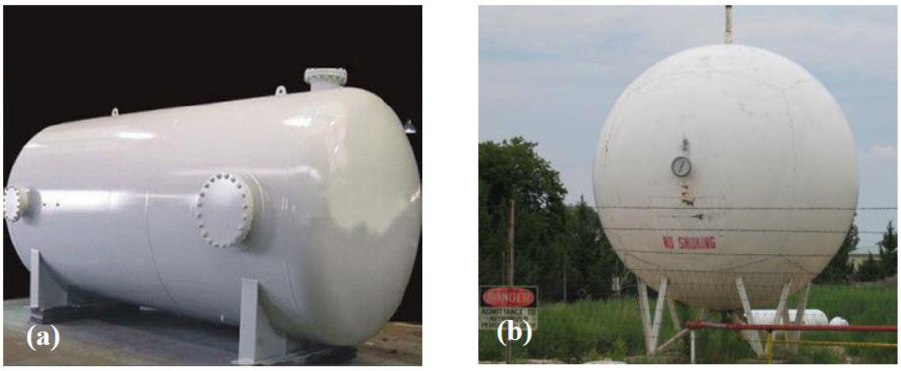

Figure 1: Pressure vessels: (a) cylindrical tank, (b) spherical tanks.

Thin Cylinder under Internal Pressure

When a thin-walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder materials, these stresses are

1. Circumferential or hoop stress
2. Radial stress
3. Longitudinal stress

Note: a cylinder is consider to be thin when the ratio $\frac{t}{d_i} < \frac{1}{20}$, where t is the thickness and d_i is the inner diameter of the cylinder.

Assumptions

Hoop and longitudinal stress are considered constant along thickness.

Radial stress is **small** for thin cylinder assumption and may **be neglected**.

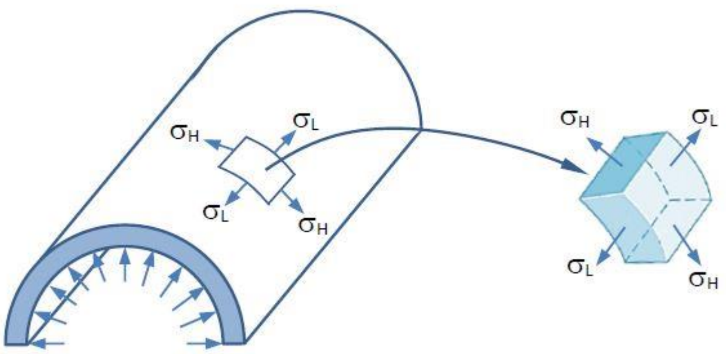

Figure 1: Thin cylindrical pressure vessel subjected to internal pressure.

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Hoop or circumferential stress

Total force on half cylinder$= p \times \ projected\ area = \ p \times (dL)$

Total resisting force $=2\sigma_H \times tL$

$2\sigma_H \times tL = pdL$

$$\text{Hoop stress} \ \ \sigma_H = \frac{pd}{2t}$$

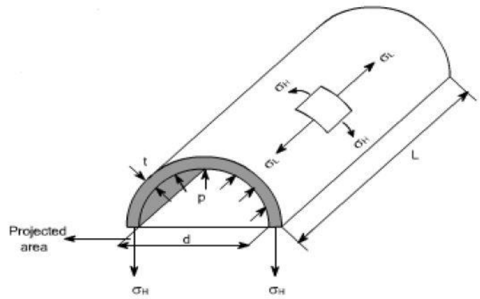

Figure 2: Balance of forces along circumferential to find hoop stress.

Longitudinal stress

Total force on the end of cylinder owing to internal pressure = pressure × area, i.e. $p \times \frac{\pi d^2}{4}$

Area of metal resisting this force$=\pi dt$

$\therefore \sigma_L = \frac{force}{area} = p\frac{\pi d^2/4}{\pi dt} = \frac{pd}{4t}$

$$\text{Longitudinal stress } \sigma_L = \frac{pd}{4t}$$

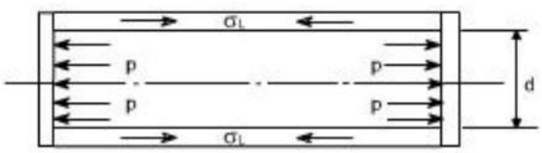

Figure 3: Balance of forces along longitudinal to find longitudinal stress.

