Al-Mustaqbal University

College of Engineering and Technology

Department of Biomedical Engineering

Stage: Second

Electric circuit II

2023-2024

Lecture (10): MESH ANALYSIS

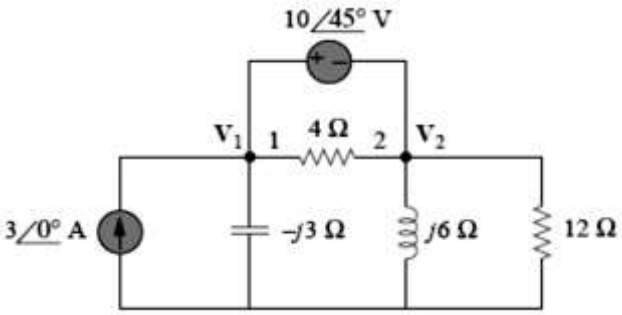

Figure 10.4

Solution: Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{V1}{-j\,3} + \frac{V2}{j6} + \frac{V2}{12}$$

or

36 = j4 V1 + (1 − j2) V2 (10.2.1)

But a voltage source is connected between nodes 1 and 2, so that

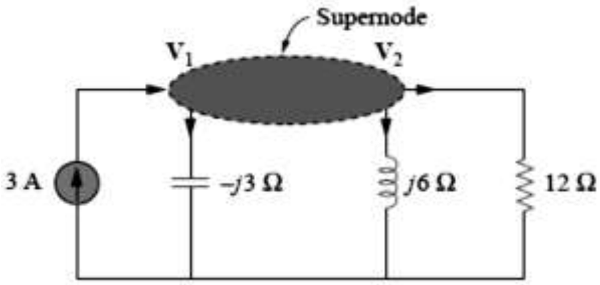

Figure 10.5 A supernode in the circuit of Fig. 10.4.

$$V1 = V2 + 10\ \angle 45^{\circ} \qquad (10.2.2)$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\ \angle 135^{\circ} = (1 + j2)\ V2 \quad \Rightarrow \quad V2 = 31.41\ \angle -87.18^{\circ}\ V$$

From Eq. (10.2.2),

$$V1 = V2 + 10\ \angle 45^{\circ} = 25.78\ \angle -70.48^{\circ}\ V$$

Practice problem 10.2: Calculate V1 and V2 in the circuit shown in Fig. 10.6.

Answer: $V1 = 19.36\ \angle 69.67^{\circ}$ V,

$V2 = 3.376\ \angle 165.7^{\circ}$ V.

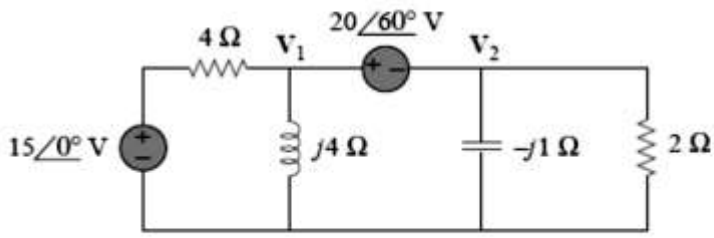

Fig. 10.6

10.3 MESH ANALYSIS

Kirchhoff's voltage law (KVL) forms the basis of mesh analysis. The validity of KVL for ac circuits was shown in Section 9.5 and is illustrated in the following examples.

Example 10.3: Determine current Io in the circuit of Fig. 10.7 using mesh analysis.

