6 ENERGY IN MAGNETIC SYSTEM

6.1 Singly excited electromechanical system (stationary)

Consider singly—excited magnetic system shown in Fig. 4. It is the magnetic system of an attracted armature relay. Here a coil of ***N*** turns wound on the magnetic core is connected to an electric source.

Let us assume that the armature is held stationary at some air gap and the current is increased from zero to some value *i*. As a result, flux Φ will be established in the magnetic system.

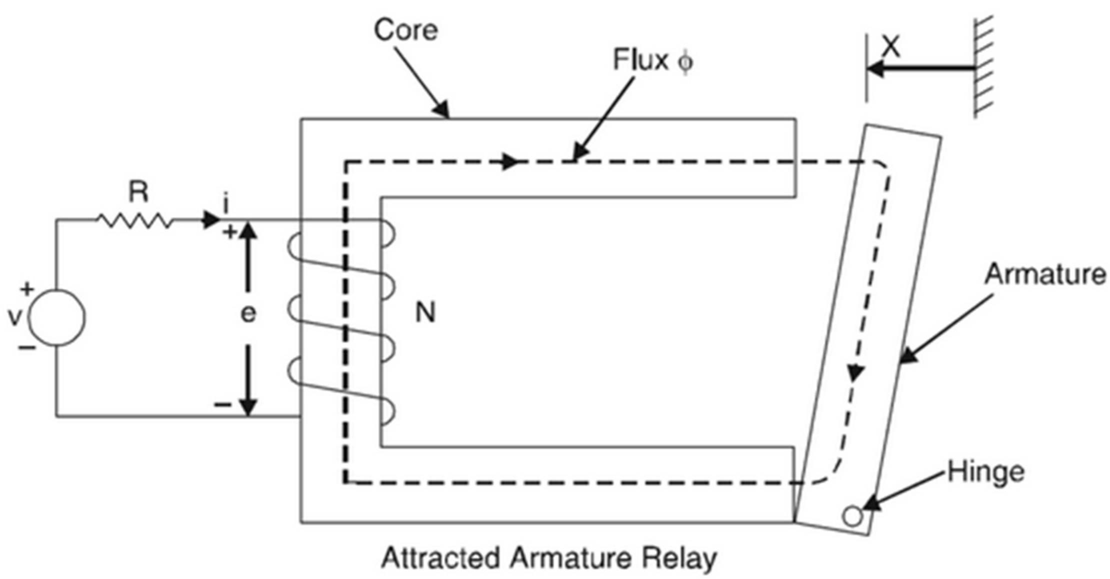

Fig.4 Example of singly excited system.

Total flux linkages, $\lambda = N\Phi$

$$e = N\frac{d\Phi}{dt} = \frac{d}{dt}N\Phi = \frac{d\lambda}{dt}$$

For the coupling device to absorb energy from the electric circuit, the coupling field must produce a reaction in the circuit. This reaction is the e.m.f. *e* produced by the magnetic field.

The incremental electrical energy due to the flow of current in time dt is

$$\mathrm{dW}_e = ei\,dt$$

The energy balance equation in differential form is

$$\mathrm{dW}_e = \mathrm{dW}_f + \mathrm{dW}_m$$

We assume there the armature is held stationary then.

$$\mathrm{dW}_e = \mathrm{dW}_f$$

$$ei\,dt = \mathrm{dW}_f$$

$$\frac{d\lambda}{dt} i dt = \mathrm{dW}_e = \mathrm{dW}_f$$

$$\mathrm{dW}_f = d\lambda\, i = Ni\, d\Phi \qquad (4)$$

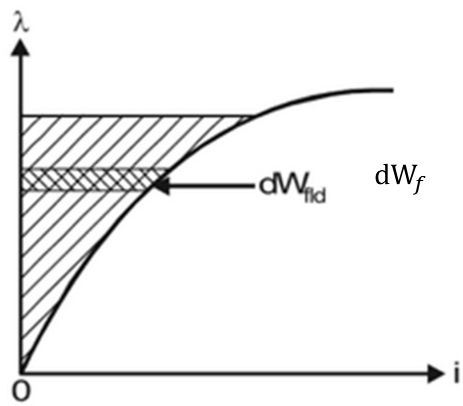

Fig.5 Example of singly excited system.

