Inverse Z-Transform

- To convert a function in the z domain into a function in the time domain requires an inverse z transform.
- This conversion is necessary, for example, to find the time domain functions like

  x[n] that correspond to the z transforms X(z)
  y[n] that correspond to the z transforms Y(z)
  h[n] impulse response from a transfer function H(z)

There are several ways of finding inverse z transforms:

A: Formal Method

- Contour Integration

B: Informal Methods

1- Inspection method using Z Transform Tables
2- Long Division (Synthetic Division or Power Series Expansion)
3- Partial Fraction Expansion

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ADVANTGES

- Relatively straight forward method
- Applicable to any rational function
- Can be use to convert improper rational function into proper rational function

DISADVANTAGES

- Sometimes will run to infinity
- General close-form solution cannot be found

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Example-14: Using long division method, determine the inverse z-transform of

$$H(z) = \frac{z^2 - 0.1z}{z^2 + 0.4z + 0.8}$$

Using long division,

$$
\begin{array}{r l}
 & 1 - 0.5z^{-1} - 0.6z^{-2} + 0.64z^{-3} - \ldots \\
z^2 + 0.4z + 0.8 \,\big)\!\! & z^2 - 0.1z \\
 & \underline{z^2 + 0.4z + 0.8} \\
 & -0.5z - 0.8 \\
 & \underline{-0.5z - 0.2 - 0.4z^{-1}} \\
 & -0.6 + 0.4z^{-1} \\
 & \underline{-0.6 - 0.24z^{-1} - 0.48z^{-2}} \\
 & 0.64z^{-1} + 0.48z^{-2} \\
 & \underline{0.64z^{-1} + 0.256z^{-2} + 0.512z^{-3}} \\
 & 0.224z^{-2} - 0.512z^{-3} \\
 & \ldots
\end{array}
$$

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Example-16: Using long division method, determine the inverse z-transform of

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

Solution: By dividing the numerator of $X(z)$ by its denominator we obtain power series

$$\frac{1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}} = 1 + \frac{3}{2}z^{-1} + \frac{7}{4}z^{-2} + \frac{15}{8}z^{-3} + \frac{31}{16}z^{-4} + \cdots$$

Using z-transform table

$$x[n] = \delta[n] + \frac{3}{2}\delta[n-1] + \frac{7}{4}\delta[n-2] + \frac{15}{8}\delta[n-3] + \frac{31}{16}\delta[n-4] + \cdots$$

or

$$x[n] = \left[\underset{\uparrow}{1}, \quad \frac{3}{2}, \quad \frac{7}{4}, \quad \frac{15}{8}, \quad \frac{31}{16}, \cdots\right]$$

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Partial Fraction Method

ADVANTGES

- It decompose the higher order system into sum of lower order system
- General close-form solution can be found

DISADVANTAGES

- Applicable to strictly proper rational function in standard form
- Getting complex by handling 3 different types of roots for a polynomial function of z, i.e.,
  1. Distinct Real Roots
  2. Repeated Real Roots
  3. Complex Conjugate Roots

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Example-17: Using partial fraction method find the inverse z-transform of the signal Y(z), if x[n] = u[n-1], h[n] = $(-0.25)^n u[n]$.

Solution

As we know that Y(z) = X(z)H(z)

where

$$X(z) = \frac{1}{z-1}$$

$$H(z) = \frac{z}{z+0.25}$$

So,

$$Y(z) = \frac{z}{(z+0.25)(z-1)}$$

$$Y(z) = \frac{z}{(z+0.25)(z-1)}$$

- **The partial fraction expansion is**

$$Y(z) = \frac{A}{z\ +\ 0.25} + \frac{B}{z\ -\ 1}$$

- **The coefficient A and B can be found using the cover-up method.**

$$A = \lim_{z\to -0.25} \frac{(z+0.25)z}{(z\ +\ 0.25)(z-1)} = \frac{-0.25}{-0.25-1} = 0.2$$

$$B = \lim_{z\to 1} \frac{(z-1)z}{(z\ +\ 0.25)(z-1)} = \frac{1}{1+0.25} = 0.8$$

$$Y(z) = \frac{0.2}{z\ +\ 0.25} + \frac{0.8}{z\ -\ 1} = z^{-1}\left(\frac{0.2z}{z+0.25} + \frac{0.8z}{z\ -1}\right)$$

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$$Y(z) = z^{-1}\left(\frac{0.2z}{z+0.25} + \frac{0.8z}{z-1}\right)$$

- **The portion inside the brackets has a inverse z transform is**

$$0.2(-0.25)^n u[n] + 0.8u[n]$$

- **The z^{-1} term outside the brackets indicates a time shift by one step.**

- **Thus, the final inverse transform is**

$$X[n] = 0.2(-0.25)^{n-1}u[n-1] + 0.8u[n-1]$$

Example-18: Using partial fraction method find the inverse z-transform of the signal

$$X(z) = \frac{5}{z^2 + 0.2z}$$

Solution

- **The denominator of X(z) can be factored to give**

$$X(z) = \frac{5}{z(z+0.2)}$$

- **The partial fraction expansion is**

$$X(z) = \frac{A}{z} + \frac{B}{z+0.2} = \frac{25}{z} + \frac{-25}{z+0.2} = z^{-1}(25 - 25\,\frac{z}{z+0.2})$$

- **Thus, the final inverse transform is**

$$X[n] = 25\delta[n-1] - 25(-0.2)^{n-1}u[n-1]$$

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