ALMUSTAQBAL UNIVERSITY
COLLEGE OF ENGINEERING AND TECHNOLOGY
COMPUTER ENGINEERING TECHNIQUE DEPARTMENT

Subject: Control Engineering Fundamentals

Tenth lecture: Transient and Steady-State Response Analyses

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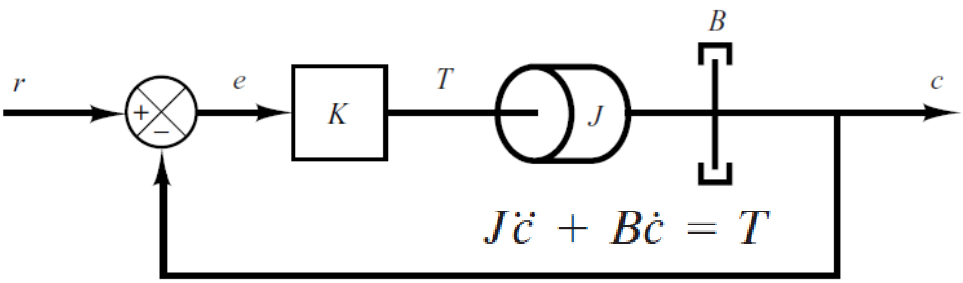
SECOND-ORDER SYSTEMS

In this section, we shall obtain the response of a typical second-order control system to a step input and impulse input. Here we consider a servo system as an example of a second-order system.

Servo System. The servo system shown in Figure (1a) consists of a proportional controller and load elements (inertia and viscous-friction elements). Suppose that we wish to control the output position c in accordance with the input position r.
The equation for the load elements is $J\ddot{c} + B\dot{c} = T$

Figure (1a)



(a)

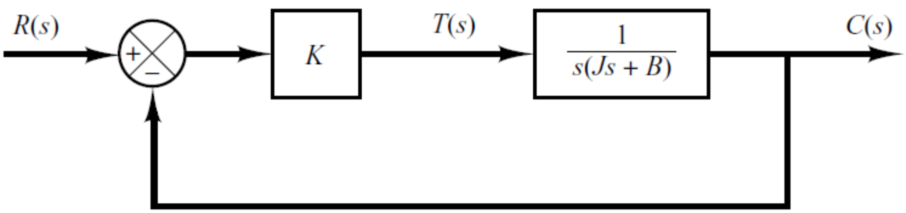
where T is the torque produced by the proportional controller whose gain is K. By taking Laplace transforms of both sides of this last equation, assuming the zero initial conditions, we obtain

$$Js^2C(s) + BsC(s) = T(s)$$

the transfer function

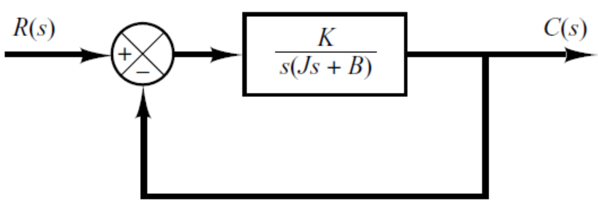
$$\frac{C(s)}{T(s)} = \frac{1}{s(Js + B)}$$

Figure (1b)



(b)

Figure (1c)



(c)

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Bs + K}$$

Step Response of Second-Order System.

The closed-loop transfer function of the system shown in Figure 1(c) is

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Bs + K}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{J}}{\left[s + \frac{B}{2J} + \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]\left[s + \frac{B}{2J} - \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right]}$$

The closed-loop poles are complex conjugates if $B^2 - 4JK < 0$ and they are real if $B^2 - 4JK \geq 0$. In the transient-response analysis, it is convenient to write

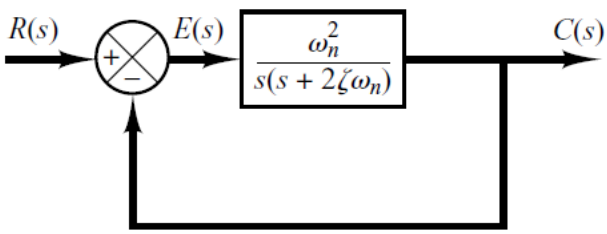
$$\frac{K}{J} = \omega_n^2, \qquad \frac{B}{J} = 2\zeta\omega_n = 2\sigma$$

where σ is called the *attenuation*; ω_n, the *undamped natural frequency*; and ζ, the *damping ratio* of the system. The damping ratio ζ is the ratio of the actual damping B to the critical damping $B_c = 2\sqrt{JK}$ or

$$\zeta = \frac{B}{B_c} = \frac{B}{2\sqrt{JK}}$$

In terms of ζ and ω_n, the system shown in Figure1(c) can be modified to that shown in Figure 2

figure 2



the closed loop transfer function is $$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

This form is called the *standard form* of the second-order system.

The dynamic behavior of the second-order system can then be described in terms of two parameters ζ and ω_n.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

1- If $0 < \zeta < 1$, the closed-loop poles are complex conjugates and lie in the left-half s plane. The system is then called **underdamped**, and the transient response is oscillatory.
2- If $\zeta = 0$, the transient response does not die out **(critical stable)**.
3- If $\zeta = 1$, the system is called **critically damped**.
4- If $\zeta > 1$ the system is called **overdamped**.

(1) **Underdamped case ($0 < \zeta < 1$):** In this case, C(s)/R(s) can be written

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \zeta\omega_n + j\omega_d)(s + \zeta\omega_n - j\omega_d)}$$

where $\omega_d = \omega_n\sqrt{1 - \zeta^2}$.

The frequency ω_d is called the *damped natural frequency*.

For a unit-step input, C(s) can be written

$$C(s) = \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)s}$$

The inverse Laplace transform can be obtained easily if C(s) is written in the following form:

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

Referring to the Laplace transform table, it can be shown that

$$\mathcal{L}^{-1}\left[\frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}\right] = e^{-\zeta\omega_n t}\cos\omega_d t$$

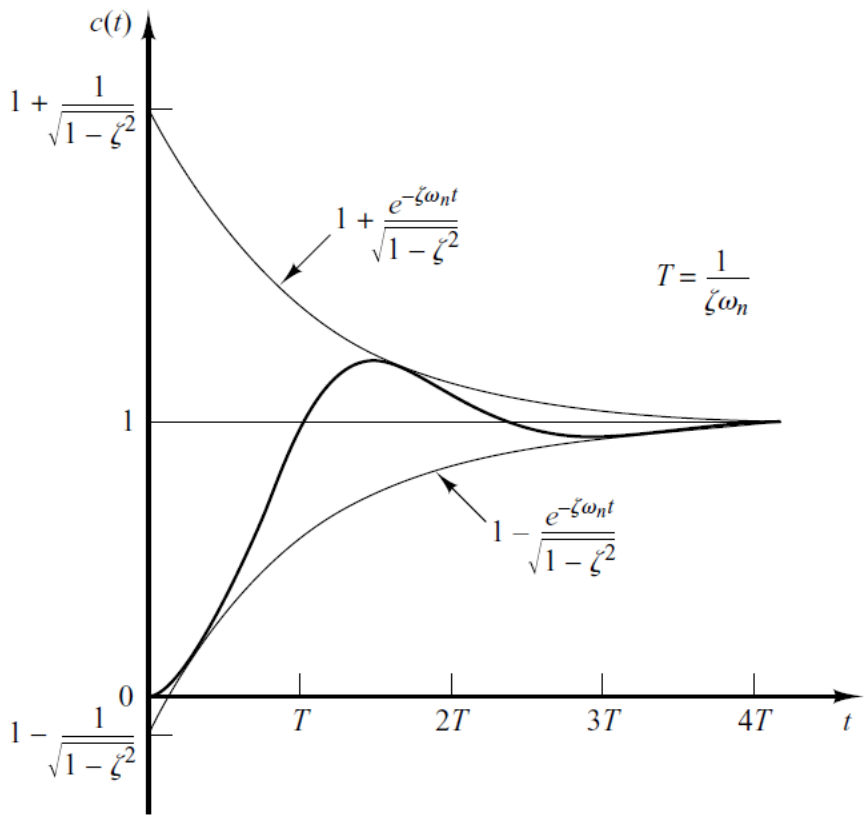
$$\mathcal{L}^{-1}\left[\frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}\right] = e^{-\zeta\omega_n t}\sin\omega_d t$$

$$\mathcal{L}^{-1}[C(s)] = c(t)$$

$$= 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right)$$

$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right), \qquad \text{for } t \geq 0$$

It can be seen that the frequency of transient oscillation is the damped natural frequency **ω_d** and thus varies with the damping ratio ζ.