The relationship between coil flux linkages λ and current i for a particular air-gap length is shown in Fig. 5 The incremental field energy dW_f, is shown as crosshatched area in this figure. When the flux linkage is increased from zero to λ (or flux from zero to Φ), the energy stored in the field is:

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$$W_f = \int_0^{\lambda} i\, d\lambda = \int_0^{\Phi} Ni\, d\Phi \qquad (5)$$

Since

$$Ni = H_g l_g + H_c l_c\,, \qquad B = \frac{\Phi}{A}\,, \qquad B = \mu H$$

Then

$$W_f = \int_0^{\Phi} Ni\, d\Phi = \int_0^{B} H_g l_g A_g dB + \int_0^{B} H_c l_c A_c dB \qquad (6)$$

$$W_f = V_g \int_0^{B} H_g dB + V_c \int_0^{B} H_c dB \qquad (7)$$

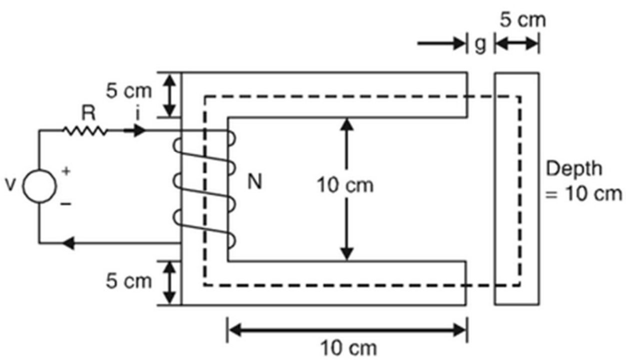
The energy stored in core equals:

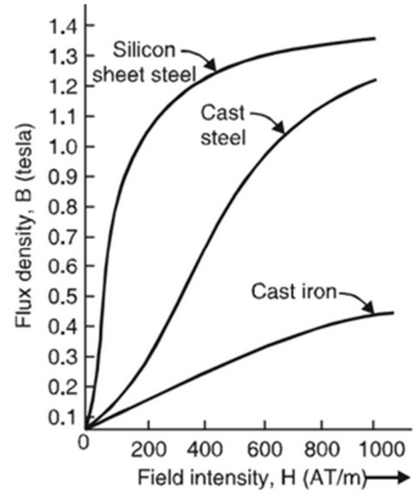
$$W_{fc} = V_c \int_0^{B} H_c dB$$

And for air gap :

$$W_{fg} = V_g \int_0^{B} H_g dB = V_g \int_0^{B} \frac{B}{\mu_0} dB = V_g \frac{B^2}{2\mu_0}$$

Example 1: The magnetic core of system below is made of cast steel whose B-H characteristic is shown below. The coil has 250 turns and the coil resistance is 5Ω. For a fixed air gap length g = 5 mm, a D.C. source is connected to the coil to produce a flux density of B=1 tesla in the air gap.





(i) Find the voltage of D.C. source. (ii) Find the stored field energy.

Solution

B = 1 T, N = 250, R = 5Ω, l_g = 5 mm

(i) From B-H figure we find $H_c = 670\ AT/m$

Length of $l_c = 2(10+5) + 2(10+5) = 60\ cm = 0.6\ m$

Magnetic intensity of air gap H_g can be found as

$$H_g = \frac{B_g}{\mu_0} = \frac{1}{4\pi \times 10^{-7}} = 795.8 \times 10^3\ AT/m$$

$$mmf = Ni = 2H_g l_g + H_c l_c$$

$$Ni = (670 \times 0.6) + (795.8 \times 10^3 \times 2 \times 5 \times 10^{-3}) = 8360\ AT$$

$$i = \frac{8360}{250} = 33.4\ A$$

Then the voltage of DC source is $V = iR = 33.4\ \times 5 = \mathbf{167.2\ V}$

(ii) Field energy stored in core:

$$W_{fc} = V_c \int_0^1 H_c dB$$

Since A=0.1 ×0.05 =0.005 m^2 then

$$V_c = 0.005 \times [(2 \times 0.2) + (2 \times 0.1)] = 0.003\ m^3$$

$$W_{fc} = 0.003 \int_0^1 H_c dB \cong 0.003 \times \frac{1}{2} \times 1 \times 670 = \mathbf{1\ J}$$

