



Al-Mustaqbal University
**College of Engineering and
Technology**
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Stage: three

Signal Processing

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**Lecture (10): Non-recursive
digital filter (FIR)**

Transfer Function of Finite Impulse Response(FIR)

An Finite Impulse Response (FIR) filter is completely specified by the following input-output relationship

$$y(n) = \sum_{k=0}^N b(k)x(n-k) \quad (1) \text{ (Difference Equation)}$$

$$y(n)=b_0x(n) + b_1x(n-1) + b_2x(n-2) + \dots + b_kx(n-k)$$

OR

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^N b(k)z^{-k} \quad (2) \text{ (Transfer Function)}$$

$$H(z)=b_0 + b_1z^{-1} + b_2z^{-2} + \dots + b_kz^{-k}$$

The length of filter =k+1

The filter coefficients b_0, b_1, b_2, b_K are the impulse response $h(n)$ of the filter .

Properties of FIR filter

- FIR filters have a finite-duration impulse response.
- FIR filters are non-recursive filters.
- No feedback
- Always stable
- Simple to implement

Example\\Given the following FIR filter

$$y(n)=0.1x(n) + 0.25x(n-1) + 0.2x(n-2)$$

Determine the **transfer function**, **filter length**, **nonzero coefficients**, and **impulse response**.

Sol\\ Applying z-transform on both sides of the difference equation yields

$$Y(z)=0.1X(z)+0.25z^{-1}X(z) +0.2z^{-2}X(z)$$

1)Then the transfer function is found to be

$$H(z) = \frac{Y(z)}{X(z)} = 0.1 + 0.25z^{-1} + 0.2z^{-2}$$

2) filter length is $k+1=3$

3) nonzero coefficients $b_0=0.1$ $b_1=0.25$ $b_2=0.2$

4) impulse response $h(n)=0.1 \delta[n]+0.25 \delta[n-1]+0.2 \delta[n-2]$

Example\\ A FIR filter has a set of filter coefficients $\{b_k\} = \{3, -1, 2, 1\}$. Determine the **difference equation** for the filter.

Sol// The length of the filter is 4.

$$y[n] = 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

Example\\ Determine the first four samples in the impulse response for the FIR filter.

$$y[n] = 0.5(x[n] + x[n-1] + x[n-2])$$

Sol\\ Substituting $\delta[n]$ for $x[n]$ and $h[n]$ for $y[n]$.

$$h[n] = 0.5(\delta[n] + \delta[n-1] + \delta[n-2])$$

$$h[0] = 0.5(\delta[0] + \delta[-1] + \delta[-2]) = 0.5(1.0 + 0.0 + 0.0) = 0.5$$

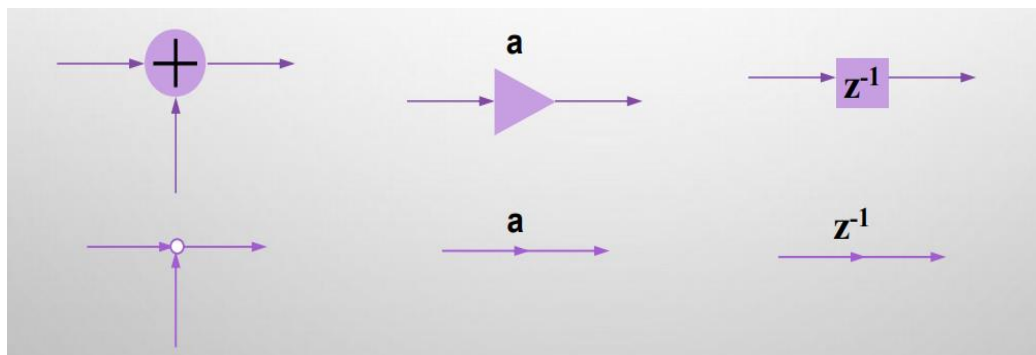
$$h[1] = 0.5(\delta[1] + \delta[0] + \delta[-1]) = 0.5(0.0 + 1.0 + 0.0) = 0.5$$

$$h[2] = 0.5(\delta[2] + \delta[1] + \delta[0]) = 0.5(0.0 + 0.0 + 1.0) = 0.5$$

$$h[3] = 0.5(\delta[3] + \delta[2] + \delta[1]) = 0.5(0.0 + 0.0 + 0.0) = 0$$

Basic elements of digital filter structures

- Adder has two inputs and one output.
- Multiplier (gain) has single-input, single-output.
- Delay element delays the signal passing through it by one sample. It is implemented by using a shift register.

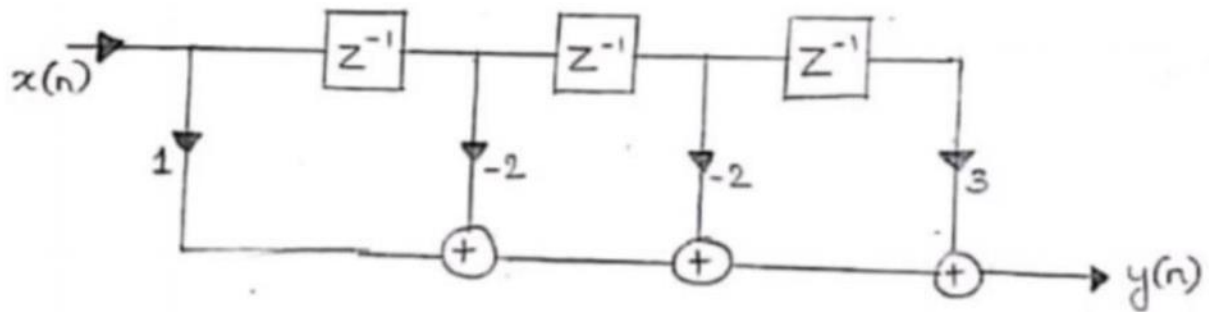


FIR Filter structures

Direct form: a direct implementation of the convolution operation . the number of delay equal to the order of the filter.

Example\\Draw the direct form structure for the FIR filter represented by the following difference equation

$$y[n] = x[n] - 2x[n-1] - 2x[n-2] + 3x[n-3]$$



Sol:

Example//Based on the transfer function, realize the digital filter using the direct form.

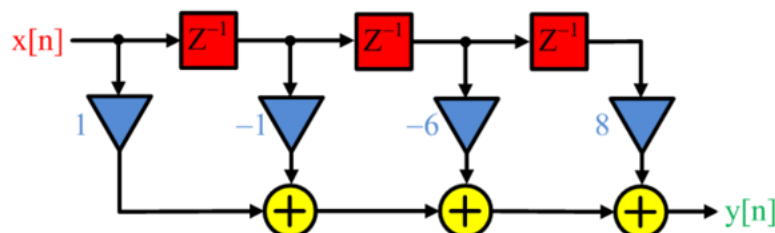
$$H(z) = (1 - 2z^{-1})(1 + z^{-1} - 4z^{-2})$$

Sol// Since the transfer function has only the numerator part or zeroes, therefore this is an FIR filter.

$$H(z) = \frac{y(z)}{x(z)} = (1 - 2z^{-1})(1 + z^{-1} - 4z^{-2})$$

$$y(z) = x(z) - z^{-1}x(z) - 6z^{-2}x(z) + 8z^{-3}x(z)$$

$$y(n) = x(n) - x(n-1) - 6x(n-2) + 8x(n-3)$$



Example// for this difference equation find the direct-form

$$y(n] = 3x[n] + 3x[n - 1] + 2x[n - 2] - 2x[n - 3]$$