The error signal for this system is the difference between the input and output and is

$$e(t) = r(t) - c(t)$$

$$= e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right), \qquad \text{for } t \geq 0$$

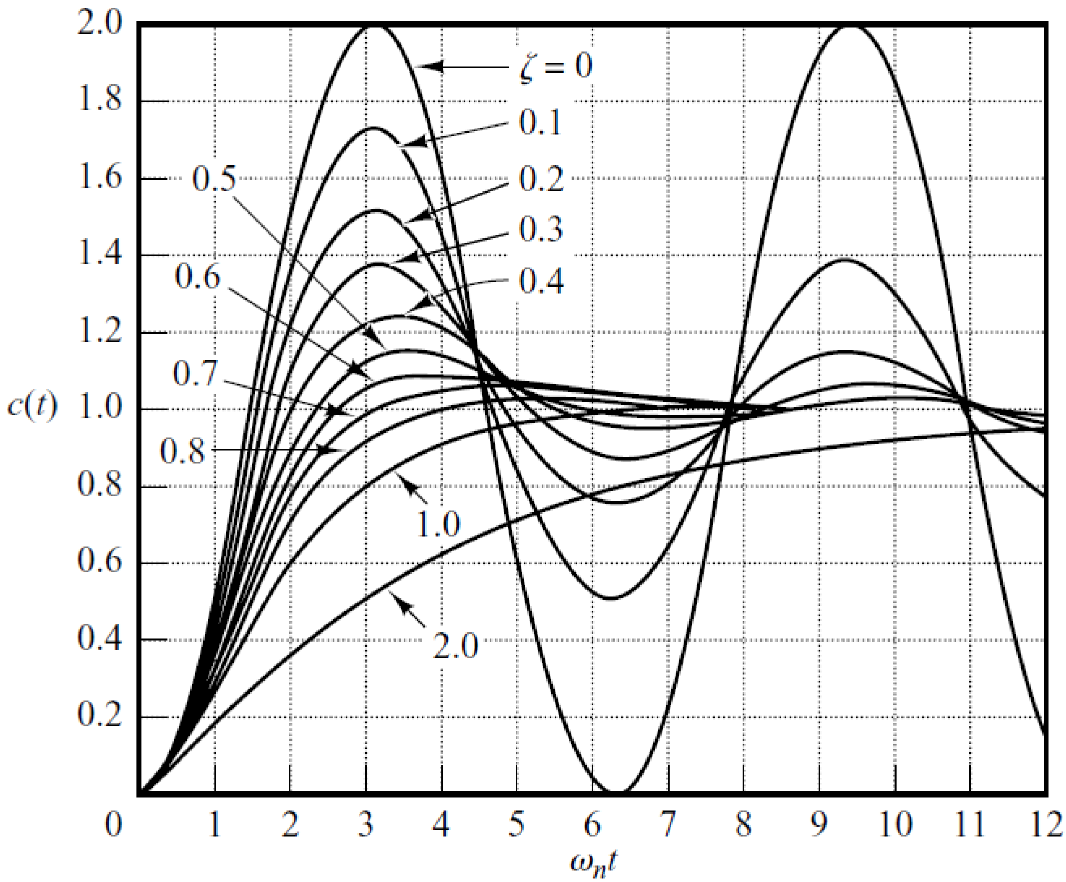
This error signal exhibits a damped sinusoidal oscillation.

At steady state, or at $t = \infty$, no error exists between the input and output.

If the damping ratio ζ is equal to zero, the response becomes undamped and oscillations continue indefinitely. The response c(t) for the zero damping case may be obtained by substituting $\zeta = 0$, yielding

$$c(t) = 1 - \cos \omega_n t, \qquad \text{for } t \geq 0$$

ω_n represents the undamped natural frequency of the system.

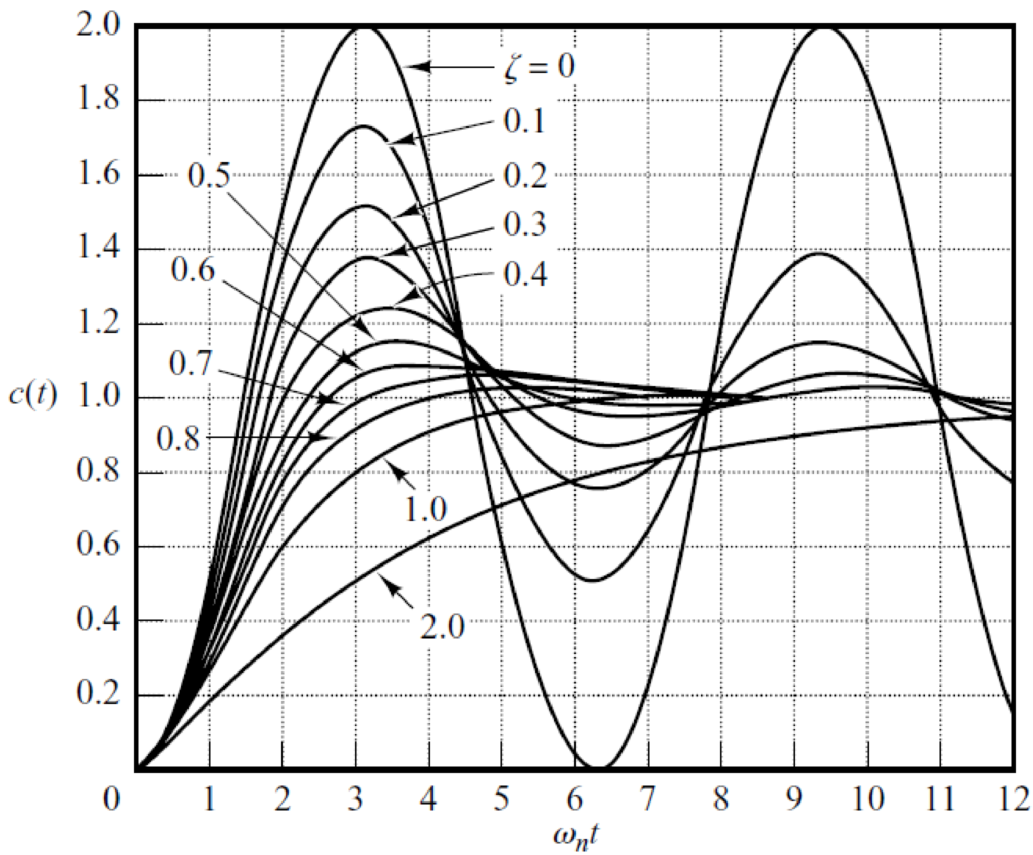


(2) *Critically damped case* (ζ = 1): If the two poles of C(s)/R(s) are equal, the system is said to be a critically damped one.