Field energy stored in air gaps is given by:

$$W_{fg} = V_g \frac{B^2}{2\mu_0} = 0.05 \times 10^{-3} \times \frac{1^2}{2 \times 4\pi \times 10^{-7}} = \mathbf{19.9\ J}$$

Then the total stored field energy is :

$$W_f = W_{fc} + W_{fg} = \mathbf{20.9\ J}$$

6.2 Singly excited electromechanical system (non-stationary)

The $\lambda - i$ characteristic of an electromagnetic system depends on the air gap length and B-H characteristic of the magnetic material. These $\lambda - i$ characteristics are shown in Fig. 6 for three values of airgap length. If the air gap length is large, the characteristic is essentially linear. The characteristic becomes nonlinear as the air gap length decreases.

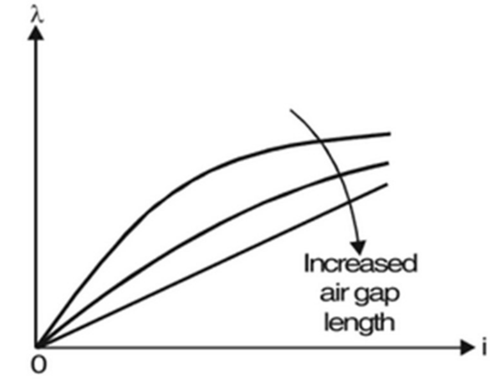


Fig.6 Effect of change in air gap over $\lambda - i$ characteristics.

Another quantity can be defined from the $\lambda - i$ characteristics is called (in Fig 7) the **Co-energy** and defined as:

$$W_f' = \int_0^i \lambda \, di$$

This quantity has no physical significance. From $\lambda - i$ characteristics, the sum of both W_f and W_f' as follows:

$$W_f + W_f' = \lambda i$$

If the relation $\lambda - i$ is linear, then $W_f = W_f'$.

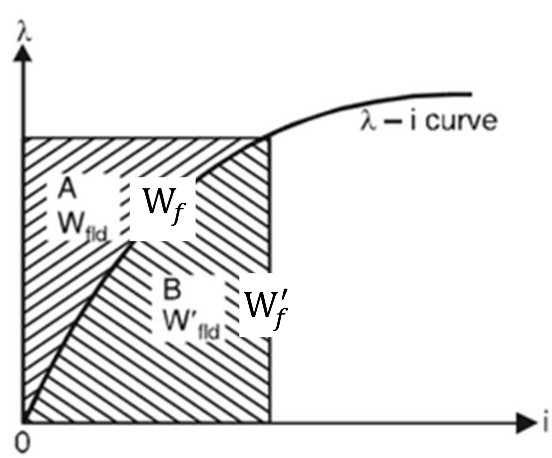


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Fig. 7 energy and Co-energy field.

Now let's consider the electromagnetic system shown in Fig. 7. Let the current through the coil be when a voltage source v is applied across its terminals. The current i sets up magnetic flux ϕ in the magnetic circuit. The flux linkages induce an e.m.f. e in the coil.

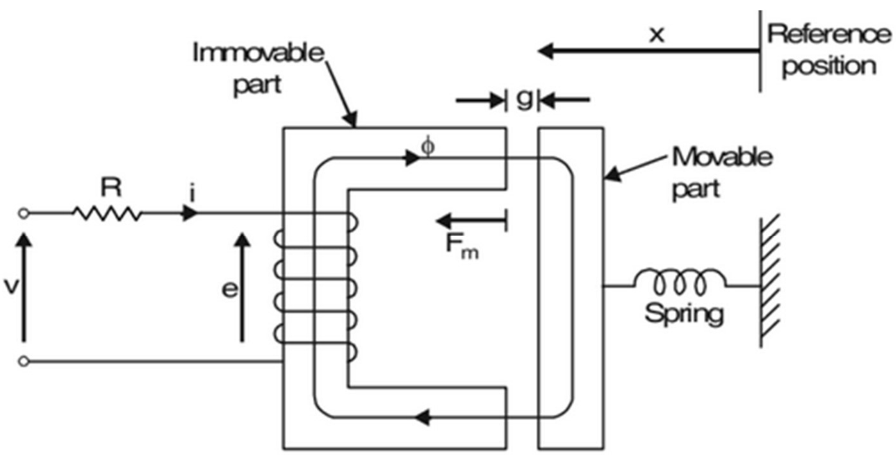

Fig. 8 non-stationary electromechanical system.

