1.1. Convolution

a mathematical operation used to combine two signals (or functions) to produce a third signal that represents how one signal modifies or "filters" the other. In signal processing and systems theory, convolution is essential for analyzing linear time-invariant (LTI) systems, which are systems that respond to an input signal in a way that doesn't change over time.

$$y(n)=x(n)*h(n)= \sum_{k=-\infty}^{\infty} x(k)\cdot h(n-k)$$

This equation can be understood as follows:

- x(k): Represents the values of the input signal.
- h(n−k): Represents the impulse response shifted by k.
- The above equation is the linear convolution of x(n) and h(n), this linear convolution gives the total response y(n).
- This equation provides the response of a Linear Shift (or Time) Invariant System (LSI or LTI) to an input x(n) using the system's impulse response h(n).

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Example1: Convolve the following two sequences x(n)&h(n) to get y(n)

$x(n)=\begin{cases}1 & for\ \ 3 \geq n \geq 0 \\ 0 & elsewhere\end{cases}$ $h(n)=\begin{cases}2 & for\ 1 \geq n \geq 0 \\ 0 & elsewhere\end{cases}$

Sol,

x(n)={1,1,1,1} h(n)={2,2}

X(0)=1 h(0)=2

X(1)=1 h(1)=2

X(2)=1

X(3)=1

- Lowest index of x(n) is $n_{xL}=0$
- Highest index of x(n) is $n_{xH}=3$
- Lowest index of h(n) is $n_{hL}=0$
- Highest index of h(n) is $n_{hH}=1$

$$y(n)=\sum_{k=-\infty}^{\infty} X(k)h(n-k) = \sum_{k=nxL=0}^{nxH=3} X(k)h(n-k)$$

y(n)=x(0)h(n)+x(1)h(n-1)+x(2)h(n-2)+x(3)h(n-3)

Range of "n"

$$(n_{xH}+n_{hH}) \geq n \geq (n_{xL}+n_{hL})$$

$$(3+1) \geq n \geq (0+0)$$

$$4 \geq n \geq 0$$

n=0

y(0)=x(0)h(0)+x(1)h(-1)+x(2)h(-2)+x(3)h(-3)=2

n=1

y(1)=x(0)h(1)+x(1)h(0)+x(2)h(-1)+x(3)h(-2)=4

n=2

y(2)=x(0)h(2)+x(1)h(1)+x(2)h(0)+x(3)h(1)=4

n=3

y(3)=x(0)h(3)+x(1)h(2)+x(2)h(1)+x(3)h(0)=4

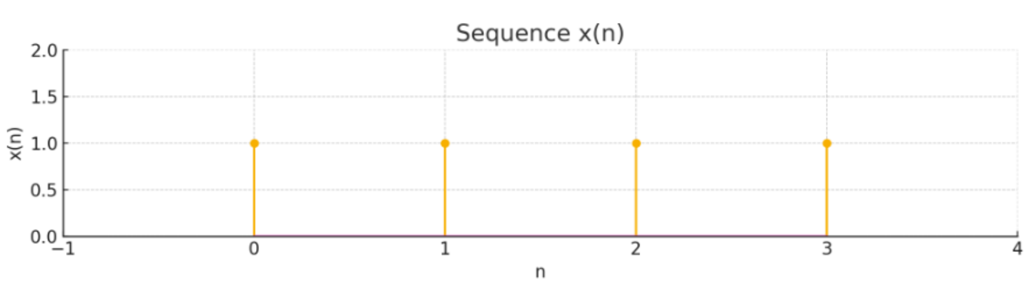
n=4

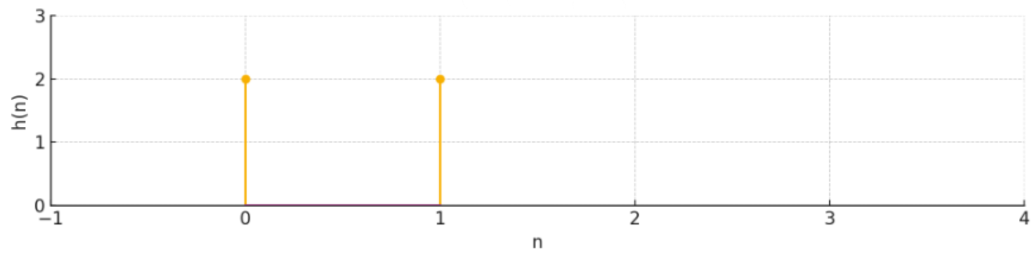
y(4)=x(0)h(4)+x(1)h(3)+x(2)h(2)+x(3)h(1)=2