For a unit-step input, R(s) = 1/s and C(s) can be written

$$C(s) = \frac{\omega_n^2}{(s + \omega_n)^2 s}$$

The inverse Laplace transform is $c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t), \quad \text{for } t \geq 0$



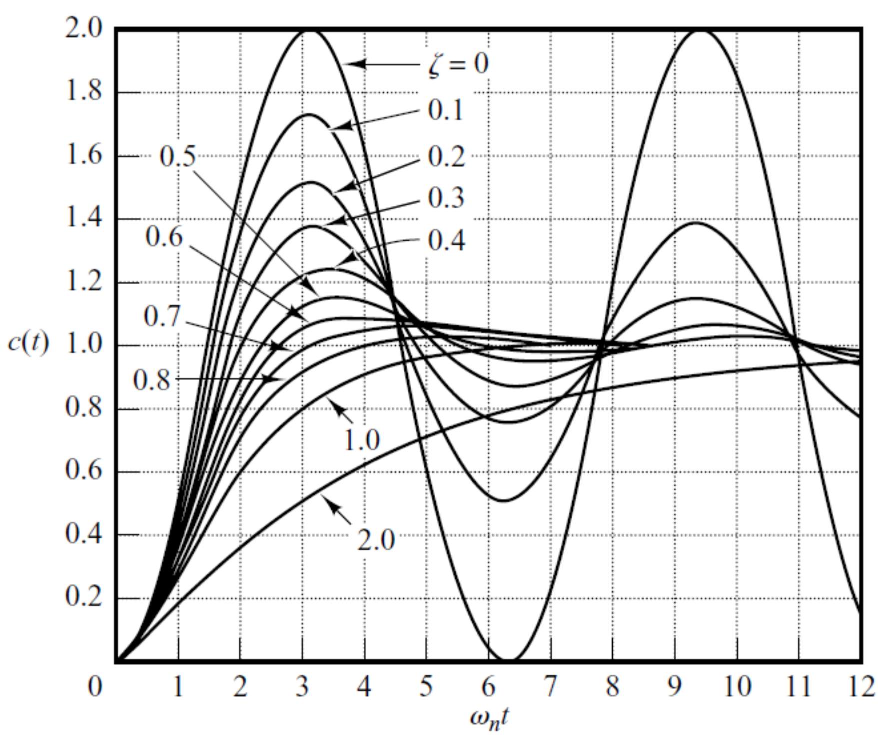
(3) *Overdamped case* (ζ > 1): In this case, the two poles of C(s)/R(s) are negative real and unequal. For a unit-step input, R(s)=1/s and C(s) can be written

$$C(s) = \frac{\omega_n^2}{\left(s + \zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\right)\left(s + \zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}\right)s}$$

The inverse Laplace transform of Equation is

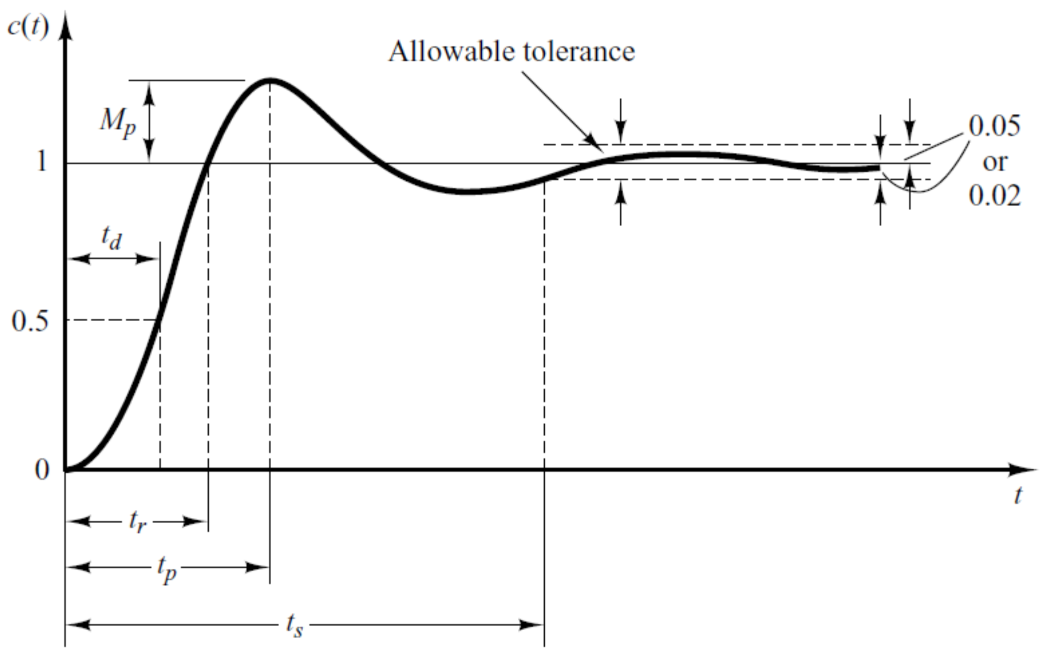
$$\begin{aligned} c(t) &= 1 + \frac{1}{2\sqrt{\zeta^2 - 1}\left(\zeta + \sqrt{\zeta^2 - 1}\right)} e^{-\left(\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} \\ &\quad - \frac{1}{2\sqrt{\zeta^2 - 1}\left(\zeta - \sqrt{\zeta^2 - 1}\right)} e^{-\left(\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t} \\ &= 1 + \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right), \qquad \text{for } t \geq 0 \end{aligned}$$

where $s_1 = \left(\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n$ and $s_2 = \left(\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n$. Thus, the response $c(t)$ includes two decaying exponential terms.



Definitions of Transient-Response Specifications

1. Delay time, t_d
2. Rise time, t_r
3. Peak time, t_p
4. Maximum overshoot, M_p
5. Settling time, t_s



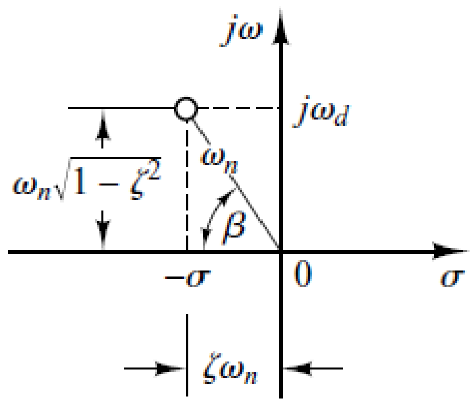
1. Delay time, t_d: The delay time is the time required for the response to reach half the final value the very first time.
2. Rise time, t_r: The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value. For underdamped second- order systems, the 0% to 100% rise time is normally used. For overdamped systems, the 10% to 90% rise time is commonly used.
3. Peak time, t_p: The peak time is the time required for the response to reach the first peak of the overshoot.
4. Maximum (percent) overshoot, M_p: The maximum overshoot is the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

5. Settling time, t_s: The settling time is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute per-centage of the final value (usually 2% or 5%). The settling time is related to the largest time constant of the control system.

Rise time t_r: by letting $c(t_r) = 1$. Thus, the rise time t_r is

$$t_r = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{-\sigma}\right) = \frac{\pi - \beta}{\omega_d}$$



Where $\omega_n\sqrt{1-\zeta^2} = \omega_d$ and $\zeta\omega_n = \sigma$

Peak time t_p:

$$t_p = \frac{\pi}{\omega_d}$$

Maximum overshoot M_p:

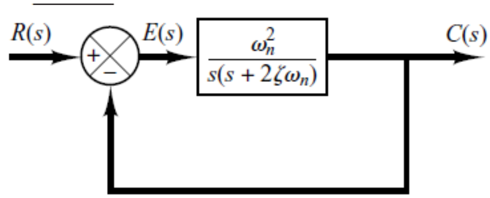
$$M_p = c(t_p) - 1 = e^{-(\sigma/\omega_d)\pi} = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$$

Settling time t_s:

$$t_s = 4T = \frac{4}{\sigma} = \frac{4}{\zeta\omega_n} \qquad \text{(2\% criterion)}$$

$$t_s = 3T = \frac{3}{\sigma} = \frac{3}{\zeta\omega_n} \qquad \text{(5\% criterion)}$$

EX.1 Consider the system in Figure below, where $\zeta = 0.6$ and $\omega_n = 5$ rad/sec. Let us obtain the rise time t_r, peak time t_p, maximum overshoot M_p, and settling time t_s when the system is subjected to a unit-step input.



