Al-Mustaqbal University

College of Engineering and Technology

Department of Biomedical Engineering

Stage: Second

Electric circuit II

2023-2024

Lecture (9): parallel AC

and the equivalent admittance is

$$\mathbf{Yeq = Y1 + Y2 + \cdots + YN} \qquad (9.47)$$

This indicates that the equivalent admittance of a parallel connection of admittances is the sum of the individual admittances.

When N = 2, as shown in Fig. 9.20, the currents in the impedances are

$$I1 = \frac{Z2}{Z1 + Z2} I, \ I2 = \frac{Z1}{Z1 + Z2} I \qquad (9.48)$$

which is the current-division principle.

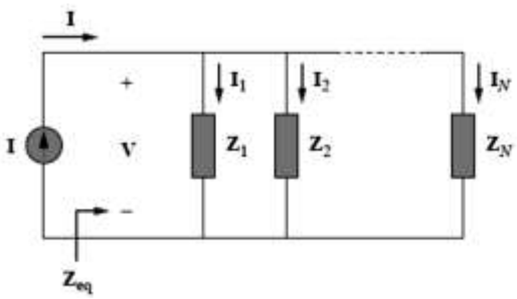


Figure 9.19 N impedances in parallel.

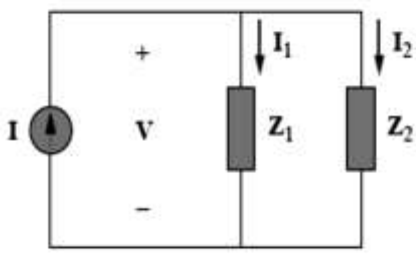


Figure 9.20 Current division.

The delta-to-wye and wye-to-delta transformations that we applied to resistive circuits are also valid for impedances. With reference to Fig. 9.21, the conversion formulas are as follows.

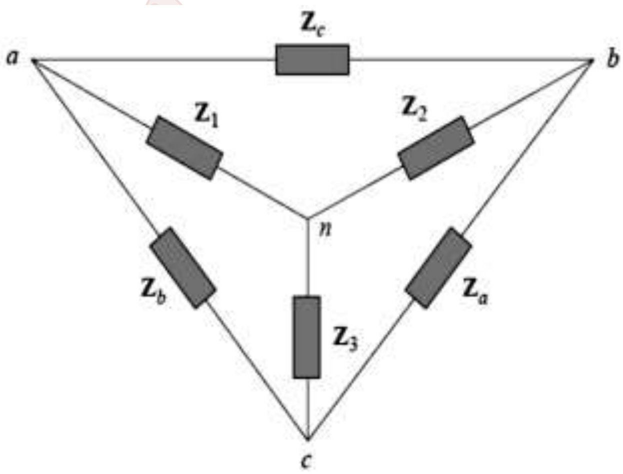


Figure 9.21 Superimposed Y and Δ networks.

Y -Δ Conversion:

$$Za = \frac{Z1Z2 + Z2Z3 + Z3Z1}{Z1}$$

$$Zb = \frac{Z1\,Z2 + Z2\,Z3 + Z3\,Z1}{Z2}$$

$$Zc = \frac{Z1\,Z2 + Z2\,Z3 + Z3\,Z1}{Z3} \qquad (9.49)$$

Δ-Y Conversion:

$$Z1 = \frac{Zb\,Zc}{Za + Zb + Zc}$$

$$Z2 = \frac{Zc\,Za}{Za + Zb + Zc}$$

$$Z3 = \frac{ZaZb}{Za + Zb + Zc} \qquad (9.50)$$

A delta or wye circuit is said to be balanced if it has equal impedances in all three branches.

When a Δ-Y circuit is balanced, Eqs. (9.49) and (9.50) become

$$Z\Delta = 3ZY \quad \text{or} \quad ZY = \frac{1}{3} Z\Delta \qquad (9.51)$$

where ZY = Z1 = Z2 = Z3 and ZΔ = Za = Zb = Zc.

Example 9.5: Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at ω = 50 rad/s.

Solution:

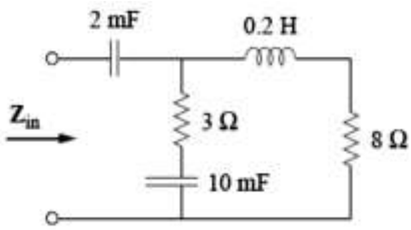


Figure 9.22

Let

Z1 = Impedance of the 2-mF capacitor

Z2 = Impedance of the 3-Ω resistor in series with the 10-mF capacitor

Z3 = Impedance of the 0.2-H inductor in series with the 8-Ω resistor

Then

$$Z1 = \frac{1}{j\omega C} = \frac{1}{j\,50 \times 2 \times 10^{-3}} = -j10\,\Omega$$

$$Z2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j\,50 \times 10 \times 10^{-3}} = (3 - j2)\,\Omega$$

$$Z3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j\,10)\,\Omega$$

The input impedance is

$$Zin = Z1 + Z2 \,||\, Z3 = -j\,10 + \frac{(3 - j2)(8 + j10)}{11 + j\,8}$$

$$= -j\,10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j\,10 + 3.22 - j1.07\,\Omega$$

Thus,

$$Zin = 3.22 - j11.07\,\Omega$$

Practice problem 9.5: Determine the input impedance of the circuit in Fig. 9.24 at ω = 10 rad/s.

Answer: 32.38 − j73.76 Ω.