Solution:

Applying KVL to mesh 1, we obtain

$$(8 + j\,10 - j2)\,I1 - (-\,j2)\,I2 - j10\,I3 = 0 \qquad (10.3.1)$$

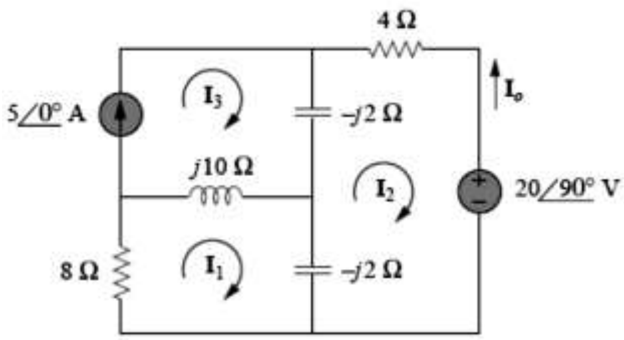

Fig. 10.7

For mesh 2,

$$(4 - j2 - j2)\,I2 - (-\,j2)\,I1 - (-\,j\,2)\,I3 + 20\,\angle 90^{\circ} = 0 \qquad (10.3.2)$$

For mesh 3, I3 = 5. Substituting this in Eqs. (10.3.1) and (10.3.2), we get

$$(8 + j\,8)\,I1 + j2\,I2 = j50 \qquad (10.3.3)$$

$$j2\,I1 + (4 - j4)\,I2 = -\,j20 - j10 \qquad (10.3.4)$$

Equations (10.3.3) and (10.3.4) can be put in matrix form as

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = D = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta 2 = D2 = \begin{vmatrix} 8+j8 & j5 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\angle -35.22°,$$

$$I2 = \frac{D2}{D} = \frac{416.17\angle -35.22°}{68} = 6.12\angle -35.22° A$$

The desired current is

Io = − I2 = 6.12 ∠144.78° A

Practice problem 10.3: Find Io in Fig. 10.8 using mesh analysis.

Answer: 1.194 ∠65.45° A.

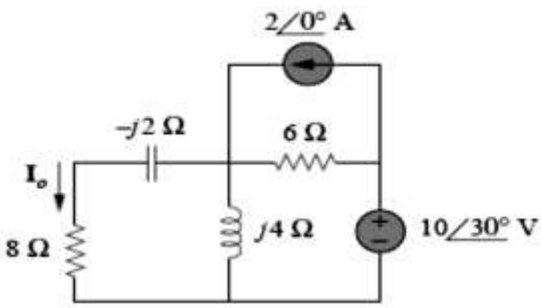

Fig. 10.7

Example 10.4: Solve for Vo in the circuit in Fig. 10.9 using mesh analysis.

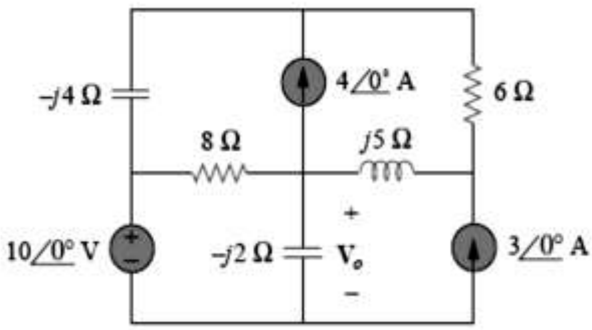

Fig. 10.9

Solution:

As shown in Fig. 10.10, meshes 3 and 4 form a supermesh due to the current source between the meshes. For mesh 1, KVL gives