From the given values of ζ and ω_n, we obtain $\omega_d = \omega_n\sqrt{1-\zeta^2} = 4$ and $\sigma = \zeta\omega_n = 3$.

Rise time t_r: The rise time is

$$t_r = \frac{\pi - \beta}{\omega_d} = \frac{3.14 - \beta}{4}$$

where β is given by

$$\beta = \tan^{-1}\frac{\omega_d}{\sigma} = \tan^{-1}\frac{4}{3} = 0.93 \text{ rad}$$

The rise time t_r is thus

$$t_r = \frac{3.14 - 0.93}{4} = 0.55 \text{ sec}$$

Peak time t_p: The peak time is

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4} = 0.785 \text{ sec}$$

Maximum overshoot M_p: The maximum overshoot is

$$M_p = e^{-(\sigma/\omega_d)\pi} = e^{-(3/4)\times 3.14} = 0.095$$

The maximum percent overshoot is thus 9.5%.

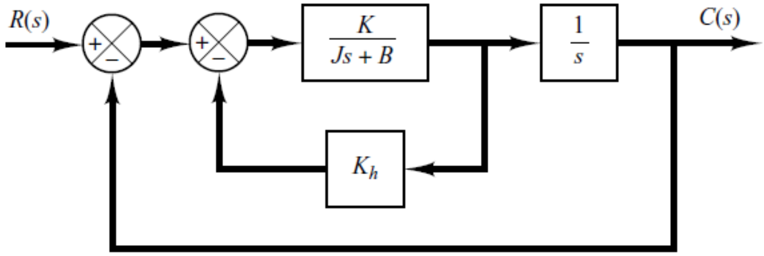
Settling time t_s: For the 2% criterion, the settling time is

$$t_s = \frac{4}{\sigma} = \frac{4}{3} = 1.33 \text{ sec}$$

For the 5% criterion,

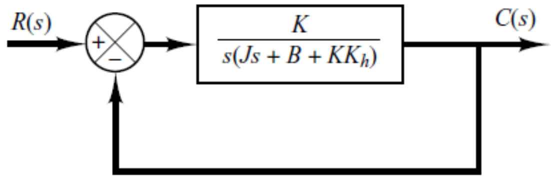
$$t_s = \frac{3}{\sigma} = \frac{3}{3} = 1 \text{ sec}$$

EX. 2 For the system in below (a), determine the values of gain K and velocity-feedback constant K_h so that the maximum overshoot in the unit-step response is 0.2 and the peak time is 1 sec. With these values of K and K_h, obtain the rise time and settling time. Assume that $J = 1$ kg-m^2 and $B = 1$ N-m/rad/sec.



(a)

(a) Block diagram of a servo system; (b) simplified block diagram.



(b)

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

By comparing with

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\boldsymbol{\omega_n = \sqrt{K/J}}$$

$$\zeta = \frac{B + KK_h}{2\sqrt{KJ}}$$

Determination of the values of K and K_h : The maximum overshoot M_p is given by Equation

$$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$$

This value must be 0.2. Thus,

$$e^{-(\zeta/\sqrt{1-\zeta^2})\pi} = 0.2$$

or

$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.61$$

which yields

$$\zeta = 0.456$$

The peak time t_p is specified as 1 sec: $t_p = \dfrac{\pi}{\omega_d} = 1$ $\omega_d = 3.14$

Since ζ is 0.456, ω_n is

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} = 3.53$$

Since the natural frequency ω_n is equal to $\sqrt{K/J}$,

$$K = J\omega_n^2 = \omega_n^2 = 12.5 \text{ N-m}$$

Then K_h is, from Equation (5–25),

$$K_h = \frac{2\sqrt{KJ}\zeta - B}{K} = \frac{2\sqrt{K}\zeta - 1}{K} = 0.178 \text{ sec}$$

Rise time t_r: From Equation (5–19), the rise time t_r is

$$t_r = \frac{\pi - \beta}{\omega_d}$$

where

$$\beta = \tan^{-1}\frac{\omega_d}{\sigma} = \tan^{-1} 1.95 = 1.10$$

Thus, t_r is

$$t_r = 0.65 \text{ sec}$$

Settling time t_s: For the 2% criterion,

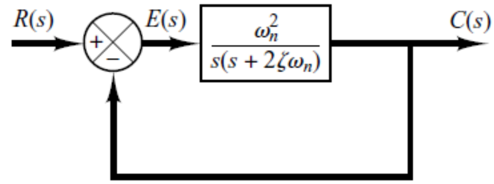
$$t_s = \frac{4}{\sigma} = 2.48 \text{ sec}$$

For the 5% criterion,

$$t_s = \frac{3}{\sigma} = 1.86 \text{ sec}$$

Impulse Response of Second-Order Systems. For a unit-impulse input $r(t)$, the corresponding Laplace transform is unity, or $R(s) = 1$. The unit-impulse response $C(s)$ of the second-order system in figure below is

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



The inverse Laplace transform of this equation yields the time solution for the response $c(t)$ as follows:

For $0 \le \zeta < 1$,

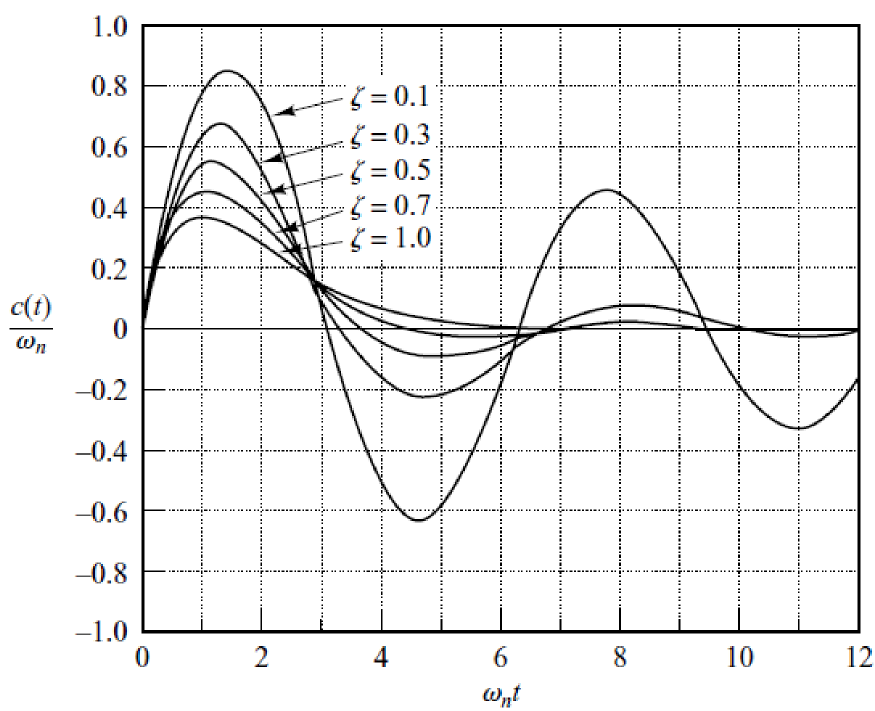
$$c(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t, \qquad \text{for } t \ge 0$$

For $\zeta = 1$,

$$c(t) = \omega_n^2 t e^{-\omega_n t}, \qquad \text{for } t \ge 0$$

For $\zeta > 1$,

$$c(t) = \frac{\omega_n}{2\sqrt{\zeta^2-1}} e^{-\left(\zeta-\sqrt{\zeta^2-1}\right)\omega_n t} - \frac{\omega_n}{2\sqrt{\zeta^2-1}} e^{-\left(\zeta+\sqrt{\zeta^2-1}\right)\omega_n t}, \qquad \text{for } t \ge 0$$



Modern Control Engineering

Third Edition

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Chapter 4 Transient-Response Analysis

EXAMPLE 4–2

Consider the system shown in Figure 4–9, where $\zeta = 0.6$ and $\omega_n = 5$ rad/sec. Let us obtain the rise time t_r, peak time t_p, maximum overshoot M_p, and settling time t_s when the system is subjected to a unit-step input.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

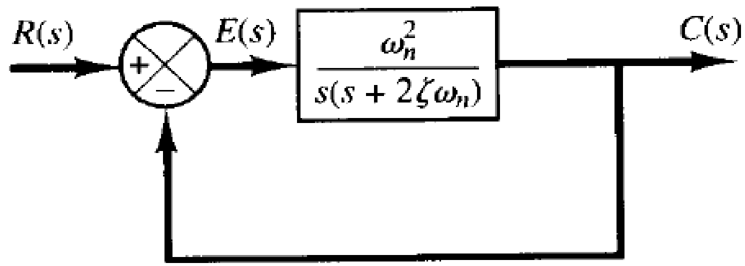


Figure 4–9
Second-order system.