❖ y(n)={2,4,4,4,2}

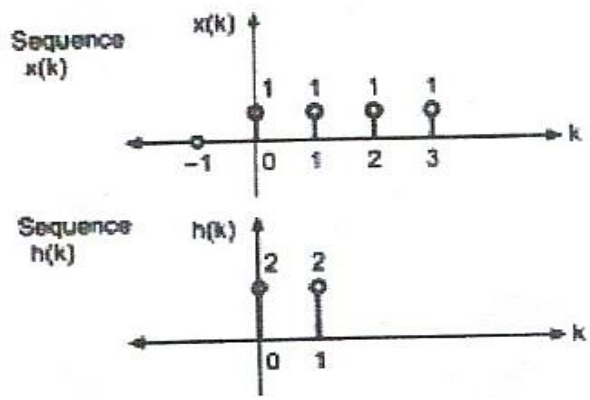
Example2: Solve the previous example by using Graphical method?

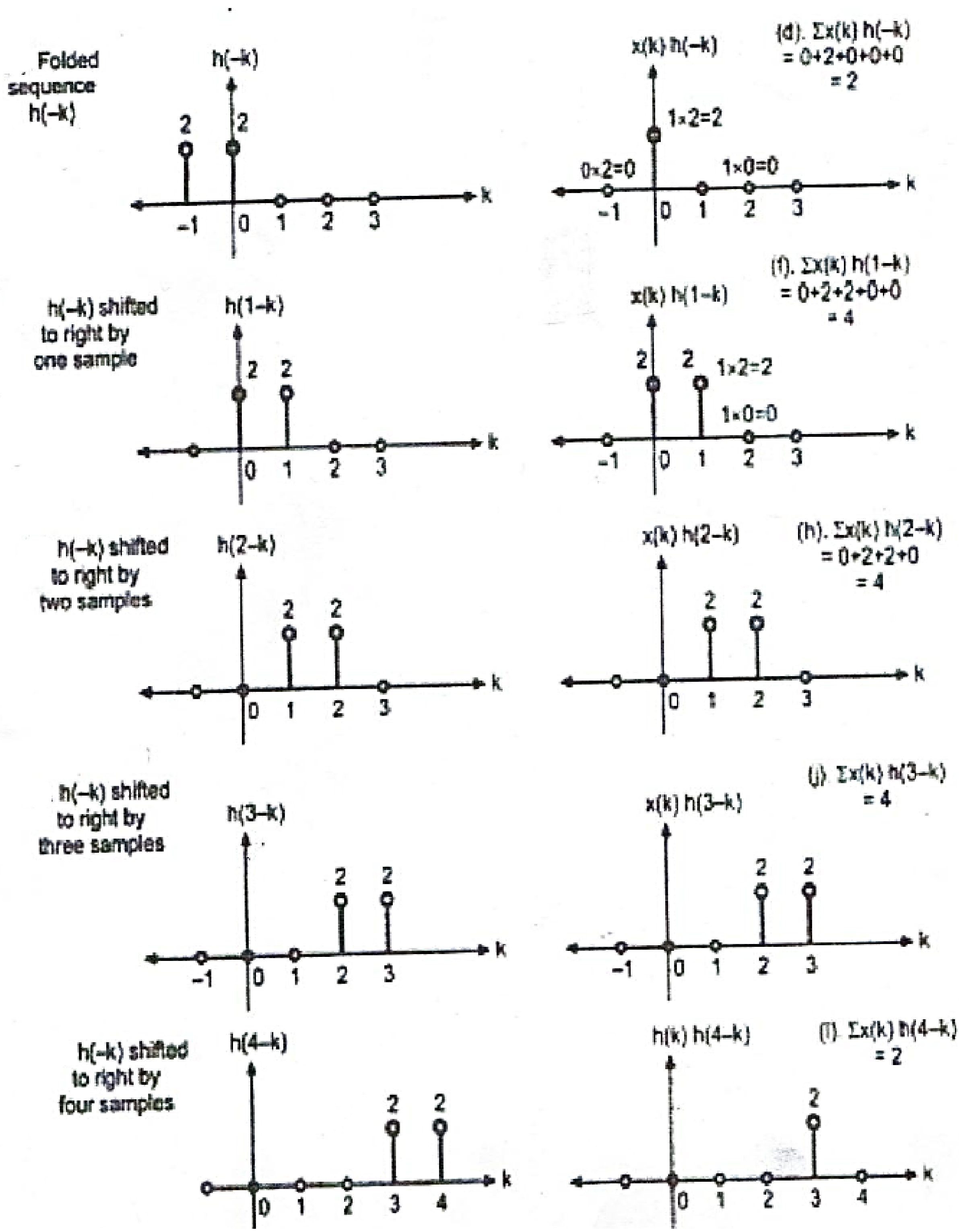
x(n)={1,1,1,1} ,   h(n)={2,2}



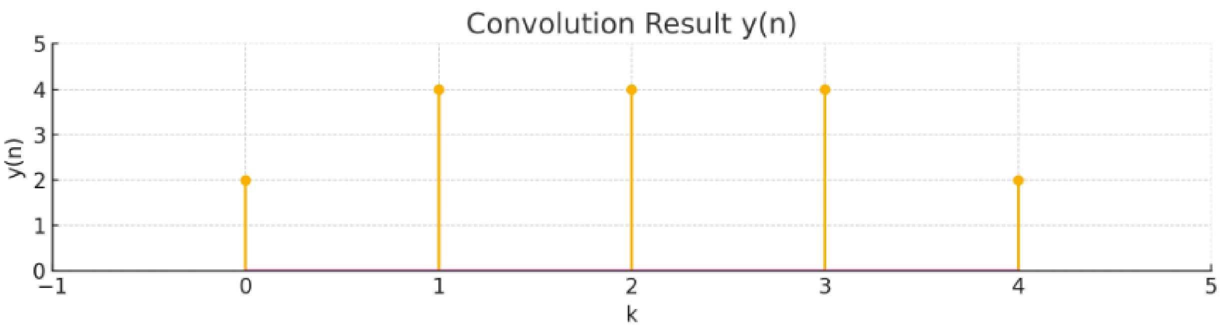


Solve:




Folded sequence h(−k)
h(−k)
x(k) h(−k)
(d). Σx(k) h(−k)
= 0+2+0+0+0
= 2
0×2=0
1×2=2
1×0=0
h(−k) shifted to right by one sample
h(1−k)
x(k) h(1−k)
(f). Σx(k) h(1−k)
= 0+2+2+0+0
= 4
1×2=2
1×0=0
h(−k) shifted to right by two samples
h(2−k)
x(k) h(2−k)
(h). Σx(k) h(2−k)
= 0+2+2+0
= 4
h(−k) shifted to right by three samples
h(3−k)
x(k) h(3−k)
(j). Σx(k) h(3−k)
= 4
h(−k) shifted to right by four samples
h(4−k)
h(k) h(4−k)
(l). Σx(k) h(4−k)
= 2

- y(n)={2,4,4,4,2}



Example3: Convolve the following two sequences to get y(n):

$$x(n)=\{1,1,0,1,1\},\ h(n)=\{1,-2,-3,4\}$$

These two sequences can also be written as:

x(−2)=1	h(-3)=1
x(−1)=1	h(-2)=-2
x(0)=0	h(-1)=-3
x(1)=1	h(0)=4
x(2)=1	

- Lowest index of x(n) is $n_{xL}=-2$
- Highest index of x(n) is $n_{xH}=2$
- Lowest index of h(n) is $n_{hL}=-3$
- Highest index of h(n) is $n_{hH}=0$

y(n)=$\sum_{k=-\infty}^{\infty} x(k)h(n-k)$=$\sum_{nxL=k=-2}^{nxH=2} x(k)h(n-k)$

y(n)=x(-2) h(n+2)+x(-1)h(n+1)+x(0)h(n)+x(1)h(n-1)+x(2)h(n-2)