$$-10 + (8 - j2) I1 - (-j2) I2 - 8 I3 = 0$$

or

$$(8 - j2) I1 + j2 I2 - 8 I3 = 10 \qquad (10.4.1)$$

For mesh 2,

$$I2 = -3 \qquad (10.4.2)$$

For the supermesh,

$$(8 - j\,4)\, I3 - 8\, I1 + (6 + j\,5)\, I4 - j5\, I2 = 0 \qquad (10.4.3)$$

Due to the current source between meshes 3 and 4, at node A,

$$I4 = I3 + 4 \qquad (10.4.4)$$

Combining Eqs. (10.4.1) and (10.4.2),

$$(8 - j\,2)\, I1 - 8\, I3 = 10 + j6 \qquad (10.4.5)$$

Combining Eqs. (10.4.2) to (10.4.4),

$$-8\, I1 + (14 + j)\, I3 = -24 - j35 \qquad (10.4.6)$$

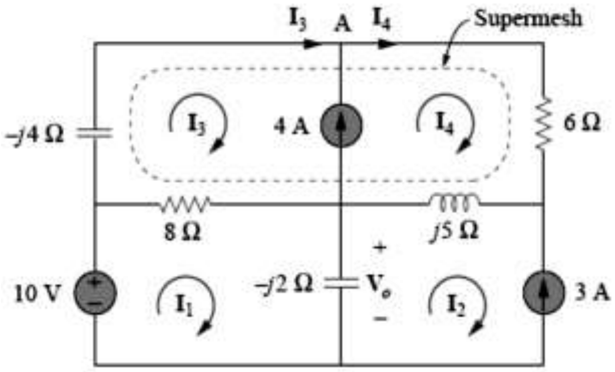

Figure 10.10 Analysis of the circuit in Fig. 10.9.

From Eqs. (10.4.5) and (10.4.6), we obtain the matrix equation

$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = D = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{vmatrix} = 112 + j\,8\ - j28\ + 2\ - 64 = 50 - j20$$

$$\Delta 1 = D1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{vmatrix} = 140 + j10 + j\,84 - 6 - 192 - j\,280$$

$$= -58 - j186$$

Current I1 is obtained as

$$I1\ = \frac{D1}{D} = \frac{-58 - j186}{50 - j20}\ = 3.618\angle 274.5°\,A$$

We obtain the following determinants

The required voltage Vo is

$$\mathbf{Vo = -\,j2\,(I1 - I2) = -\,j2\,(3.618\,\angle 274.5^\circ + 3)}$$

$$\mathbf{= -\,7.2134 - j6.568 = 9.756\,\angle 222.32^\circ\,V}$$

Practice problem 10.4: Calculate current Io in the circuit of Fig. 10.11.

Answer: **5.075∠ 5.943**° A.

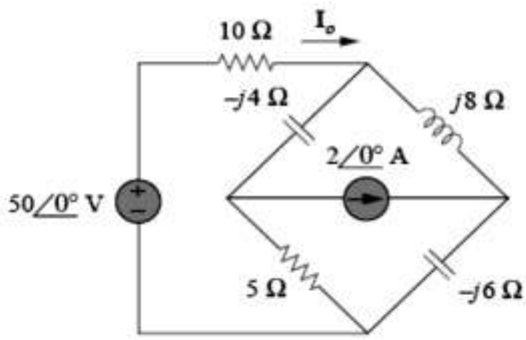

Fig. 10.11

10.4 SUPERPOSITION THEOREM

Since ac circuits are linear, the superposition theorem applies to ac circuits the same way it applies to dc circuits. The theorem becomes important if the circuit has sources operating at different frequencies. In this case, since the impedances depend on frequency, we must have a different frequency-domain circuit for each frequency. The total response must be obtained by adding the individual responses in the time domain. It is incorrect to try to add the responses in the phasor or frequency domain. Why? Because the exponential factor e jωt is implicit in sinusoidal analysis, and that factor would change for every angular frequency ω. It would therefore not make sense to add responses at different frequencies in the phasor domain. Thus, when a circuit has sources operating at different frequencies, one must add the responses due to the individual frequencies in the time domain.

Example 10.5: Use the superposition theorem to find Io in the circuit in Fig. 10.7.

Solution:

Let

$$\mathbf{Io = I'o + I''o} \qquad (10.5.1)$$

where I'o and I"o are due to the voltage and current sources, respectively. To find I'o , consider the circuit in Fig. 10.12(a). If we let Z be the parallel combination of − j2 and 8 + j 10, then

$$\boldsymbol{Z = \frac{-j2(8+j10)}{-2j+8+j10} = 0.25 - j2.25}$$