From the given values of ζ and ω_n, → $\omega_d = \omega_n\sqrt{1-\zeta^2} = 4$ and $\sigma = \zeta\omega_n = 3$.

Rise time t_r: The rise time is

$$t_r = \frac{\pi - \beta}{\omega_d} = \frac{3.14 - \beta}{4}$$

where β is given by

$$\beta = \tan^{-1}\frac{\omega_d}{\sigma} = \tan^{-1}\frac{4}{3} = 0.93 \text{ rad}$$

The rise time t_r is thus

$$t_r = \frac{3.14 - 0.93}{4} = 0.55 \text{ sec}$$

Peak time t_p: The peak time is

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4} = 0.785 \text{ sec}$$

Maximum overshoot M_p: The maximum overshoot is

$$M_p = e^{-(\sigma/\omega_d)\pi} = e^{-(3/4)\times 3.14} = 0.095$$

The maximum percent overshoot is thus 9.5%

Settling time t_s: For the 2% criterion, the settling time is

$$t_s = \frac{4}{\sigma} = \frac{4}{3} = 1.33 \text{ sec}$$

For the 5% criterion,

$$t_s = \frac{3}{\sigma} = \frac{3}{3} = 1 \text{ sec}$$

EXAMPLE 4–3

For the system shown in Figure 4–17(a), determine the values of gain K and velocity feedback constant K_h so that the maximum overshoot in the unit-step response is 0.2 and the peak time is 1 sec. With these values of K and K_h, obtain the rise time and settling time. Assume that $J = 1$ kg-m² and $B = 1$ N-m/rad/sec.

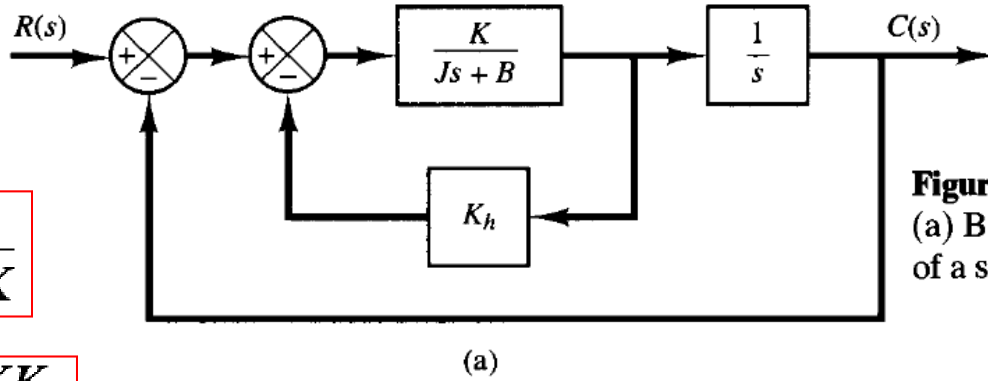


Figure 4–17 (a) Block diagram of a servo system;

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

$$\omega_n = \sqrt{K/J} \qquad \zeta = \frac{B + KK_h}{2\sqrt{KJ}}$$

Determination of the values of K and K_h:

$$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi} \Rightarrow e^{-(\zeta/\sqrt{1-\zeta^2})\pi} = 0.2 \Rightarrow \frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.61 \Rightarrow \zeta = 0.456$$

The peak time t_p is specified as 1 sec;

$$t_p = \frac{\pi}{\omega_d} = 1 \Rightarrow \omega_d = 3.14$$

Since ζ is 0.456, ω_n is

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 3.53$$

Since the natural frequency ω_n is equal to $\sqrt{K/J}$, ➡ $K = J\omega_n^2 = \omega_n^2 = 12.5$ N-m

Then, K_h is, ➡ $\zeta = \dfrac{B + KK_h}{2\sqrt{KJ}}$ ➡ $K_h = \dfrac{2\sqrt{KJ}\zeta - B}{K} = \dfrac{2\sqrt{K}\zeta - 1}{K} = 0.178$ sec

Rise time t_r: ➡ $t_r = \dfrac{\pi - \beta}{\omega_d}$ ➡ $\beta = \tan^{-1}\dfrac{\omega_d}{\sigma} = \tan^{-1} 1.95 = 1.10$

Thus, t_r is $t_r = 0.65$ sec

Settling time t_s: For the 2% criterion, ➡ $t_s = \dfrac{4}{\sigma} = 2.48$ sec

For the 5% criterion, ➡ $t_s = \dfrac{3}{\sigma} = 1.86$ sec

A-4-2.

Consider the mechanical system shown in Figure 4–37. Suppose that the system is at rest initially [$x(0) = 0, \dot{x}(0) = 0$], and at $t = 0$ it is set into motion by a unit-impulse force. Obtain a mathematical model for the system. Then find the motion of the system.

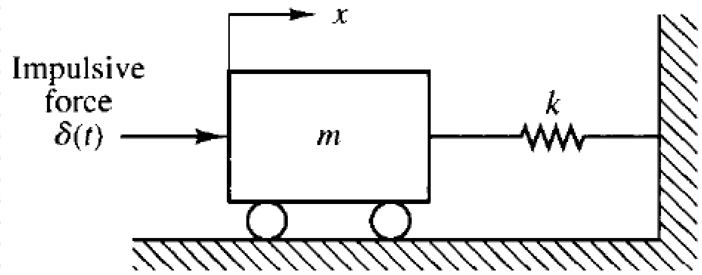


Figure 4–37 Mechanical system.

$$m\ddot{x} + kx = \delta(t)$$

$$m[s^2X(s) - sx(0) - \dot{x}(0)] + kX(s) = 1$$

By substituting the initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$ into this last equation and solving for $X(s)$, we obtain

$$X(s) = \frac{1}{ms^2 + k}$$

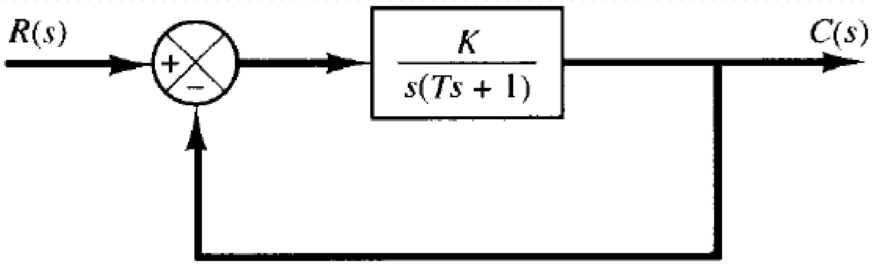
The inverse Laplace transform of $X(s)$ becomes