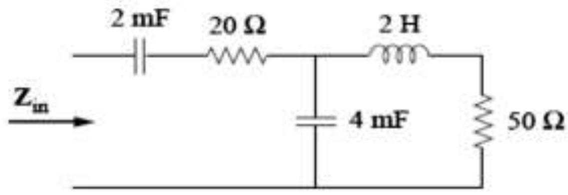


Fig. 9.24

Example 9.6: Determine vo(t) in the circuit in Fig. 9.25.

Solution: To do the analysis in the frequency domain, we must first transform the time domain circuit in Fig. 9.25 to the phasor-domain equivalent in Fig. 9.26. The transformation produces

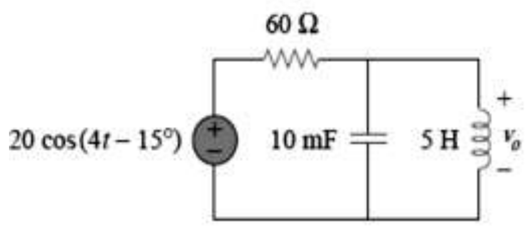


Figure 9.25 For Example 9.11.

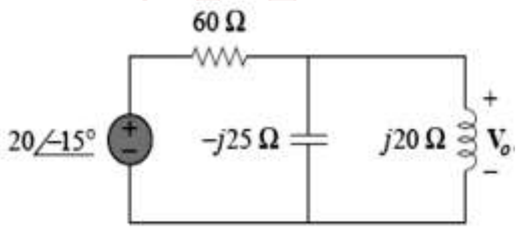


Figure 9.26 The frequency-domain equivalent of the circuit in Fig. 9.25.

vs = 20 cos(4t − 15°) ⇒ Vs = 20 ∠−15° V, ω = 4

$$10\text{ mF} \Rightarrow \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j\,25\ \Omega$$

5H ⇒ jωL = j 4 × 5 = j 20 Ω

Let

Z1 = Impedance of the 60-Ω resistor

Z2 = Impedance of the parallel combination of the 10-mF capacitor and the 5-H inductor

Then Z 1 = 60 Ω and

$$Z2 = -j\,25 \,||\, j\,20 = \frac{-j25 \times j\,20}{-j25 + j\,20} = j\,100\ \Omega$$

By the voltage-division principle,

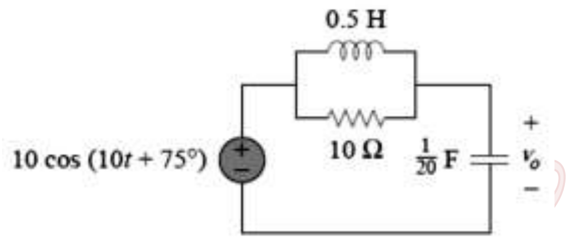
$$Vo = \frac{Z2}{Z1 + Z2}\,Vs = \frac{j100}{60 + j\,100}\,(20\ -15°)$$

$$= (0.8575\angle 30.96^\circ)(20\angle -15^\circ) = 17.15\angle 15.96^\circ\text{ V.}$$

We convert this to the time domain and obtain vo (t) = 17.15 cos(4t + 15.96°)V

Practice problem 9.6: Calculate vo in the circuit in Figure below.

Answer: **vo (t) = 7.071 cos(10t − 60°) V.**



Example 9.7: Find current I in the circuit in Fig. 9.27.

Solution:

The delta network connected to nodes a, b, and c can be converted to the Y network of Fig. 9.28. We obtain the Y impedances as follows using Eq. (9.49):

$$Zan = \frac{j4(2-j4)}{j4+2-j4+8} = \frac{4(4+j2)}{10} = (1.6 + j0.8)\ \Omega$$

$$Zbn = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad Zcn = \frac{8(2-j4)}{10} = (1.6 - j\,3.2)\ \Omega$$

The total impedance at the source terminals is

$$Z = 12 + Zan + (Zbn - j3) \parallel (Zcn + j6 + 8)$$
$$= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)$$
$$= 13.6 + j0.8 + \frac{j\,0.2(9.6+j2.8)}{9.6+j3}$$
$$= 13.6 + j1 = 13.64\ \angle 4.204^\circ$$

The desired current is

$$I = \frac{V}{Z} = \frac{50\ \angle 0^\circ}{13.64\ \angle 4.204^\circ} = 3.666\ \angle -4.204^\circ\ A$$

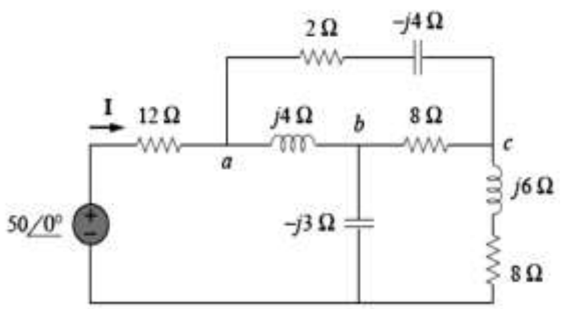


Figure 9.27

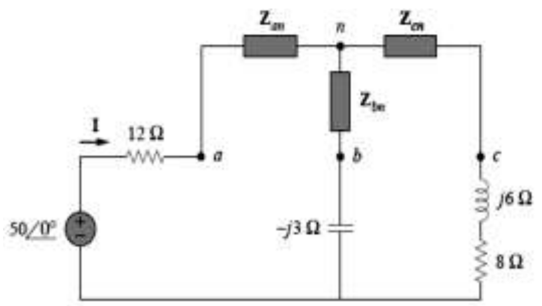


Figure 9.28 The circuit in Fig. 9.27 after delta-to-wye transformation.

Practice problem 9.8: Find I in the circuit in Figure below.

Answer: **6.364 $\angle 3.802^{\circ}$ A.**