Thin Spherical Shell under Internal Pressure

Because of the symmetry of the sphere, the stresses set up owing to internal pressure will be two mutually perpendicular hoop or circumferential stresses of equal value and a radial stress.

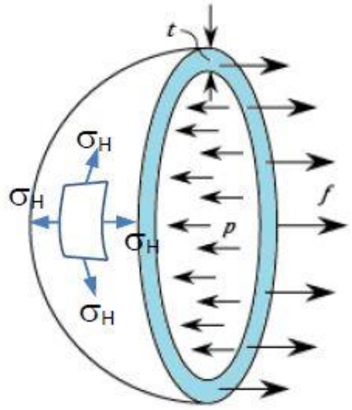

Figure 4: Thin spherical shell subjected to internal pressure.

Note: for $\frac{t}{d} < \frac{1}{20}$, the spherical vessel is considered thin, and the radial stress, σ_R, can be neglected

Force on half-sphere owing to internal pressure = *pressure* × *projected area*
$=p\,\frac{\pi d^2}{4}$

Resisting force $=\sigma_H \times \pi d t$

$$p\frac{\pi d^2}{4} = \sigma_H \times \pi d t$$

Circumferential or hoop stress$=\frac{pd}{4t}$

Example 1: A water tank of 8 m diameter and 12 m high. If the tank is to be completely filled, determine the thickness of the tank plating if the stress is limited to 40 MPa.

Solution/

$d = 8\ m$

$h = 12\ m$

$t = ?$

$\sigma_H = 40\ MPa$

$$\sigma_H = \frac{Pd}{2t} \quad ; \quad P = \rho g h$$

$$* \ P = 1000 * 9.81 * 12 = 117720\ Pa = 0.118\ MPa$$

$$\Rightarrow 40 = \frac{0.118 * 8000}{2 * t}$$

$$\Rightarrow \boxed{t = 11.8\ mm}$$

$\longrightarrow$ Ans.

Example 2: A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m^2. (a) Calculate the tangential and longitudinal stresses in the steel. (b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m^2.

Solution

$d = 400\ mm$

$t = 20\ mm$

$P = 4.5\ MPa$

(a) $\sigma_H = ?$

(b) $P = ?$ when $\sigma_H = 120\ MPa$

(a)

$$\sigma_H = \frac{Pd}{2t} = \frac{4.5 * 400}{2 * 20}$$

$$\boxed{\sigma_H = 45\ MPa}$$

$\longrightarrow$ Ans.

(b)

$$\sigma_H = \frac{Pd}{2t}$$

$$120 = \frac{P * 400}{2 * 20}$$

$$\boxed{P = 12\ MPa}$$

$\longrightarrow$ Ans. (b)

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Example 3: The wall thickness of a 1.2 m diameter spherical tank is 8 mm. Calculate the allowable internal pressure if the stress is limited to 0.5 MPa.

Solution

$d = 1200 \ mm$

$t = 8 \ mm$

$P = ?$

$\sigma_H = 0.5 MPa$

$$\sigma_H = \frac{Pd}{4t}$$

$$0.5 = \frac{P * 1200}{4 * 8}$$

$$\Rightarrow P = 0.0133 MPa \longrightarrow Ans.$$

Example 4*:* Calculate the minimum wall thickness for a cylindrical vessel that is to carry a gas at a pressure of 10 MPa. The diameter of the vessel is 0.6 m and the stress is limited to 83 MPa.

$t = ?$

$P = 10 MPa$

$d = 600 mm$

$\sigma_H = 83 \ MPa$

$\sigma_L = 40 MPa$

ملاحظة / في هذه الحالة يتم حساب ال (t) من ال (σ_H) وكذلك من ال (σ_L) ويتم المفاضلة بينهما

For σ_H

$$\sigma_H = \frac{Pd}{2t}$$

$$83 = \frac{10 * 600}{2 * t}$$

$$\Rightarrow \boxed{t = 36.14 \ mm}$$

For σ_L

$$\sigma_L = \frac{Pd}{4t}$$

$$\Rightarrow 40 = \frac{10 * 600}{4 * t}$$

$$\boxed{t = 37.5 \ mm}$$

∴ The minumium wall thickness of the cylinder is

$$\boxed{t = 37.5 \ mm}$$