$$X(t) = \frac{1}{\sqrt{mk}} \sin \sqrt{\frac{k}{m}}\, t$$

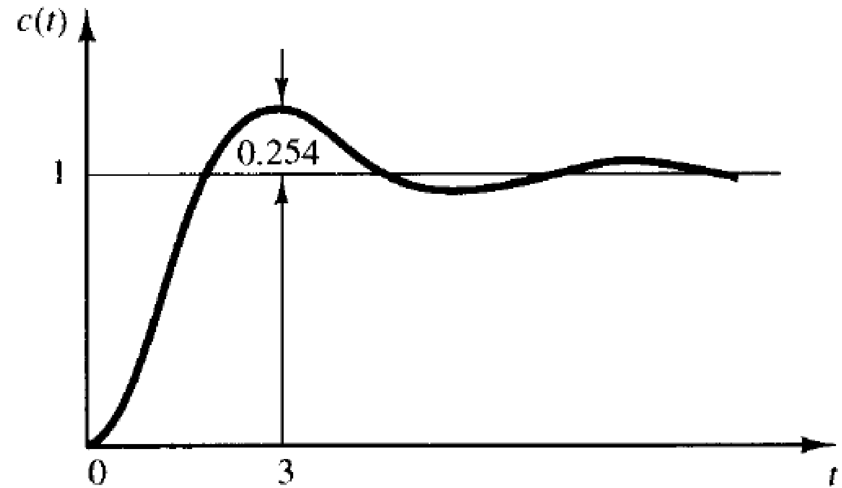
The oscillation is a simple harmonic motion. The amplitude of the oscillation is $1/\sqrt{mk}$.

A-4-6.

When the system shown in Figure 4–41(a) is subjected to a unit-step input, the system output responds as shown in Figure 4–41(b). Determine the values of K and T from the response curve.



(a)



(b)

Figure 4–41

Solution. The maximum overshoot of 25.4% corresponds to $\zeta = 0.4$. From the response curve we have $t_p = 3$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_n\sqrt{1-0.4^2}} = 3 \quad \rightarrow \quad \omega_n = 1.14$$

From the block diagram we have

$$\frac{C(s)}{R(s)} = \frac{K}{Ts^2 + s + K}$$

from which

$$\omega_n = \sqrt{\frac{K}{T}}, \qquad 2\zeta\omega_n = \frac{1}{T}$$

Therefore, the values of T and K are determined as

$$T = \frac{1}{2\zeta\omega_n} = \frac{1}{2 \times 0.4 \times 1.14} = 1.09$$

$$K = \omega_n^2 T = 1.14^2 \times 1.09 = 1.42$$

A-4-7.

Determine the values of K and k of the closed-loop system shown in Figure 4–42 so that the maximum overshoot in unit-step response is 25% and the peak time is 2 sec. Assume that $J = 1$ kg-m².

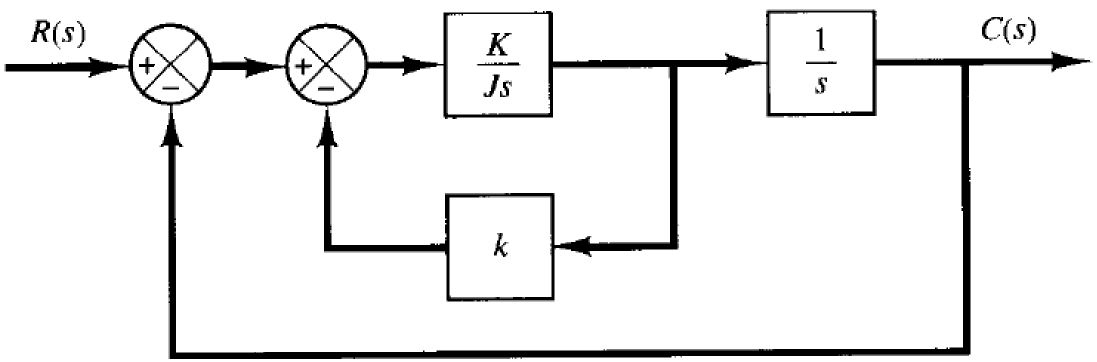


Figure 4–42
Closed-loop system.

Solution.

The closed-loop transfer function is →

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Kks + K}$$

By substituting $J = 1$ kg-m² →

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + Kks + K}$$

$$\omega_n = \sqrt{K}, \qquad 2\zeta\omega_n = Kk$$

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \rightarrow e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.25 \rightarrow \frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.386$$

$$\boxed{\zeta = 0.404}$$

The peak time t_p is specified as 2 sec.

$$t_p = \frac{\pi}{\omega_d} = 2$$

$$\boxed{\omega_d = 1.57}$$

the undamped natural frequency ω_n is

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{1.57}{\sqrt{1-0.404^2}} = 1.72$$

Therefore, we obtain

$$K = \omega_n^2 = 1.72^2 = 2.95 \text{ N-m}$$

$$k = \frac{2\zeta\omega_n}{K} = \frac{2 \times 0.404 \times 1.72}{2.95} = 0.471 \text{ sec}$$

A-4-8. What is the unit-step response of the system shown in Figure 4–43?

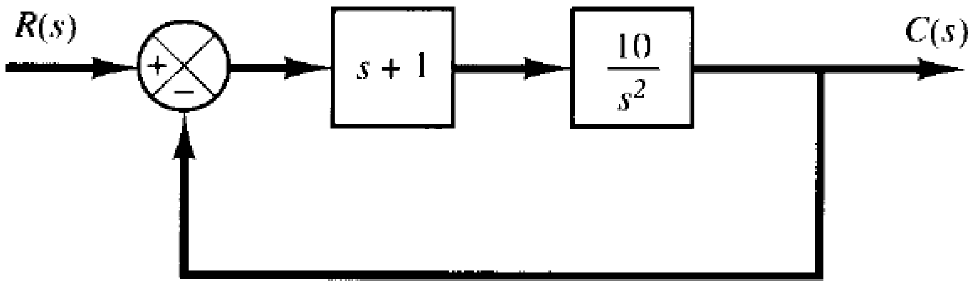


Figure 4–43
Closed-loop system.

Solution. The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{10s + 10}{s^2 + 10s + 10}$$

$$C(s) = \frac{10s + 10}{s^2 + 10s + 10}\frac{1}{s} = \frac{10s + 10}{(s + 5 + \sqrt{15})(s + 5 - \sqrt{15})s}$$

$$= \frac{-4 - \sqrt{15}}{3 + \sqrt{15}}\frac{1}{s + 5 + \sqrt{15}} + \frac{-4 + \sqrt{15}}{3 - \sqrt{15}}\frac{1}{s + 5 - \sqrt{15}} + \frac{1}{s}$$

$$c(t) = -\frac{4 + \sqrt{15}}{3 + \sqrt{15}}e^{-(5+\sqrt{15})t} + \frac{4 - \sqrt{15}}{-3 + \sqrt{15}}e^{-(5-\sqrt{15})t} + 1$$

$$= -1.1455e^{-8.87t} + 0.1455e^{-1.13t} + 1$$

Clearly, the output will not exhibit any oscillation.