Range of "n"

$(n_{xL}+n_{hL}) \leq n \leq (n_{xH}+n_{xH})$

$(-2-3) \leq n \leq (2+0)$

$-5 \leq n \leq 2$

n=-5

y(-5)=x(-2)h(-5+2)+x(-1)h(-5+1)+x(0)h(-5)+x(1)h(-5-1)+x(2)h(-5-2)=1

n=-4

y(-4)=x(-2)h(-2)+x(-1)h(-3)+x(0)h(-4)+x(1)h(-5)+x(2)h(-6)=-2+1=-1

n=-3

y(-3)=x(-2)h(-3+2)+x(-1)h(-3+1)+x(0)h(-3)+x(1)h(-3-1)+x(2)h(-3-2)=-5

n=-2

y(-2)=x(-2)h(-2+2)+x(-1)h(-2+1)+x(0)h(-2)+x(1)h(-2-1)+x(2)h(-2-2)=2

n=-1

y(-1)=x(-2)h(-1+2)+x(-1)h(-1+1)+x(0)h(-1)+x(1)h(-1-1)+x(2)h(-1-2)=3

n=0

y(0)=x(-2)h(0+2)+x(-1)h(0+1)+x(0)h(0)+x(1)h(0-1)+x(2)h(0-2)= -5

n=1

y(1)=x(-2)h(1+2)+x(-1)h(1+1)+x(0)h(1)+x(1)h(1-1)+x(2)h(1-2)= 1

n=2

y(2)=x(-2)h(2+2)+x(-1)h(2+1)+x(0)h(2)+x(1)h(2-1)+x(2)h(2-2)= 4

- y(n)={−5,2,3,−5,1,4}

Example4: Determine the convolution of the following two sequences by using direct method?

x(n)={1,2,3,1}, h(n)={2,0,2}

y(n)=$\sum_{k=-\infty}^{\infty} X(k)h(n-k) = \sum_{k=0}^{5} X(k)h(n-k)$

y(n)=x(0)h(n)+x(1)h(n-1)+x(2)h(n-2)+x(3)h(n-3)+x(4)h(n-4)+x(5)h(n-5)

n=0

y(0)=x(0)h(0)+x(1)h(-1)+x(2)h(-2)+x(3)h(-3)+x(4)h(-4)+x(5)h(-5)=2

n=1

y(1)=x(0)h(1-0)+x(1)h(1-1)+x(2)h(1-2)+x(3)h(1-3)+x(4)h(1-4)+x(5)h(1-5)=4

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n=2

y(2)=x(0)h(2-0)+x(1)h(2-1)+x(2)h(2-2)+x(3)h(2-3)+x(4)h(2-4)+x(5)h(2-5)=8

n=3

y(3)=x(0)h(3-0)+x(1)h(3-1)+x(2)h(3-2)+x(3)h(3-3)+x(4)h(3-4)+x(5)h(3-5)=6

n=4

y(4)=x(0)h(4-0)+x(1)h(4-1)+x(2)h(4-2)+x(3)h(4-3)+x(4)h(4-4)+x(5)h(4-5)=6

n=5

y(5)=x(0)h(5-0)+x(1)h(5-1)+x(2)h(5-2)+x(3)h(5-3)+x(4)h(5-4)+x(5)h(5-5)=2

- y(n)={2,4,8,6,6,2}

H.W/ Determine the convolution between x(n)&h(n) as given below:

h(n)=u(n)

x(n)={1,1,2} using graphical method?

H.W/ Determine the response the system whose input x(n)&h(n) is given as following:

$$x(n)=\begin{cases}\frac{1}{3}n & for\ 0\le n\le 6\\ 0 & elsewhere\end{cases}\qquad h(n)=\begin{cases}1 & for\ -2\le n\le 2\\ 0 & elsewhere\end{cases}$$

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