Suppose the movable part moves from one position (say $x = x_1$.) to another position ($x = x_2$,) so that at the end of the movement, the air gap decreases. The $\lambda - i$ characteristics of the system for these two positions are shown in Fig. 9(i). Note that operating points of the system are **a** when $x = x_1$, and **b** when $x = x_2$. The current i(= v/R) will remain the same at both the positions in the steady state.

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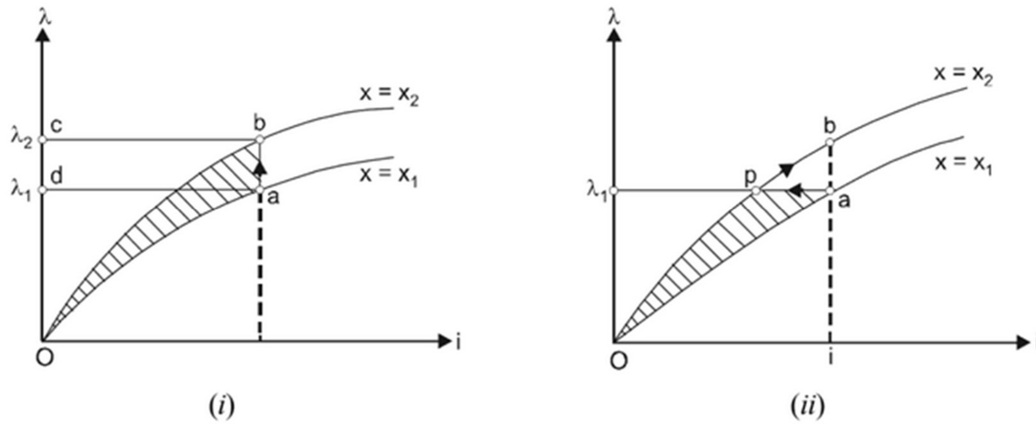

Fig. 9 $\lambda - i$ at different x.

If we assume the system have a linear $\lambda - i$ relation then:

$$\lambda = \mathrm{L(x)}\, i \qquad (8)$$

Where L(x) is the inductance of the coil which depends on length of the air gap x. The energy field is given by:

$$\mathrm{W}_f = \int_0^{\lambda} i\, d\lambda = \int_0^{\lambda} \frac{\lambda}{\mathrm{L(x)}}\, d\lambda = \frac{\lambda^2}{2\,\mathrm{L(x)}} = \frac{1}{2}\,\mathrm{L(x)} i^2 \qquad (9)$$

From Eq 3 we know that:

$$\mathrm{dW}_f = \mathrm{dW}_e - \mathrm{dW}_m = i d\lambda - \mathrm{F\,dx}$$

Since W_f is a function of both λ and x then

$$\mathrm{dW}_f(\lambda, \mathrm{x}) = \frac{\partial \mathrm{W}_f(\lambda, \mathrm{x})}{\partial \lambda} d\lambda + \frac{\partial \mathrm{W}_f(\lambda, \mathrm{x})}{\partial \mathrm{x}} dx$$

By comparing the above two equations we conclude:

$$i = \frac{\partial \mathrm{W}_f(\lambda, \mathrm{x})}{\partial \lambda} \; and \; \mathrm{F} = -\frac{\partial \mathrm{W}_f(\lambda, \mathrm{x})}{\partial \mathrm{x}}$$

From Eq 9 we can find F :

$$F = -\frac{\partial W_f(\lambda, x)}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{\lambda^2}{2\,L(x)}\right)\Big|_{\lambda = \text{constant}}$$

$$= \frac{i^2}{2}\frac{dL(x)}{dx} \qquad (10)$$