Q1: For the system shown in figure (1)

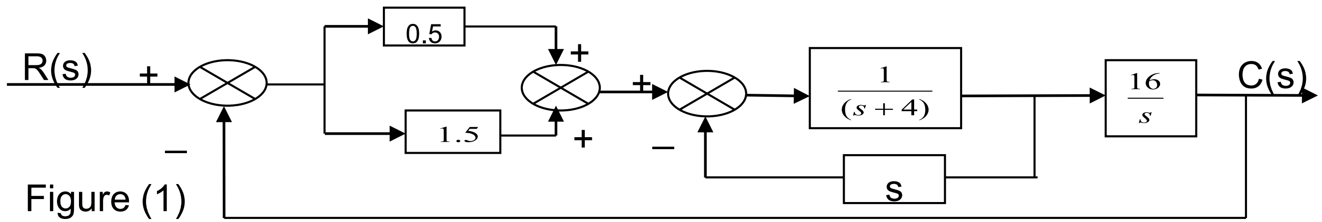
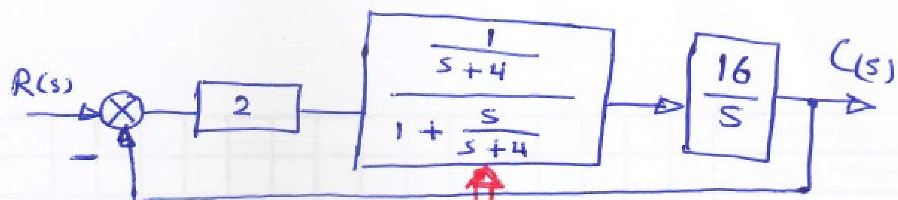


Figure (1)

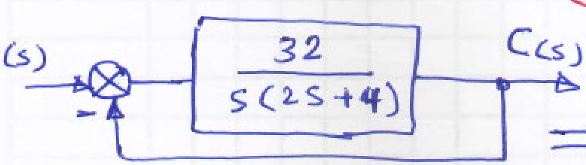
(1) obtain the maximum over shoot, settling time for approximately 5% error, rise time and peak time when the input is a step function

(2) sketch the output response for step input.

Answer



$$\frac{1}{s+4+s} = \frac{1}{2s+4}$$



$$\Rightarrow \frac{C(s)}{R(s)} = \frac{32}{2s^2+4s+32}$$

ـ يجب ان نجعله يساوي واحد لغرض مقارنة المقام بـ $1s^2+2\zeta\omega_n s+\omega_n^2$

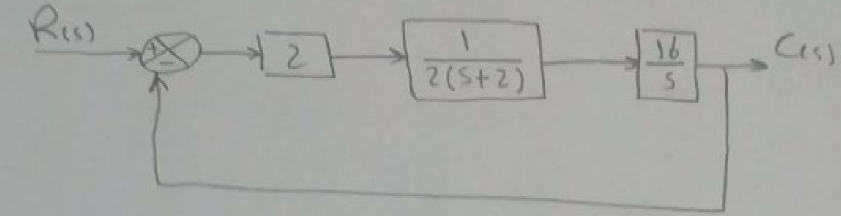
16 ← ليس دائماً يساوي ω_n^2

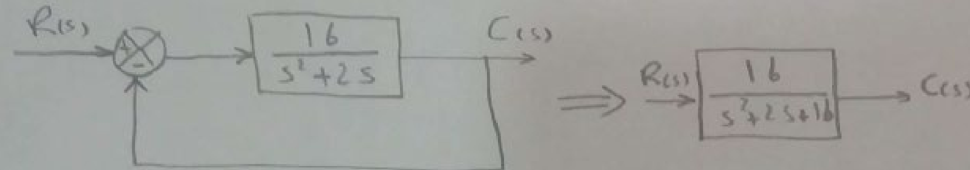
$$\frac{C(s)}{R(s)} = \frac{16}{1s^2+2s+16} = \frac{\omega_n^2}{1s^2+2\zeta\omega_n s+\omega_n^2}$$

$2 \rightarrow 2\zeta\omega_n$

$16 \rightarrow \omega_n^2$

او يمكن ان يكون الحل كالآتي





$$\therefore \frac{C(s)}{R(s)} = \frac{16}{s^2+2s+16} \Rightarrow W_n^2 = 16 \Rightarrow W_n = 4 \text{ rad/s}$$

$$2\zeta W_n = 2 \Rightarrow \zeta = 0.25$$

$$W_d = b = W_n\sqrt{1-\zeta^2} = 4\sqrt{1-0.25^2} = 3.873 \text{ rad/s}$$

$$\text{max. overshoot} \Rightarrow M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{\frac{-0.25\pi}{\sqrt{1-0.25^2}}} = 0.4442 = 44.41\%$$

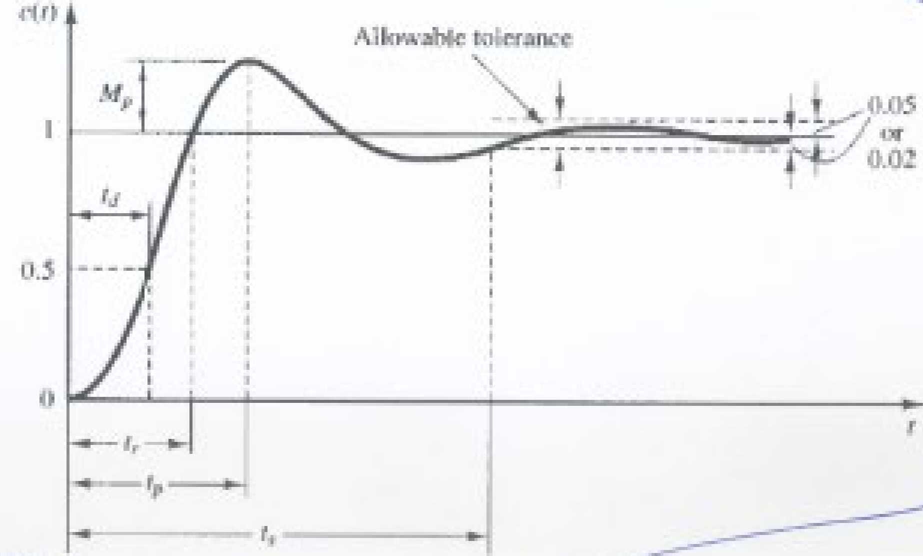
$$\text{settling Time} = t_s \geqslant \frac{3}{\zeta W_n} \Rightarrow t_s \geqslant \frac{3}{0.25\times 4} \Rightarrow t_s \geqslant 3 \text{ sec}$$

$$\text{Rise Time} = t_r = \frac{\pi - \beta}{W_d} = \frac{\pi - \cos^{-1}\zeta}{W_d} = \frac{\pi - 75.52^\circ}{3.873} = \frac{\pi - \frac{\pi}{2.383}}{3.873} = 0.47 \text{ sec}$$

$$\text{Peak Time} = t_p = \frac{\pi}{W_d} = \frac{\pi}{3.873} = 0.811 \text{ sec}$$

Q1

(2)



هذا رسم غير مطلوب رسم هنا للتوضيح فقط

قيم M_p, t_s, t_r, t_p اخذت من الفرع (1) الاول

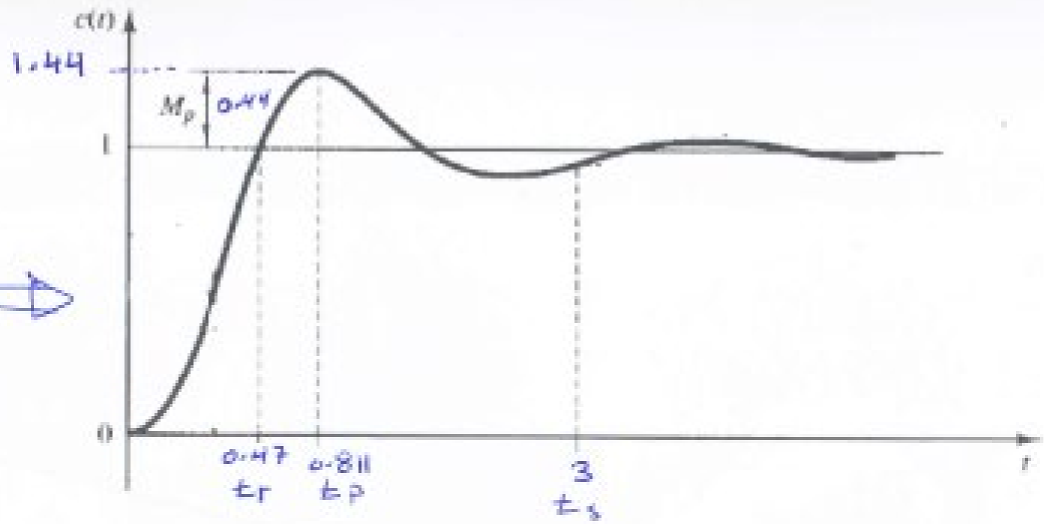
$M_p = 0.44$

$t_s \geq 3$

$t_r = 0.47$

$t_p = 0.811$

هذا الرسم المقرر ⟹



يتضح من ذلك عند ايجاد قيم وصف transient response وهي M_p t_r, t_p, t_s يكون رسم الاداء سهل . ويمكن تصور الاداء من خلال تلك المواصفات

Q2: (A) Define **two** of the following statements, (1) transient response, (2) steady state response, (3) impulse response

(B) For the system shown in figure (2) which employs derivative of feedback in addition to unity feedback

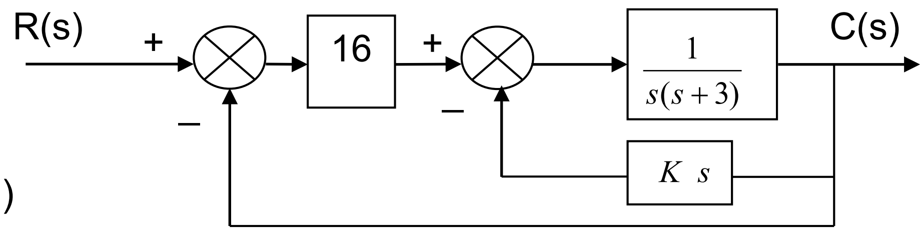


Figure (2)

(1) Determine the value of K so that the damping ratio ($\zeta = 0.5$)

(2) Find the rise time, settling time with approximate 5% error, damped natural frequency, and maximum overshoot. Use the damping ratio ($\zeta = 0.5$)

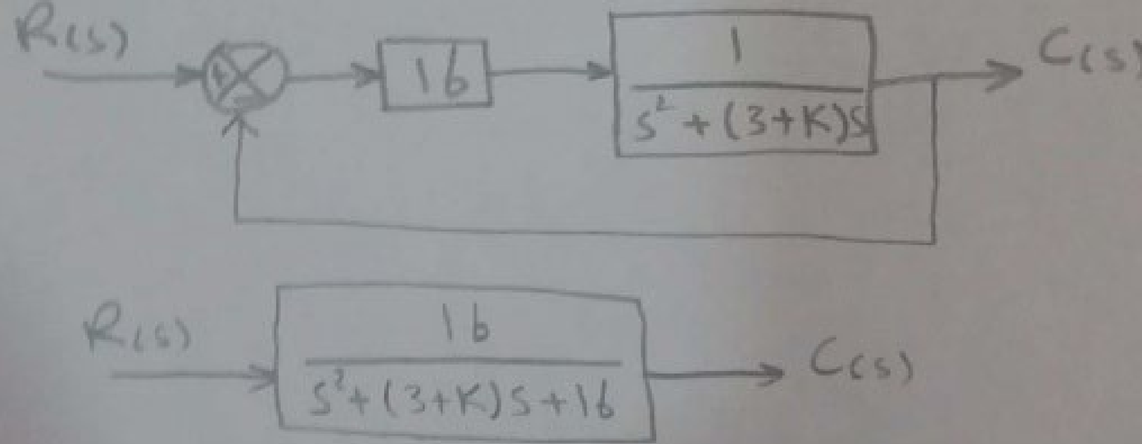
(3) Sketch the output c(t) when the input r(t) is unit step function

A) 1) Transient response:- response goes from initial state to final state.

2) steady-state response:- The manner in which the system output behaves as t approaches infinity.

3) Impulse response:- is the response of system to a single pulse of infinitely small duration and unit energy

B)



1) $w_n^2 = 16 \Rightarrow w_n = 4$ rad/s , $2\xi w_n = 3+K \Rightarrow K = 1$

2) $w_d = w_n\sqrt{1-\xi^2} = 4\sqrt{1-0.5^2} = 3.464$ rad/s = Damped Natural frequency

$$t_r = \frac{\pi - \beta}{w_d} = \frac{\pi - \cos^{-1}\xi}{w_d} = \frac{\pi - 60^\circ}{3.464} = \frac{\pi - \frac{\pi}{3}}{3.464} = 0.604 \text{ sec}$$

$$t_s \geqslant \frac{3}{\xi w_n} \Rightarrow t_s \geqslant \frac{3}{0.5\times 4} \Rightarrow t_s \geqslant 1.5 \text{ sec}$$

$$M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = e^{\frac{-0.5\pi}{\sqrt{1-0.5^2}}} = 0.1629 = 16.29\,\%$$

Q3: For the system shown in figure (3),

(1) Find the value of k_m which gives step response of damping ratio (ζ=0.4)

(2) Obtain maximum over shoot, settling time with approximately 5% error, damped natural frequency, rise time, and peak time for damping ratio (ζ=0.4).

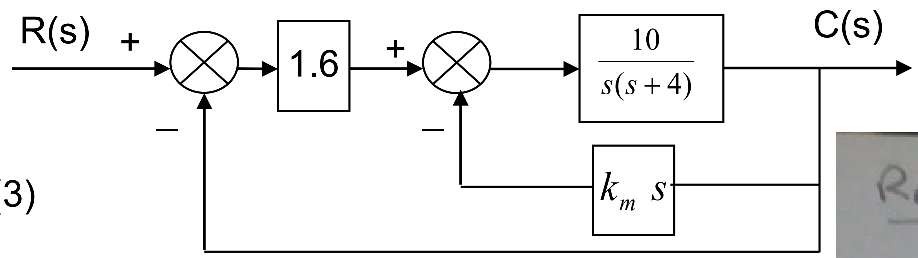
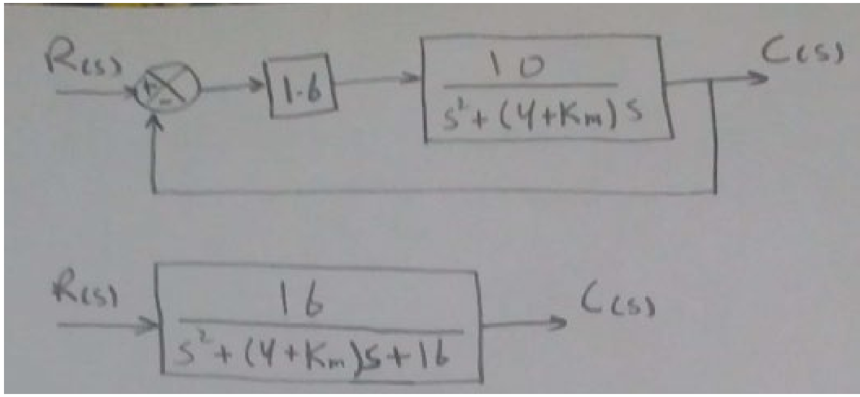


Figure (3)



1) $w_n^2 = 16 \Rightarrow w_n = 4$ rad/s

$2\zeta w_n = 4 + K_m \Rightarrow K_m = -0.8$

2) $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{\frac{-0.4\pi}{\sqrt{1-0.4^2}}} = 0.254 = 25.4\%$

$t_s \geq \frac{3}{\zeta w_n} \Rightarrow t_s \geq \frac{3}{0.4 \times 4} \Rightarrow t_s \geq 1.875$ sec

$w_d = w_n\sqrt{1-\zeta^2} = 4\sqrt{1-0.4^2} = 3.66$ rad/s

$$t_r = \frac{\pi - \beta}{\omega_d} = \frac{\pi - \cos^{-1}\zeta}{\omega_d} = \frac{\pi - 66.92^\circ}{3.66} = \frac{\pi - \frac{\pi}{2.704}}{3.66} = 0.541 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.66} = 0.858 \text{ sec}$$